Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/121-4.5.2.1-a+b-sec-^m-c+d-sec-^n

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 6:41pm

Contents

1 Introduction **3**

2 detailed summary tables of results **19**

3 Listing of integrals **83**

4 Appendix **1289**

Chapter 1

Introduction

Local contents

1.1 Listing of CAS systems tested 4
1.2 Results 5
1.3 Time and leaf size Performance 9
1.4 list of integrals that has no closed form antiderivative 11
1.5 List of integrals solved by CAS but has no known antiderivative 12
1.6 list of integrals solved by CAS but failed verification 13
1.7 Timing 13
1.8 Verification 14
1.9 Important notes about some of the results 14
1.10 Design of the test system 17

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [241]. This is test number [121].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.59 (240)	0.41 (1)
Mathematica	94.19 (227)	5.81 (14)
Maple	90.46 (218)	9.54 (23)
Fricas	60.17 (145)	39.83 (96)
Giac	53.94 (130)	46.06 (111)
Maxima	39.83 (96)	60.17 (145)
Mupad	23.24 (56)	76.76 (185)
Sympy	2.49 (6)	97.51 (235)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

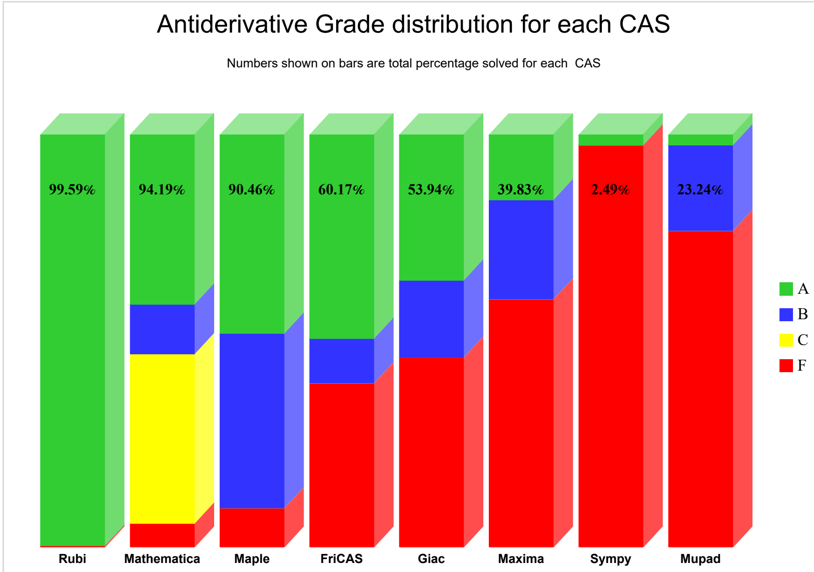
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

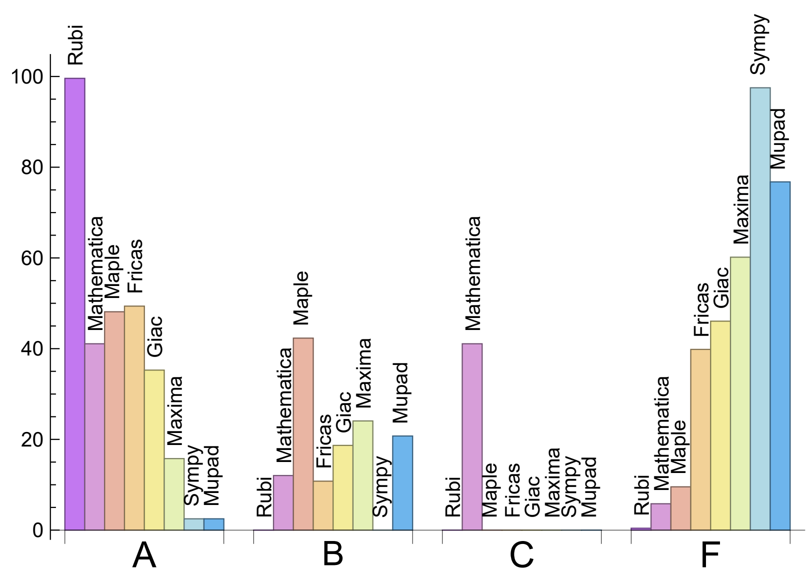
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.59	0.00	0.00	0.41
Fricas	49.38	10.79	0.00	39.83
Maple	48.13	42.32	0.00	9.54
Mathematica	41.08	12.03	41.08	5.81
Giac	35.27	18.67	0.00	46.06
Maxima	15.77	24.07	0.00	60.17
Mupad	N/A	20.75	0.00	76.76
Sympy	2.49	0.00	0.00	97.51

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	14	100.00 %	0.00 %	0.00 %
Maple	23	100.00 %	0.00 %	0.00 %
Fricas	96	68.75 %	31.25 %	0.00 %
Giac	111	48.65 %	0.00 %	51.35 %
Maxima	145	68.28 %	14.48 %	17.24 %
Sympy	235	84.68 %	5.53 %	9.79 %
Mupad	185	92.97 %	7.03 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

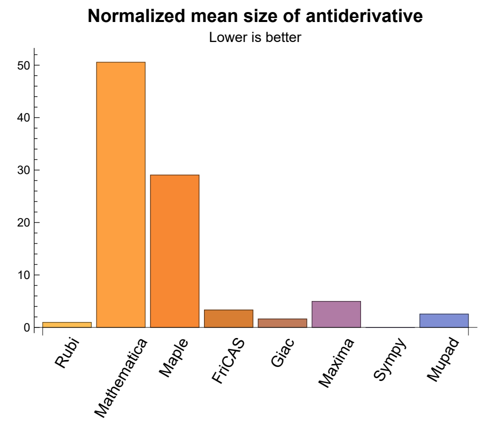
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

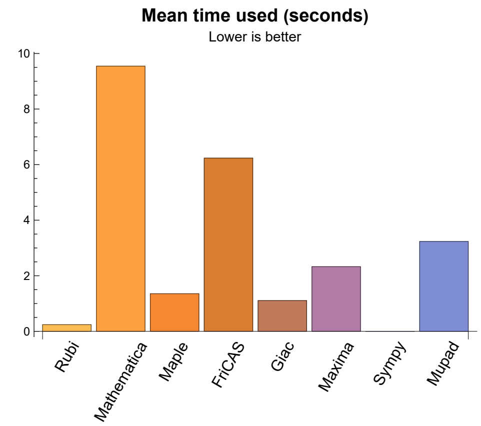
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	211.50	0.96	153.00	1.00
Mathematica	9.54	25083.28	50.57	171.00	1.23
Maple	1.35	14584.05	29.05	282.50	1.76
Maxima	2.33	833.58	4.96	264.00	2.53
Fricas	6.23	576.68	3.32	437.00	3.08
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	1.11	279.01	1.61	171.00	1.39
Mupad	3.23	459.00	2.54	122.00	1.08

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
10
20
30
40
50
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{221, 222, 223, 224, 225, 226, 227, 228, 229, 236\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {216}

Mathematica {62, 63, 64, 70, 71, 77, 78, 85, 144, 147, 148, 149, 151, 152, 153, 158, 159, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 200, 202, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 230, 231, 240, 241}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

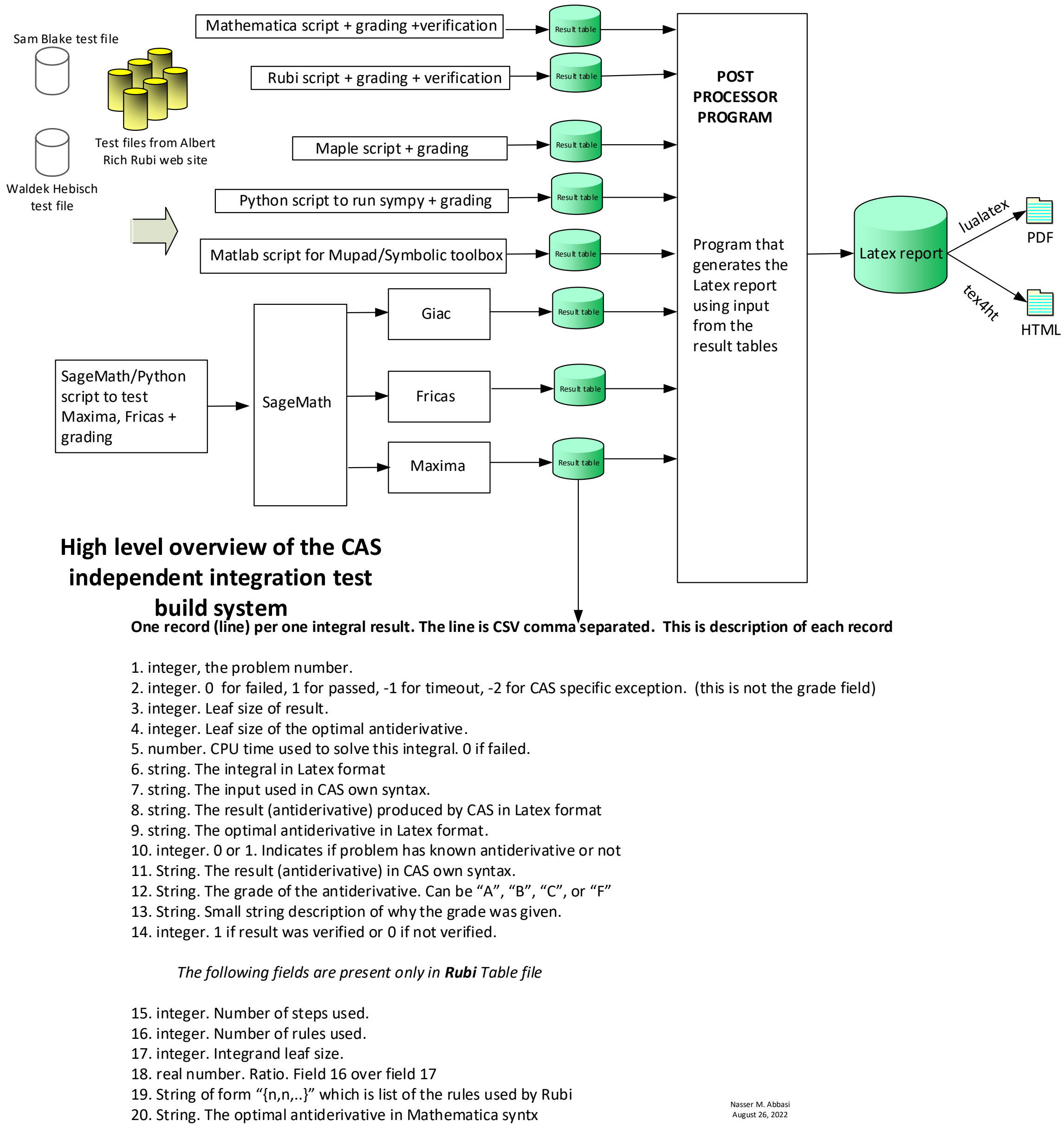
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS 20
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 74

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi . . . 21
2.1.2 Mathematica . . . 21
2.1.3 Maple . . . 21
2.1.4 Maxima . . . 22
2.1.5 FriCAS . . . 22
2.1.6 Sympy . . . 23
2.1.7 Giac . . . 23
2.1.8 Mupad . . . 24

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241 }

B grade: { }

C grade: { }

F grade: { 217 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 24, 25, 26, 29, 30, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 145, 146, 150, 154, 155, 156, 157, 160, 161, 162, 168, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 199, 201, 203, 204, 208, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 233, 236, 237, 238, 239 }

B grade: { 6, 16, 17, 21, 22, 23, 28, 31, 37, 39, 40, 144, 193, 195, 197, 200, 202, 209, 210, 211, 212, 213, 214, 215, 216, 220, 230, 240, 241 }

C grade: { 7, 18, 27, 38, 47, 48, 49, 55, 56, 62, 63, 64, 70, 71, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 147, 148, 149, 151, 152, 153, 158, 159, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 198, 205, 206, 207, 217, 219, 231, 232 }

F grade: { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 234, 235 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 51, 60, 68, 69, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 144, 145, 146, 149, 189, 190, 191, 192, 193, 194, 195, 196, 201, 203, 204, 207, 208, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 12, 42, 43, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 105, 106, 107, 114, 118, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220 }

C grade: { }

F grade: { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 13, 15, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 89, 90, 97, 98, 106, 112, 113, 114, 119, 125, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 3, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 45, 52, 59, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 109, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 150, 156, 162 }

C grade: { }

F grade: { 42, 43, 44, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 104, 105, 110, 111, 118, 124, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 94, 95, 96, 101, 102, 103, 130, 143, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 178, 179, 184, 185, 187, 188, 189, 236 }

B grade: { 5, 22, 23, 61, 62, 63, 74, 75, 83, 105, 114, 118, 122, 153, 159, 174, 180, 186, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 90, 91, 92, 93, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 170, 171, 175, 176, 177, 181, 182, 183, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.6 Sympy

A grade: { 221, 222, 224, 225, 227, 236 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 50, 57, 58, 59, 90, 91, 92, 93, 98, 99, 100, 106, 107, 108, 109, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 148, 160, 161, 190, 192, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 5, 27, 38, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 143, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 165, 189, 191, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 101, 102, 103, 104, 105, 110, 111, 112, 117, 118, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.8 Mupad

A grade: { 221, 222, 224, 225, 227, 236 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	165	268	361	191	0	191	228
N.S.	1	1.00	0.84	1.37	1.84	0.97	0.00	0.97	1.16
time (sec)	N/A	0.224	2.101	0.177	0.318	3.412	0.000	0.567	2.534

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	146	206	259	174	0	172	195
N.S.	1	1.00	1.04	1.47	1.85	1.24	0.00	1.23	1.39
time (sec)	N/A	0.151	1.187	0.100	0.298	2.654	0.000	0.519	2.300

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	122	171	219	157	0	153	163
N.S.	1	1.00	1.26	1.76	2.26	1.62	0.00	1.58	1.68
time (sec)	N/A	0.083	0.788	0.083	0.285	2.384	0.000	0.525	2.189

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	58	61	69	0	48	84
N.S.	1	1.00	0.96	1.23	1.30	1.47	0.00	1.02	1.79
time (sec)	N/A	0.052	0.036	0.052	0.297	2.504	0.000	0.479	3.709

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	72	84	103	111	0	103	91
N.S.	1	1.00	1.31	1.53	1.87	2.02	0.00	1.87	1.65
time (sec)	N/A	0.046	0.321	0.045	0.279	2.733	0.000	0.482	1.513

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	169	64	165	94	0	77	46
N.S.	1	1.00	3.02	1.14	2.95	1.68	0.00	1.38	0.82
time (sec)	N/A	0.122	0.312	0.123	0.511	3.148	0.000	0.503	1.484

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	47	188	94	0	57	40
N.S.	1	1.00	0.75	0.66	2.65	1.32	0.00	0.80	0.56
time (sec)	N/A	0.187	0.065	0.128	0.521	1.816	0.000	0.483	1.389

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	171	63	233	137	0	72	96
N.S.	1	1.00	1.68	0.62	2.28	1.34	0.00	0.71	0.94
time (sec)	N/A	0.273	0.653	0.142	0.500	2.681	0.000	0.516	1.450

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	227	77	320	184	0	88	124
N.S.	1	1.00	1.71	0.58	2.41	1.38	0.00	0.66	0.93
time (sec)	N/A	0.334	0.676	0.165	0.503	3.958	0.000	0.519	1.496

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	283	90	365	227	0	104	146
N.S.	1	1.00	1.73	0.55	2.23	1.38	0.00	0.63	0.89
time (sec)	N/A	0.417	0.930	0.174	0.518	3.218	0.000	0.549	1.536

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	189	288	385	208	0	210	259
N.S.	1	1.00	1.01	1.53	2.05	1.11	0.00	1.12	1.38
time (sec)	N/A	0.185	2.327	0.138	0.287	2.933	0.000	0.584	2.619

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	165	267	361	191	0	191	227
N.S.	1	1.00	1.25	2.02	2.73	1.45	0.00	1.45	1.72
time (sec)	N/A	0.111	1.968	0.111	0.283	2.353	0.000	0.578	2.593

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	93	101	86	0	65	122
N.S.	1	1.00	0.90	1.37	1.49	1.26	0.00	0.96	1.79
time (sec)	N/A	0.057	0.040	0.075	0.271	2.929	0.000	0.538	4.959

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	122	169	219	157	0	153	163
N.S.	1	1.00	1.26	1.74	2.26	1.62	0.00	1.58	1.68
time (sec)	N/A	0.088	0.857	0.077	0.305	3.365	0.000	0.551	2.133

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	101	96	116	127	0	104	104
N.S.	1	1.00	1.31	1.25	1.51	1.65	0.00	1.35	1.35
time (sec)	N/A	0.107	0.486	0.056	0.283	2.709	0.000	0.545	1.552

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	240	94	296	137	0	111	85
N.S.	1	1.00	3.08	1.21	3.79	1.76	0.00	1.42	1.09
time (sec)	N/A	0.152	2.798	0.122	0.505	3.542	0.000	0.507	1.479

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	177	66	296	168	0	80	45
N.S.	1	1.00	2.01	0.75	3.36	1.91	0.00	0.91	0.51
time (sec)	N/A	0.266	1.282	0.144	0.513	2.878	0.000	0.510	1.437

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	53	60	306	137	0	73	96
N.S.	1	1.00	0.52	0.59	3.00	1.34	0.00	0.72	0.94
time (sec)	N/A	0.338	0.093	0.154	0.503	2.114	0.000	0.528	1.378

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	227	76	417	184	0	88	122
N.S.	1	1.00	1.71	0.57	3.14	1.38	0.00	0.66	0.92
time (sec)	N/A	0.435	0.644	0.165	0.511	4.925	0.000	0.547	1.438

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	283	90	439	227	0	104	146
N.S.	1	1.00	1.73	0.55	2.68	1.38	0.00	0.63	0.89
time (sec)	N/A	0.517	0.957	0.189	0.521	3.104	0.000	0.583	1.461

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	153	384	137	653	260	0	153	145
N.S.	1	1.12	2.82	1.01	4.80	1.91	0.00	1.12	1.07
time (sec)	N/A	0.284	3.437	0.169	0.493	2.311	0.000	0.569	1.495

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	753	105	447	237	0	134	112
N.S.	1	1.00	7.38	1.03	4.38	2.32	0.00	1.31	1.10
time (sec)	N/A	0.220	6.281	0.138	0.498	2.283	0.000	0.512	1.471

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	216	66	290	185	0	80	46
N.S.	1	1.00	2.54	0.78	3.41	2.18	0.00	0.94	0.54
time (sec)	N/A	0.247	1.174	0.135	0.512	2.166	0.000	0.510	1.410

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	47	184	100	0	60	38
N.S.	1	1.00	1.00	0.70	2.75	1.49	0.00	0.90	0.57
time (sec)	N/A	0.167	0.054	0.131	0.534	2.864	0.000	0.468	1.375

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	113	46	129	92	0	53	41
N.S.	1	1.00	1.85	0.75	2.11	1.51	0.00	0.87	0.67
time (sec)	N/A	0.108	0.342	0.113	0.497	3.316	0.000	0.487	1.336

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	135	60	110	76	0	81	69
N.S.	1	1.00	1.96	0.87	1.59	1.10	0.00	1.17	1.00
time (sec)	N/A	0.085	0.585	0.105	0.498	2.397	0.000	0.494	1.422

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	32	49	87	0	95	58
N.S.	1	1.00	0.85	0.70	1.07	1.89	0.00	2.07	1.26
time (sec)	N/A	0.051	0.054	0.118	0.510	2.140	0.000	0.504	1.468

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	257	88	159	166	0	110	161
N.S.	1	1.00	2.62	0.90	1.62	1.69	0.00	1.12	1.64
time (sec)	N/A	0.107	1.349	0.126	0.521	3.027	0.000	0.498	1.544

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	315	101	181	179	0	122	185
N.S.	1	1.00	1.90	0.61	1.09	1.08	0.00	0.73	1.11
time (sec)	N/A	0.162	1.360	0.134	0.494	2.029	0.000	0.506	1.635

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	383	114	202	250	0	135	209
N.S.	1	1.00	1.82	0.54	0.96	1.19	0.00	0.64	1.00
time (sec)	N/A	0.215	1.313	0.141	0.528	2.778	0.000	0.551	1.779

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	557	118	610	311	0	154	134
N.S.	1	1.00	3.44	0.73	3.77	1.92	0.00	0.95	0.83
time (sec)	N/A	0.325	6.169	0.166	0.502	2.620	0.000	0.557	1.490

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	231	77	430	259	0	102	50
N.S.	1	1.00	1.56	0.52	2.91	1.75	0.00	0.69	0.34
time (sec)	N/A	0.441	1.325	0.161	0.504	3.190	0.000	0.559	1.421

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	90	60	301	147	0	80	93
N.S.	1	1.00	0.94	0.62	3.14	1.53	0.00	0.83	0.97
time (sec)	N/A	0.288	0.083	0.145	0.527	3.117	0.000	0.545	1.404

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	171	61	229	147	0	80	93
N.S.	1	1.00	1.78	0.64	2.39	1.53	0.00	0.83	0.97
time (sec)	N/A	0.221	0.519	0.139	0.504	2.569	0.000	0.519	1.396

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	169	58	173	133	0	71	85
N.S.	1	1.00	1.92	0.66	1.97	1.51	0.00	0.81	0.97
time (sec)	N/A	0.155	0.500	0.132	0.502	3.646	0.000	0.493	1.377

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	197	73	132	118	0	102	82
N.S.	1	1.00	1.56	0.58	1.05	0.94	0.00	0.81	0.65
time (sec)	N/A	0.144	1.029	0.124	0.480	3.565	0.000	0.557	1.425

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	257	88	158	166	0	116	161
N.S.	1	1.00	2.57	0.88	1.58	1.66	0.00	1.16	1.61
time (sec)	N/A	0.106	1.115	0.125	0.487	4.164	0.000	0.557	1.504

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	48	60	127	0	129	94
N.S.	1	1.00	0.58	0.72	0.90	1.90	0.00	1.93	1.40
time (sec)	N/A	0.064	0.077	0.133	0.494	4.073	0.000	0.551	1.569

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	362	114	203	250	0	142	209
N.S.	1	1.00	2.81	0.88	1.57	1.94	0.00	1.10	1.62
time (sec)	N/A	0.129	1.482	0.141	0.504	4.052	0.000	0.574	1.852

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	441	127	223	292	0	154	233
N.S.	1	1.00	2.10	0.60	1.06	1.39	0.00	0.73	1.11
time (sec)	N/A	0.180	1.799	0.161	0.498	3.021	0.000	0.591	2.041

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	499	140	247	334	0	169	257
N.S.	1	1.00	1.98	0.56	0.98	1.33	0.00	0.67	1.02
time (sec)	N/A	0.229	2.308	0.161	0.509	3.295	0.000	0.608	2.328

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	121	391	0	404	0	295	-1
N.S.	1	1.00	0.69	2.23	0.00	2.31	0.00	1.69	-0.01
time (sec)	N/A	0.134	1.131	1.240	0.000	3.915	0.000	1.546	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	111	302	0	376	0	263	-1
N.S.	1	1.00	0.79	2.16	0.00	2.69	0.00	1.88	-0.01
time (sec)	N/A	0.115	1.242	0.237	0.000	2.925	0.000	1.347	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	142	0	340	0	232	-1
N.S.	1	1.00	0.92	1.35	0.00	3.24	0.00	2.21	-0.01
time (sec)	N/A	0.109	0.876	0.220	0.000	2.684	0.000	1.271	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	115	159	255	0	193	-1
N.S.	1	1.00	1.06	1.74	2.41	3.86	0.00	2.92	-0.02
time (sec)	N/A	0.072	0.353	0.171	0.543	2.311	0.000	1.187	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	90	116	0	290	0	196	-1
N.S.	1	1.00	1.30	1.68	0.00	4.20	0.00	2.84	-0.01
time (sec)	N/A	0.100	0.661	0.204	0.000	2.849	0.000	1.105	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	78	214	0	369	0	296	-1
N.S.	1	1.00	0.75	2.06	0.00	3.55	0.00	2.85	-0.01
time (sec)	N/A	0.108	0.254	0.222	0.000	3.781	0.000	1.118	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	78	311	0	441	0	391	-1
N.S.	1	1.00	0.56	2.24	0.00	3.17	0.00	2.81	-0.01
time (sec)	N/A	0.116	0.276	0.273	0.000	3.814	0.000	1.178	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	78	402	0	517	0	487	-1
N.S.	1	1.00	0.45	2.31	0.00	2.97	0.00	2.80	-0.01
time (sec)	N/A	0.121	0.256	0.304	0.000	4.245	0.000	1.235	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	392	0	416	0	297	-1
N.S.	1	1.00	0.69	2.21	0.00	2.35	0.00	1.68	-0.01
time (sec)	N/A	0.128	1.021	0.228	0.000	3.880	0.000	1.608	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	112	232	0	384	0	265	-1
N.S.	1	1.00	0.79	1.63	0.00	2.70	0.00	1.87	-0.01
time (sec)	N/A	0.116	0.873	0.209	0.000	2.645	0.000	1.465	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	96	212	1076	330	0	227	-1
N.S.	1	1.00	0.95	2.10	10.65	3.27	0.00	2.25	-0.01
time (sec)	N/A	0.083	0.704	0.175	0.593	3.453	0.000	1.205	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	194	0	293	0	198	-1
N.S.	1	1.00	1.33	2.77	0.00	4.19	0.00	2.83	-0.01
time (sec)	N/A	0.110	0.602	0.175	0.000	4.121	0.000	1.161	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	113	215	0	381	0	318	-1
N.S.	1	1.00	1.11	2.11	0.00	3.74	0.00	3.12	-0.01
time (sec)	N/A	0.112	0.684	0.214	0.000	2.972	0.000	1.283	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	102	304	0	453	0	428	-1
N.S.	1	1.00	0.74	2.22	0.00	3.31	0.00	3.12	-0.01
time (sec)	N/A	0.119	0.762	0.244	0.000	2.773	0.000	1.455	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	102	401	0	537	0	555	-1
N.S.	1	1.00	0.59	2.33	0.00	3.12	0.00	3.23	-0.01
time (sec)	N/A	0.132	1.318	0.264	0.000	3.087	0.000	1.602	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	134	483	0	474	0	329	-1
N.S.	1	1.00	0.63	2.28	0.00	2.24	0.00	1.55	-0.00
time (sec)	N/A	0.133	1.279	0.303	0.000	2.752	0.000	1.766	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	124	323	0	440	0	297	-1
N.S.	1	1.00	0.70	1.82	0.00	2.49	0.00	1.68	-0.01
time (sec)	N/A	0.121	0.966	0.213	0.000	2.864	0.000	1.516	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	303	1501	382	0	227	-1
N.S.	1	1.00	0.83	2.30	11.37	2.89	0.00	1.72	-0.01
time (sec)	N/A	0.102	0.840	0.183	0.613	3.348	0.000	1.429	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	96	120	0	315	0	268	-1
N.S.	1	1.00	0.93	1.17	0.00	3.06	0.00	2.60	-0.01
time (sec)	N/A	0.120	0.740	0.182	0.000	3.147	0.000	1.249	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	351	0	367	0	243	-1
N.S.	1	1.00	1.38	4.74	0.00	4.96	0.00	3.28	-0.01
time (sec)	N/A	0.109	4.359	0.203	0.000	3.541	0.000	1.447	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	196	306	0	477	0	428	-1
N.S.	1	1.00	1.88	2.94	0.00	4.59	0.00	4.12	-0.01
time (sec)	N/A	0.122	5.470	0.237	0.000	3.512	0.000	1.413	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	361	395	0	569	0	556	-1
N.S.	1	1.00	2.58	2.82	0.00	4.06	0.00	3.97	-0.01
time (sec)	N/A	0.124	8.039	0.250	0.000	3.187	0.000	1.603	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	205	492	0	649	0	672	-1
N.S.	1	1.00	1.19	2.86	0.00	3.77	0.00	3.91	-0.01
time (sec)	N/A	0.135	3.595	0.299	0.000	4.352	0.000	1.835	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	153	544	0	597	0	0	-1
N.S.	1	1.00	0.83	2.94	0.00	3.23	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.592	0.260	0.000	4.980	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	166	372	0	561	0	0	-1
N.S.	1	1.00	1.09	2.45	0.00	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.160	1.365	0.230	0.000	3.661	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	124	329	0	473	0	0	-1
N.S.	1	1.00	1.04	2.76	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.480	0.226	0.000	3.780	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	144	0	321	0	0	-1
N.S.	1	1.00	0.94	1.66	0.00	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.284	0.180	0.000	4.267	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	101	195	0	474	0	0	-1
N.S.	1	1.00	0.83	1.61	0.00	3.92	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.578	0.218	0.000	3.771	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	5576	377	0	566	0	0	-1
N.S.	1	1.00	34.63	2.34	0.00	3.52	0.00	0.00	-0.01
time (sec)	N/A	0.158	24.248	0.247	0.000	4.864	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	5592	545	0	662	0	0	-1
N.S.	1	1.00	28.53	2.78	0.00	3.38	0.00	0.00	-0.01
time (sec)	N/A	0.186	24.130	0.341	0.000	2.297	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	196	552	0	685	0	0	-1
N.S.	1	1.00	0.97	2.72	0.00	3.37	0.00	0.00	-0.00
time (sec)	N/A	0.194	1.557	0.245	0.000	3.662	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	132	377	0	593	0	0	-1
N.S.	1	1.00	0.78	2.23	0.00	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.163	1.884	0.226	0.000	4.448	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	134	128	369	0	585	0	0	-1
N.S.	1	1.13	1.08	3.10	0.00	4.92	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.119	0.183	0.000	2.813	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	130	130	371	0	548	0	0	-1
N.S.	1	1.15	1.15	3.28	0.00	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.057	0.155	0.000	4.323	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	154	377	0	560	0	0	-1
N.S.	1	1.00	0.87	2.13	0.00	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.162	1.249	0.208	0.000	3.476	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	5612	387	0	608	0	0	-1
N.S.	1	1.00	26.22	1.81	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	0.183	24.317	0.264	0.000	2.991	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	5629	725	0	776	0	0	-1
N.S.	1	1.00	22.61	2.91	0.00	3.12	0.00	0.00	-0.00
time (sec)	N/A	0.230	24.234	0.446	0.000	3.135	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	180	726	0	801	0	0	-1
N.S.	1	1.00	0.69	2.79	0.00	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.232	3.700	0.272	0.000	7.112	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	164	550	0	706	0	0	-1
N.S.	1	1.00	0.72	2.40	0.00	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.201	2.722	0.236	0.000	4.729	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	136	553	0	696	0	0	-1
N.S.	1	1.00	0.71	2.90	0.00	3.64	0.00	0.00	-0.01
time (sec)	N/A	0.163	1.614	0.230	0.000	7.149	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	136	545	0	696	0	0	-1
N.S.	1	1.00	0.72	2.88	0.00	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.153	1.526	0.195	0.000	5.080	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	181	134	543	0	656	0	0	-1
N.S.	1	1.22	0.91	3.67	0.00	4.43	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.485	0.169	0.000	4.862	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	158	545	0	662	0	0	-1
N.S.	1	1.00	0.69	2.37	0.00	2.88	0.00	0.00	-0.00
time (sec)	N/A	0.206	1.511	0.237	0.000	3.689	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	5650	725	0	768	0	0	-1
N.S.	1	1.00	21.00	2.70	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.227	24.263	0.295	0.000	4.659	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	149	194	1393	497	0	0	-1
N.S.	1	1.00	0.81	1.05	7.53	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.242	6.433	0.730	0.630	3.994	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	162	184	767	461	0	0	-1
N.S.	1	1.00	1.17	1.32	5.52	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.177	2.237	0.270	0.590	3.407	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	99	164	264	380	0	0	-1
N.S.	1	1.00	1.06	1.76	2.84	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.333	0.250	0.569	2.711	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	102	127	42	216	0	0	-1
N.S.	1	1.00	2.12	2.65	0.88	4.50	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.600	0.253	0.541	2.750	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	86	100	69	0	0	72	-1
N.S.	1	1.00	1.69	1.96	1.35	0.00	0.00	1.41	-0.02
time (sec)	N/A	0.061	0.926	0.268	0.494	0.000	0.000	1.446	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	107	162	440	0	0	147	-1
N.S.	1	1.00	1.11	1.69	4.58	0.00	0.00	1.53	-0.01
time (sec)	N/A	0.131	1.124	0.250	0.570	0.000	0.000	1.573	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	152	226	1288	0	0	178	-1
N.S.	1	1.00	1.07	1.59	9.07	0.00	0.00	1.25	-0.01
time (sec)	N/A	0.183	1.362	0.270	0.749	0.000	0.000	1.854	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	198	288	2681	0	0	208	-1
N.S.	1	1.00	1.05	1.53	14.26	0.00	0.00	1.11	-0.01
time (sec)	N/A	0.243	1.951	0.274	3.315	0.000	0.000	1.927	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	157	189	1460	505	0	0	-1
N.S.	1	1.00	0.83	0.99	7.68	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.245	1.356	0.266	0.642	3.440	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	159	171	518	376	0	0	-1
N.S.	1	1.00	1.54	1.66	5.03	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.077	1.455	0.257	0.571	2.720	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	128	149	264	377	0	0	-1
N.S.	1	1.00	1.38	1.60	2.84	4.05	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.813	0.246	0.576	2.958	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	105	149	65	0	0	0	-1
N.S.	1	1.00	1.01	1.43	0.62	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	1.254	0.233	0.563	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	115	161	101	0	0	120	-1
N.S.	1	1.00	1.15	1.61	1.01	0.00	0.00	1.20	-0.01
time (sec)	N/A	0.127	0.726	0.233	0.526	0.000	0.000	1.677	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	153	227	1937	0	0	170	-1
N.S.	1	1.00	1.05	1.55	13.27	0.00	0.00	1.16	-0.01
time (sec)	N/A	0.197	1.239	0.248	0.809	0.000	0.000	2.040	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	199	289	3777	0	0	200	-1
N.S.	1	1.00	1.02	1.47	19.27	0.00	0.00	1.02	-0.01
time (sec)	N/A	0.255	2.212	0.290	3.335	0.000	0.000	1.717	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	164	191	1738	437	0	0	-1
N.S.	1	1.00	1.07	1.25	11.36	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.590	0.280	0.728	2.920	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	149	199	1460	505	0	0	-1
N.S.	1	1.00	0.78	1.05	7.68	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.262	1.245	0.274	0.676	2.852	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	164	179	767	456	0	0	-1
N.S.	1	1.00	1.18	1.29	5.52	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.175	1.334	0.269	0.612	3.161	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	292	183	0	0	0	0	-1
N.S.	1	1.00	1.92	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	6.814	0.263	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	111	237	0	478	0	0	-1
N.S.	1	1.00	1.16	2.47	0.00	4.98	0.00	0.00	-0.01
time (sec)	N/A	0.125	1.277	0.252	0.000	3.463	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	155	229	147	0	0	170	-1
N.S.	1	1.00	1.55	2.29	1.47	0.00	0.00	1.70	-0.01
time (sec)	N/A	0.124	1.442	0.249	0.520	0.000	0.000	2.036	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	202	281	4035	0	0	200	-1
N.S.	1	1.00	1.36	1.90	27.26	0.00	0.00	1.35	-0.01
time (sec)	N/A	0.186	2.677	0.268	3.345	0.000	0.000	3.346	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	285	353	6623	0	0	230	-1
N.S.	1	1.00	1.47	1.82	34.14	0.00	0.00	1.19	-0.01
time (sec)	N/A	0.253	5.770	0.282	23.478	0.000	0.000	2.159	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	299	415	9881	0	0	260	-1
N.S.	1	1.00	1.23	1.70	40.50	0.00	0.00	1.07	-0.00
time (sec)	N/A	0.323	6.243	0.299	140.960	0.000	0.000	1.782	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	153	189	0	0	0	0	-1
N.S.	1	1.00	0.75	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	17.001	0.273	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	181	169	0	0	0	0	-1
N.S.	1	1.00	1.20	1.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	4.009	0.262	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	103	93	65	0	0	0	-1
N.S.	1	1.00	1.01	0.91	0.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	10.324	0.262	0.589	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	127	75	36	0	0	36	-1
N.S.	1	1.00	2.59	1.53	0.73	0.00	0.00	0.73	-0.02
time (sec)	N/A	0.055	0.989	0.245	0.498	0.000	0.000	1.224	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	104	101	42	292	0	67	-1
N.S.	1	1.00	2.26	2.20	0.91	6.35	0.00	1.46	-0.02
time (sec)	N/A	0.063	1.147	0.313	0.580	3.447	0.000	1.292	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	217	143	167	889	0	0	112	-1
N.S.	1	1.29	0.85	0.99	5.29	0.00	0.00	0.67	-0.01
time (sec)	N/A	0.097	9.499	0.267	0.585	0.000	0.000	1.996	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	194	229	2398	0	0	144	-1
N.S.	1	1.00	0.71	0.84	8.75	0.00	0.00	0.53	-0.00
time (sec)	N/A	0.101	2.008	0.280	0.775	0.000	0.000	1.760	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	204	275	2583	0	0	0	-1
N.S.	1	1.00	0.95	1.28	12.01	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.091	2.218	0.259	0.819	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	116	236	0	489	0	0	-1
N.S.	1	1.00	1.21	2.46	0.00	5.09	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.781	0.253	0.000	4.202	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	114	106	74	0	0	73	-1
N.S.	1	1.00	1.16	1.08	0.76	0.00	0.00	0.74	-0.01
time (sec)	N/A	0.124	1.208	0.243	0.493	0.000	0.000	1.469	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	119	436	0	0	83	-1
N.S.	1	1.00	1.13	1.27	4.64	0.00	0.00	0.88	-0.01
time (sec)	N/A	0.120	0.598	0.257	0.542	0.000	0.000	1.348	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	141	159	889	0	0	101	-1
N.S.	1	1.00	0.66	0.74	4.13	0.00	0.00	0.47	-0.00
time (sec)	N/A	0.095	1.490	0.268	0.580	0.000	0.000	2.396	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	121	175	527	532	0	153	-1
N.S.	1	1.00	1.20	1.73	5.22	5.27	0.00	1.51	-0.01
time (sec)	N/A	0.076	1.607	0.256	0.573	2.341	0.000	2.180	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	275	291	4644	0	0	185	-1
N.S.	1	1.00	0.79	0.84	13.38	0.00	0.00	0.53	-0.00
time (sec)	N/A	0.125	2.680	0.302	3.373	0.000	0.000	1.991	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	157	335	0	0	0	93	-1
N.S.	1	1.00	0.71	1.52	0.00	0.00	0.00	0.42	-0.00
time (sec)	N/A	0.093	2.543	0.257	0.000	0.000	0.000	2.502	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	154	144	108	0	0	75	-1
N.S.	1	1.00	1.57	1.47	1.10	0.00	0.00	0.77	-0.01
time (sec)	N/A	0.122	1.640	0.249	0.509	0.000	0.000	1.764	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	152	152	1937	0	0	111	-1
N.S.	1	1.00	1.06	1.06	13.45	0.00	0.00	0.77	-0.01
time (sec)	N/A	0.185	0.875	0.250	0.774	0.000	0.000	1.676	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	151	152	1280	0	0	124	-1
N.S.	1	1.00	1.08	1.09	9.14	0.00	0.00	0.89	-0.01
time (sec)	N/A	0.182	0.685	0.307	0.731	0.000	0.000	1.576	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	195	223	2398	0	0	144	-1
N.S.	1	1.00	0.72	0.83	8.88	0.00	0.00	0.53	-0.00
time (sec)	N/A	0.103	1.742	0.290	0.771	0.000	0.000	1.749	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	275	284	4644	0	0	193	-1
N.S.	1	1.00	0.80	0.82	13.46	0.00	0.00	0.56	-0.00
time (sec)	N/A	0.118	2.440	0.274	3.355	0.000	0.000	2.024	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	237	1505	610	0	225	-1
N.S.	1	1.00	0.99	1.57	9.97	4.04	0.00	1.49	-0.01
time (sec)	N/A	0.085	2.259	0.292	0.788	5.302	0.000	2.037	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.099	0.184	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.460	0.169	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	3.406	0.172	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.566	0.152	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	1.708	0.136	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.990	0.151	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	1.601	0.150	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	3.519	0.173	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.825	0.167	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.131	0.171	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	1.211	0.178	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.129	1.493	0.172	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	133	141	0	316	0	189	-1
N.S.	1	1.00	1.46	1.55	0.00	3.47	0.00	2.08	-0.01
time (sec)	N/A	0.062	0.687	0.472	0.000	2.779	0.000	1.104	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	810	295	0	0	0	0	-1
N.S.	1	1.00	3.51	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	33.953	0.530	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	178	285	0	0	0	0	-1
N.S.	1	1.00	0.79	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	8.349	0.250	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	187	327	0	0	0	0	-1
N.S.	1	1.00	0.59	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	12.321	0.312	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	587	546	0	503	0	535	-1
N.S.	1	1.00	2.17	2.01	0.00	1.86	0.00	1.97	-0.00
time (sec)	N/A	0.120	14.457	0.296	0.000	3.051	0.000	1.525	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	517	389	0	421	0	365	-1
N.S.	1	1.00	2.52	1.90	0.00	2.05	0.00	1.78	-0.00
time (sec)	N/A	0.104	14.329	0.246	0.000	3.083	0.000	1.375	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	444	248	0	347	0	268	-1
N.S.	1	1.00	3.08	1.72	0.00	2.41	0.00	1.86	-0.01
time (sec)	N/A	0.077	6.637	0.207	0.000	3.614	0.000	1.261	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	159	256	0	193	-1
N.S.	1	1.00	1.15	1.79	2.41	3.88	0.00	2.92	-0.02
time (sec)	N/A	0.062	0.341	0.173	0.542	2.357	0.000	1.145	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	2650	501	0	715	0	277	-1
N.S.	1	1.00	25.24	4.77	0.00	6.81	0.00	2.64	-0.01
time (sec)	N/A	0.162	26.107	2.894	0.000	2.590	0.000	1.194	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	2907	97143	0	1499	0	617	-1
N.S.	1	1.00	13.27	443.58	0.00	6.84	0.00	2.82	-0.00
time (sec)	N/A	0.159	28.734	0.921	0.000	7.844	0.000	1.393	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	3070	330372	0	2470	0	1381	-1
N.S.	1	1.00	10.70	1151.12	0.00	8.61	0.00	4.81	-0.00
time (sec)	N/A	0.217	24.457	3.273	0.000	13.970	0.000	1.803	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	219	539	0	513	0	525	-1
N.S.	1	1.00	0.91	2.24	0.00	2.13	0.00	2.18	-0.00
time (sec)	N/A	0.135	3.907	3.298	0.000	3.407	0.000	1.867	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	145	382	0	427	0	359	-1
N.S.	1	1.00	0.82	2.17	0.00	2.43	0.00	2.04	-0.01
time (sec)	N/A	0.096	1.384	2.807	0.000	2.994	0.000	1.564	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	1076	343	0	263	-1
N.S.	1	1.00	0.97	2.26	10.25	3.27	0.00	2.50	-0.01
time (sec)	N/A	0.104	0.596	2.914	0.607	1.993	0.000	1.319	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	135	868	0	777	0	285	-1
N.S.	1	1.00	1.23	7.89	0.00	7.06	0.00	2.59	-0.01
time (sec)	N/A	0.163	0.521	4.649	0.000	4.005	0.000	1.190	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	2862	62283	0	1726	0	688	-1
N.S.	1	1.00	12.50	271.98	0.00	7.54	0.00	3.00	-0.00
time (sec)	N/A	0.170	24.925	9.263	0.000	16.611	0.000	1.663	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	3166	234363	0	2831	0	1592	-1
N.S.	1	1.00	10.21	756.01	0.00	9.13	0.00	5.14	-0.00
time (sec)	N/A	0.249	24.616	11.862	0.000	31.324	0.000	2.372	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	286	677	0	653	0	613	-1
N.S.	1	1.00	0.85	2.01	0.00	1.94	0.00	1.82	-0.00
time (sec)	N/A	0.143	6.377	1.556	0.000	3.168	0.000	1.753	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	191	504	0	531	0	441	-1
N.S.	1	1.00	0.74	1.95	0.00	2.06	0.00	1.71	-0.00
time (sec)	N/A	0.120	2.783	1.483	0.000	3.921	0.000	3.454	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1501	419	0	309	-1
N.S.	1	1.00	0.90	2.40	10.57	2.95	0.00	2.18	-0.01
time (sec)	N/A	0.165	0.996	1.325	0.783	2.384	0.000	1.619	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	343	1490	0	1210	0	412	-1
N.S.	1	1.00	1.69	7.34	0.00	5.96	0.00	2.03	-0.00
time (sec)	N/A	0.161	6.610	4.518	0.000	7.950	0.000	2.031	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	3026	46082	0	2117	0	661	-1
N.S.	1	1.00	9.20	140.07	0.00	6.43	0.00	2.01	-0.00
time (sec)	N/A	0.234	24.376	7.294	0.000	47.671	0.000	2.018	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	3344	209489	0	3453	0	1606	-1
N.S.	1	1.00	6.24	390.84	0.00	6.44	0.00	3.00	-0.00
time (sec)	N/A	0.342	25.426	13.279	0.000	83.951	0.000	2.356	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	787	907	0	662	0	0	-1
N.S.	1	1.00	3.05	3.52	0.00	2.57	0.00	0.00	-0.00
time (sec)	N/A	0.137	7.264	1.531	0.000	23.422	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	295	358	0	516	0	0	-1
N.S.	1	1.00	1.61	1.96	0.00	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.103	2.562	1.511	0.000	7.485	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	337	0	0	-1
N.S.	1	1.00	1.01	2.13	0.00	3.70	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.312	1.310	0.000	4.452	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	431980	663	0	1120	0	0	-1
N.S.	1	1.00	2602.29	3.99	0.00	6.75	0.00	0.00	-0.01
time (sec)	N/A	0.249	39.278	4.649	0.000	26.867	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	473385	117715	0	0	0	0	-1
N.S.	1	1.00	1137.94	282.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	36.269	9.065	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	654358	402966	0	0	0	0	-1
N.S.	1	1.00	1002.08	617.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	38.867	15.823	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	21121	957	0	744	0	0	-1
N.S.	1	1.00	65.19	2.95	0.00	2.30	0.00	0.00	-0.00
time (sec)	N/A	0.162	29.475	1.724	0.000	34.808	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	16153	756	0	663	0	0	-1
N.S.	1	1.00	55.70	2.61	0.00	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.148	27.990	1.569	0.000	26.044	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	11183	552	0	591	0	0	-1
N.S.	1	1.00	88.06	4.35	0.00	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.123	26.890	1.507	0.000	8.806	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	378865	2080	0	0	0	0	-1
N.S.	1	1.00	961.59	5.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	35.315	5.355	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	582620	164978	0	0	0	0	-1
N.S.	1	1.00	1040.39	294.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	37.906	8.362	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	802	802	776222	480553	0	0	0	0	-1
N.S.	1	1.00	967.86	599.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	40.798	15.050	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	21194	1444	0	931	0	0	-1
N.S.	1	1.00	44.15	3.01	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.221	29.387	2.093	0.000	83.551	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	16249	1133	0	833	0	0	-1
N.S.	1	1.00	34.72	2.42	0.00	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.197	28.028	1.576	0.000	31.176	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	11243	824	0	721	0	0	-1
N.S.	1	1.00	68.55	5.02	0.00	4.40	0.00	0.00	-0.01
time (sec)	N/A	0.182	27.003	1.622	0.000	8.981	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	486155	3863	0	0	0	0	-1
N.S.	1	1.00	821.21	6.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	37.040	5.309	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	756	756	688080	197500	0	0	0	0	-1
N.S.	1	1.00	910.16	261.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	39.495	8.948	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	999	999	893714	556423	0	0	0	0	-1
N.S.	1	1.00	894.61	556.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.663	42.756	19.138	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	240	1551	0	868	0	0	-1
N.S.	1	1.00	1.95	12.61	0.00	7.06	0.00	0.00	-0.01
time (sec)	N/A	0.213	20.508	2.070	0.000	12.179	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	102	189	0	221	0	0	-1
N.S.	1	1.00	1.67	3.10	0.00	3.62	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.216	2.128	0.000	5.618	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	135	377	0	552	0	0	-1
N.S.	1	1.00	1.22	3.40	0.00	4.97	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.980	1.977	0.000	3.026	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	184	482	0	945	0	0	-1
N.S.	1	1.00	1.30	3.42	0.00	6.70	0.00	0.00	-0.01
time (sec)	N/A	0.234	14.459	1.960	0.000	2.955	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	171	415	0	975	0	0	-1
N.S.	1	1.00	1.21	2.94	0.00	6.91	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.345	2.020	0.000	4.736	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	258	0	274	573
N.S.	1	1.00	1.01	1.09	0.00	3.85	0.00	4.09	8.55
time (sec)	N/A	0.090	0.165	0.167	0.000	2.587	0.000	0.505	2.676

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	155	168	0	577	0	201	2500
N.S.	1	1.00	1.26	1.37	0.00	4.69	0.00	1.63	20.33
time (sec)	N/A	0.183	0.668	0.221	0.000	4.423	0.000	0.510	9.327

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	267	287	0	1176	0	457	2500
N.S.	1	1.00	1.31	1.41	0.00	5.76	0.00	2.24	12.25
time (sec)	N/A	0.366	1.320	0.340	0.000	3.036	0.000	0.577	11.353

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	136	195	0	687	0	237	2500
N.S.	1	1.00	1.02	1.47	0.00	5.17	0.00	1.78	18.80
time (sec)	N/A	0.203	0.725	0.252	0.000	2.833	0.000	0.571	10.205

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	493	386	0	1433	0	658	2500
N.S.	1	1.00	2.08	1.63	0.00	6.05	0.00	2.78	10.55
time (sec)	N/A	0.559	2.051	0.396	0.000	3.085	0.000	0.611	12.136

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	438	635	0	2394	0	1201	2500
N.S.	1	1.00	1.16	1.68	0.00	6.35	0.00	3.19	6.63
time (sec)	N/A	1.370	3.310	0.596	0.000	4.919	0.000	0.639	15.074

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	517	458	0	1653	0	818	2500
N.S.	1	1.00	2.04	1.80	0.00	6.51	0.00	3.22	9.84
time (sec)	N/A	0.786	2.208	0.507	0.000	3.799	0.000	0.635	14.496

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	459	785	0	2808	0	1572	2500
N.S.	1	1.00	1.11	1.91	0.00	6.82	0.00	3.82	6.07
time (sec)	N/A	0.768	3.766	0.747	0.000	4.176	0.000	0.668	16.091

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	1285	1345	0	4386	0	3173	2500
N.S.	1	1.00	2.07	2.16	0.00	7.05	0.00	5.10	4.02
time (sec)	N/A	1.227	6.822	1.079	0.000	4.835	0.000	0.770	16.348

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	913	1372	0	0	0	0	-1
N.S.	1	1.00	2.85	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	17.792	3.540	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	225	443	0	0	0	0	-1
N.S.	1	1.00	1.02	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	7.286	2.740	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	6063	2340	0	0	0	0	-1
N.S.	1	1.00	15.96	6.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	24.504	3.528	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	230	581	0	0	0	0	-1
N.S.	1	1.00	0.71	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	8.805	2.869	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	7138	3285	0	0	0	0	-1
N.S.	1	1.00	16.15	7.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	25.487	3.454	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	145	215	0	0	0	0	-1
N.S.	1	1.00	0.70	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.080	2.449	2.897	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	251	318	0	0	0	0	-1
N.S.	1	1.00	1.16	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	27.701	3.185	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	1491	2010	0	0	0	0	-1
N.S.	1	1.00	3.97	5.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	14.390	3.342	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	2083	5710	0	0	0	0	-1
N.S.	1	1.00	4.21	11.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	16.792	3.251	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	39925	543	0	0	0	0	-1
N.S.	1	1.00	102.63	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	35.542	2.441	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	336	352	0	0	0	0	-1
N.S.	1	1.00	1.70	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	5.461	2.645	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	1708	2847	0	0	0	0	-1
N.S.	1	1.00	2.86	4.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	9.307	2.745	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	899	899	1990	15724	0	0	0	0	-1
N.S.	1	1.00	2.21	17.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.518	6.873	2.707	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	744	744	1750	4302	0	0	0	0	-1
N.S.	1	1.00	2.35	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	9.548	2.596	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	919	919	1960	13060	0	0	0	0	-1
N.S.	1	1.00	2.13	14.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.447	6.715	2.516	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2385	39418	0	0	0	0	-1
N.S.	1	1.00	2.13	35.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.160	7.517	3.485	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	891	891	2026	15922	0	0	0	0	-1
N.S.	1	1.00	2.27	17.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.340	6.737	2.655	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1150	2344	32283	0	0	0	0	-1
N.S.	1	1.00	2.04	28.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.360	7.372	3.398	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1428	1428	2979	75468	0	0	0	0	-1
N.S.	1	1.00	2.09	52.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.709	8.358	4.484	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F	F(-1)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	0	49385	491	0	0	0	0	-1
N.S.	1	0.00	75.74	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.056	35.694	2.795	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	325	352	0	0	0	0	-1
N.S.	1	1.00	1.64	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	3.385	2.737	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	249	292	0	0	0	0	-1
N.S.	1	1.00	0.63	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	2.487	3.102	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	763	1761	3451	0	0	0	0	-1
N.S.	1	1.23	2.83	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	9.686	2.898	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	2.366	0.161	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	48.513	0.146	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-2)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	78.301	0.146	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	2.316	0.145	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	49.131	0.151	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-2)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	84.591	0.151	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	56.514	0.148	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-2)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	91.258	0.145	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	130.037	0.165	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	2425	0	0	0	0	0	-1
N.S.	1	1.00	22.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	14.544	0.188	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	343	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	2.080	0.140	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	299	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	2.817	0.130	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.222	0.128	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	1.079	0.124	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	9.832	0.134	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	2.464	0.204	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	278	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	1.092	0.145	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	200	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.514	0.118	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	125	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.225	0.114	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	5411	0	0	0	0	0	-1
N.S.	1	1.00	26.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	25.231	0.112	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	14108	0	0	0	0	0	-1
N.S.	1	1.00	43.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	44.058	0.124	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [86] had the largest ratio of [30]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	9	1.00	26	0.346
2	A	11	8	1.00	26	0.308
3	A	5	3	1.00	26	0.115
4	A	4	3	1.00	26	0.115
5	A	4	3	1.00	24	0.125
6	A	8	6	1.00	26	0.231
7	A	9	6	1.00	26	0.231
8	A	12	7	1.00	26	0.269
9	A	15	7	1.00	26	0.269
10	A	18	7	1.00	26	0.269
11	A	13	8	1.00	26	0.308
12	A	6	3	1.00	26	0.115
13	A	5	3	1.00	26	0.115
14	A	5	3	1.00	26	0.115
15	A	9	8	1.00	24	0.333
16	A	15	11	1.00	26	0.423
17	A	13	9	1.00	26	0.346
18	A	15	9	1.00	26	0.346
19	A	19	9	1.00	26	0.346
20	A	23	9	1.00	26	0.346
21	A	26	14	1.12	26	0.538
22	A	21	13	1.00	26	0.500
23	A	13	9	1.00	26	0.346
24	A	9	6	1.00	26	0.231
25	A	7	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
26	A	4	3	1.00	26	0.115
27	A	4	3	1.00	26	0.115
28	A	5	3	1.00	26	0.115
29	A	13	8	1.00	26	0.308
30	A	17	9	1.00	26	0.346
31	A	29	15	1.00	26	0.577
32	A	20	13	1.00	26	0.500
33	A	15	9	1.00	26	0.346
34	A	12	7	1.00	26	0.269
35	A	9	6	1.00	24	0.250
36	A	12	8	1.00	26	0.308
37	A	5	3	1.00	26	0.115
38	A	5	3	1.00	26	0.115
39	A	6	3	1.00	26	0.115
40	A	14	8	1.00	26	0.308
41	A	18	9	1.00	26	0.346
42	A	5	4	1.00	28	0.143
43	A	5	4	1.00	28	0.143
44	A	5	4	1.00	28	0.143
45	A	4	4	1.00	26	0.154
46	A	4	4	1.00	28	0.143
47	A	5	4	1.00	28	0.143
48	A	6	4	1.00	28	0.143
49	A	7	4	1.00	28	0.143
50	A	6	5	1.00	28	0.179
51	A	6	5	1.00	28	0.179
52	A	5	5	1.00	26	0.192
53	A	4	4	1.00	28	0.143
54	A	5	5	1.00	28	0.179
55	A	6	5	1.00	28	0.179
56	A	7	5	1.00	28	0.179
57	A	5	4	1.00	28	0.143
58	A	5	4	1.00	28	0.143
59	A	5	4	1.00	26	0.154
60	A	5	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	4	1.00	28	0.143
62	A	5	4	1.00	28	0.143
63	A	5	4	1.00	28	0.143
64	A	5	4	1.00	28	0.143
65	A	8	6	1.00	28	0.214
66	A	7	6	1.00	28	0.214
67	A	6	5	1.00	28	0.179
68	A	5	4	1.00	26	0.154
69	A	6	5	1.00	28	0.179
70	A	7	6	1.00	28	0.214
71	A	8	6	1.00	28	0.214
72	A	8	6	1.00	28	0.214
73	A	7	6	1.00	28	0.214
74	A	7	6	1.13	28	0.214
75	A	6	5	1.15	26	0.192
76	A	7	6	1.00	28	0.214
77	A	8	6	1.00	28	0.214
78	A	9	6	1.00	28	0.214
79	A	9	7	1.00	28	0.250
80	A	8	7	1.00	28	0.250
81	A	7	6	1.00	28	0.214
82	A	7	6	1.00	28	0.214
83	A	7	6	1.22	26	0.231
84	A	8	7	1.00	28	0.250
85	A	9	7	1.00	28	0.250
86	A	5	3	1.00	30	0.100
87	A	4	3	1.00	30	0.100
88	A	3	3	1.00	30	0.100
89	A	2	2	1.00	30	0.067
90	A	2	2	1.00	30	0.067
91	A	3	3	1.00	30	0.100
92	A	4	3	1.00	30	0.100
93	A	5	3	1.00	30	0.100
94	A	5	4	1.00	30	0.133
95	A	3	3	1.00	30	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	30	0.100
97	A	3	2	1.00	30	0.067
98	A	3	3	1.00	30	0.100
99	A	4	4	1.00	30	0.133
100	A	5	4	1.00	30	0.133
101	A	4	3	1.00	30	0.100
102	A	5	4	1.00	30	0.133
103	A	4	3	1.00	30	0.100
104	A	3	2	1.00	30	0.067
105	A	3	3	1.00	30	0.100
106	A	3	3	1.00	30	0.100
107	A	4	4	1.00	30	0.133
108	A	5	4	1.00	30	0.133
109	A	6	4	1.00	30	0.133
110	A	3	2	1.00	30	0.067
111	A	3	2	1.00	30	0.067
112	A	3	2	1.00	30	0.067
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	30	0.067
115	A	3	2	1.29	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	30	0.067
118	A	3	3	1.00	30	0.100
119	A	3	3	1.00	30	0.100
120	A	3	3	1.00	30	0.100
121	A	3	2	1.00	30	0.067
122	A	3	3	1.00	30	0.100
123	A	3	2	1.00	30	0.067
124	A	3	2	1.00	30	0.067
125	A	3	3	1.00	30	0.100
126	A	4	4	1.00	30	0.133
127	A	4	3	1.00	30	0.100
128	A	3	2	1.00	30	0.067
129	A	3	2	1.00	30	0.067
130	A	4	3	1.00	30	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	24	0.083
132	A	3	3	1.00	26	0.115
133	A	3	3	1.00	26	0.115
134	A	3	3	1.00	26	0.115
135	A	3	3	1.00	24	0.125
136	A	3	3	1.00	26	0.115
137	A	3	3	1.00	26	0.115
138	A	4	3	1.00	28	0.107
139	A	3	3	1.00	28	0.107
140	A	2	2	1.00	28	0.071
141	A	4	4	1.00	28	0.143
142	A	5	5	1.00	28	0.179
143	A	6	5	1.00	27	0.185
144	A	3	3	1.00	27	0.111
145	A	3	3	1.00	27	0.111
146	A	5	5	1.00	27	0.185
147	A	5	4	1.00	27	0.148
148	A	5	4	1.00	27	0.148
149	A	5	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	5	5	1.00	27	0.185
152	A	7	6	1.00	27	0.222
153	A	8	7	1.00	27	0.259
154	A	6	5	1.00	27	0.185
155	A	5	5	1.00	27	0.185
156	A	5	5	1.00	25	0.200
157	A	5	5	1.00	27	0.185
158	A	7	6	1.00	27	0.222
159	A	8	6	1.00	27	0.222
160	A	5	4	1.00	27	0.148
161	A	5	4	1.00	27	0.148
162	A	6	5	1.00	25	0.200
163	A	7	5	1.00	27	0.185
164	A	10	6	1.00	27	0.222
165	A	14	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	9	5	1.00	27	0.185
167	A	7	4	1.00	27	0.148
168	A	5	4	1.00	25	0.160
169	A	8	7	1.00	27	0.259
170	A	12	6	1.00	27	0.222
171	A	16	6	1.00	27	0.222
172	A	10	5	1.00	27	0.185
173	A	10	5	1.00	27	0.185
174	A	6	5	1.00	25	0.200
175	A	12	6	1.00	27	0.222
176	A	15	6	1.00	27	0.222
177	A	19	6	1.00	27	0.222
178	A	14	5	1.00	27	0.185
179	A	14	5	1.00	27	0.185
180	A	7	5	1.00	25	0.200
181	A	16	6	1.00	27	0.222
182	A	19	6	1.00	27	0.222
183	A	23	6	1.00	27	0.222
184	A	5	5	1.00	29	0.172
185	A	2	2	1.00	29	0.069
186	A	5	5	1.00	29	0.172
187	A	5	4	1.00	29	0.138
188	A	5	4	1.00	29	0.138
189	A	4	4	1.00	23	0.174
190	A	5	5	1.00	23	0.217
191	A	6	6	1.00	23	0.261
192	A	5	5	1.00	25	0.200
193	A	6	6	1.00	25	0.240
194	A	7	7	1.00	25	0.280
195	A	6	6	1.00	25	0.240
196	A	7	7	1.00	25	0.280
197	A	8	8	1.00	25	0.320
198	A	5	5	1.00	25	0.200
199	A	3	3	1.00	27	0.111
200	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	5	1.00	27	0.185
202	A	7	7	1.00	25	0.280
203	A	3	3	1.00	25	0.120
204	A	3	3	1.00	27	0.111
205	A	6	6	1.00	25	0.240
206	A	7	7	1.00	25	0.280
207	A	3	3	1.00	29	0.103
208	A	1	1	1.00	29	0.034
209	A	5	5	1.00	29	0.172
210	A	7	7	1.00	29	0.241
211	A	6	6	1.00	29	0.207
212	A	7	7	1.00	29	0.241
213	A	8	8	1.00	29	0.276
214	A	7	7	1.00	29	0.241
215	A	8	8	1.00	29	0.276
216	A	9	8	1.00	29	0.276
217	F	0	0	N/A	0.	N/A
218	A	1	1	1.00	29	0.034
219	A	3	3	1.00	29	0.103
220	A	6	6	1.23	29	0.207
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000
224	A	0	0	0.00	0	0.000
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	0	0	0.00	0	0.000
229	A	0	0	0.00	0	0.000
230	A	4	4	1.00	27	0.148
231	A	8	6	1.00	27	0.222
232	A	7	5	1.00	27	0.185
233	A	6	4	1.00	25	0.160
234	A	7	5	1.00	27	0.185
235	A	8	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	0	0	0.00	0	0.000
237	A	8	6	1.00	27	0.222
238	A	7	5	1.00	27	0.185
239	A	6	4	1.00	25	0.160
240	A	7	5	1.00	27	0.185
241	A	10	5	1.00	27	0.185

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^5 \, dx$ 84
3.2 $\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^4 \, dx$ 89
3.3 $\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^3 \, dx$ 94
3.4 $\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^2 \, dx$ 98
3.5 $\int (a + a\sec(e + fx))^2(c - c\sec(e + fx)) \, dx$ 102
3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)} \, dx$ 106
3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2} \, dx$ 110
3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3} \, dx$ 114
3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} \, dx$ 119
3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} \, dx$ 124
3.11 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^5 \, dx$ 130
3.12 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^4 \, dx$ 135
3.13 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^3 \, dx$ 139
3.14 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^2 \, dx$ 143
3.15 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx)) \, dx$ 147
3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)} \, dx$ 151
3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2} \, dx$ 156
3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} \, dx$ 161
3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} \, dx$ 166
3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5} \, dx$ 172
3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2} \, dx$ 178
3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2} \, dx$ 184
3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2} \, dx$ 190

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$ . . . 195
3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$ . . . 199
3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$ . . . 203
3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$ . . . 207
3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$ . . . 211
3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$ . . . 215
3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$ . . . 220
3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$ . . . 225
3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$ . . . 231
3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$ . . . 237
3.34 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$ . . . 242
3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$ . . . 247
3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$ . . . 252
3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$ . . . 257
3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$ . . . 261
3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$ . . . 265
3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$ . . . 269
3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$ . . . 275
3.42 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^4\,dx$ . . . 281
3.43 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^3\,dx$ . . . 287
3.44 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^2\,dx$ . . . 293
3.45 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))\,dx$ . . . 299
3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$ . . . 303
3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$ . . . 307
3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$ . . . 312
3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$ . . . 317
3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$ . . . 322
3.51 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx$ . . . 328
3.52 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx$ . . . 334
3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$ . . . 339
3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$ . . . 343
3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$ . . . 348
3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$ . . . 353
3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$ . . . 359
3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$ . . . 365

3.59 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx$ . . . 371
3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$ . . . 376
3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$ . . . 380
3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$ . . . 385
3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$ . . . 390
3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$ . . . 395
3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 400
3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 406
3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 411
3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 416
3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$ . . . 420
3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^2}\,dx$ . . . 425
3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^3}\,dx$ . . . 430
3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 436
3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 442
3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 447
3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 452
3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$ . . . 457
3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$ . . . 462
3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$ . . . 467
3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 473
3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 479
3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 485
3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 491
3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 497
3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$ . . . 503
3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$ . . . 509
3.86 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\,dx$ . . . 515
3.87 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx$ . . . 520
3.88 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx$ . . . 524
3.89 $\int \sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}\,dx$ . . . 528
3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 532

3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 535
3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 539
3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$ . . . 544
3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$ . . . 550
3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$ . . . 555
3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$ . . . 559
3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 563
3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 567
3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 571
3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$ . . . 576
3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$ . . . 582
3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$ . . . 587
3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$ . . . 592
3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 596
3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 600
3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 604
3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$ . . . 608
3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$ . . . 614
3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$ . . . 620
3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 626
3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 630
3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 634
3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 638
3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 641
3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 645
3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 649
3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 654
3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 659
3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 663

3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 667
3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 671
3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 675
3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 679
3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 684
3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 688
3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 692
3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$ . . . 697
3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$ . . . 702
3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$ . . . 707
3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$ . . . 712
3.131 $\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$ . . . 717
3.132 $\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$ . . . 720
3.133 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$ . . . 723
3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$ . . . 726
3.135 $\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$ . . . 729
3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$ . . . 732
3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$ . . . 735
3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$ . . . 738
3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$ . . . 741
3.140 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^n\,dx$ . . . 744
3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$ . . . 747
3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$ . . . 751
3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$ . . . 755
3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$ . . . 759
3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$ . . . 763
3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$ . . . 767
3.147 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^4\,dx$ . . . 772
3.148 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^3\,dx$ . . . 779
3.149 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2\,dx$ . . . 785
3.150 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))\,dx$ . . . 791
3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$ . . . 795
3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$ . . . 801

3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$ 808
3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$ 816
3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$ 823
3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$ 830
3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$ 835
3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$ 840
3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$ 847
3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$ 855
3.161 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx$ 860
3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$ 866
3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$ 871
3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$ 877
3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$ 885
3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$ 894
3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$ 900
3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$ 905
3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$ 909
3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2}\,dx$ 914
3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^3}\,dx$ 919
3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$ 924
3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$ 930
3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$ 936
3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$ 941
3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$ 947
3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$ 952
3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$ 957
3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$ 963
3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$ 969
3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx$ 974
3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$ 980
3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$ 985
3.184 $\int \sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}\,dx$ 990
3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$ 995

3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$ 999

3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$ 1004

3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$ 1009

3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$ 1014

3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$ 1018

3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$ 1024

3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$ 1031

3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$ 1037

3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$ 1045

3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$ 1054

3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$ 1062

3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$ 1071

3.198 $\int \sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\,dx$ 1082

3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$ 1087

3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$ 1091

3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$ 1097

3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$ 1101

3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$ 1107

3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$ 1111

3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$ 1115

3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$ 1121

3.207 $\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$ 1127

3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$ 1131

3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$ 1135

3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$ 1142

3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$ 1149

3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$ 1156

3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$ 1163

3.214 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$ 1171

3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$ 1178

3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$ 1186
3.217 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$ 1194
3.218 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$ 1197
3.219 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\,dx$ 1201
3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))^{3/2}}\,dx$ 1205
3.221 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$ 1212
3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$ 1215
3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$ 1218
3.224 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$ 1221
3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$ 1224
3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$ 1227
3.227 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$ 1230
3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$ 1233
3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$ 1236
3.230 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^m\,dx$ 1239
3.231 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^3\,dx$ 1244
3.232 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^2\,dx$ 1249
3.233 $\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))\,dx$ 1253
3.234 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$ 1257
3.235 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$ 1261
3.236 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^m\,dx$ 1265
3.237 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^3\,dx$ 1268
3.238 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^2\,dx$ 1273
3.239 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\,dx$ 1277
3.240 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$ 1280
3.241 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$ 1284

3.1 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=196

$$a^2c^5x-\frac{19a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan(e+fx)}{f}+\frac{17a^2c^5\sec(e+fx)\tan(e+fx)}{16f}+\frac{a^2c^5\sec^3(e+fx)\,t}{8f}$$

```
[Out] a^2*c^5*x-19/16*a^2*c^5*arctanh(sin(f*x+e))/f-a^2*c^5*tan(f*x+e)/f+17/16*a^
2*c^5*sec(f*x+e)*tan(f*x+e)/f+1/8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/3*a^2
*c^5*tan(f*x+e)^3/f-3/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^2*c^5*sec(f
*x+e)^3*tan(f*x+e)^3/f+3/5*a^2*c^5*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.22, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\frac{3a^2c^5\tan^5(e+fx)}{5f}+\frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{a^2c^5\tan(e+fx)}{f}-\frac{19a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{6f}+\frac{a^2c^5\tan(e+fx)\sec^3(e+fx)}{8f}-\frac{3a^2c^5\tan^3(e+fx)\sec(e+fx)}{4f}+\frac{17a^2c^5\tan(e+fx)\sec(e+fx)}{16f}+a^2c^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] a^2*c^5*x - (19*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (a^2*c^5*Tan[e + f*
x])/f + (17*a^2*c^5*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (a^2*c^5*Sec[e + f*
x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Tan[e + f*x]^3)/(3*f) - (3*a^2*c^5*Sec[
e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6
*f) + (3*a^2*c^5*Tan[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^5\,dx = (a^2c^2)\int (c - c\sec(e + fx))^3\tan^4(e + fx)\,dx$$

$$= (a^2c^2)\int \left(c^3\tan^4(e + fx) - 3c^3\sec(e + fx)\tan^4(e + fx) -\right.$$

$$= (a^2c^5)\int \tan^4(e + fx)\,dx - (a^2c^5)\int \sec^3(e + fx)\tan^4(e +$$

$$= \frac{a^2c^5\tan^3(e + fx)}{3f} - \frac{3a^2c^5\sec(e + fx)\tan^3(e + fx)}{4f} - \frac{a^2c^5}{}$$

$$= -\frac{a^2c^5\tan(e + fx)}{f} + \frac{9a^2c^5\sec(e + fx)\tan(e + fx)}{8f} + \frac{a^2c^5}{}$$

$$= a^2c^5x - \frac{9a^2c^5\tanh^{-1}(\sin(e + fx))}{8f} - \frac{a^2c^5\tan(e + fx)}{f} + \frac{1}{}$$

$$= a^2c^5x - \frac{19a^2c^5\tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^2c^5\tan(e + fx)}{f} +$$

Mathematica [A]
time = 2.10, size = 165, normalized size = 0.84

$$\frac{a^2c^5\sec^6(e+fx)\left(1200e+1200fx-4560\tanh^{-1}(\sin(e+fx))\cos^6(e+fx)+1800(e+fx)\cos(2(e+fx))+720e\cos(4(e+fx))+720fx\cos(4(e+fx))+120e\cos(6(e+fx))+120fx\cos(6(e+fx))-210\sin(e+fx)-120\sin(2(e+fx))+865\sin(3(e+fx))-768\sin(4(e+fx))+435\sin(5(e+fx))-88\sin(6(e+fx))\right)}{3840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*Sec[e + f*x]^6*(1200*e + 1200*f*x - 4560*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 72
0*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)]
- 210*Sin[e + f*x] - 120*Sin[2*(e + f*x)] + 865*Sin[3*(e + f*x)] - 768*Sin[
4*(e + f*x)] + 435*Sin[5*(e + f*x)] - 88*Sin[6*(e + f*x)]))/(3840*f)
```

Maple [A]
time = 0.18, size = 268, normalized size = 1.37

method	result
risch	$a^2c^5x - \frac{ic^5a^2\left(435\,e^{11i(fx+e)}-240\,e^{10i(fx+e)}+865\,e^{9i(fx+e)}+1200\,e^{8i(fx+e)}-210\,e^{7i(fx+e)}+1760\,e^{6i(fx+e)}+210\,e^{\dots}\right.}{120f\left(e^{2i(fx+e)}+1\right)^6}$
derivativedivides	$\frac{-c^5a^2\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)-3c^5a^2\left(-\frac{8}{15}-\frac{\left(\sec^4\right.}{}\right.}{}$
default	$\frac{-c^5a^2\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)-3c^5a^2\left(-\frac{8}{15}-\frac{\left(\sec^4\right.}{}\right.}{}$

norman	$\frac{a^2c^5x+a^2c^5x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-6a^2c^5x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15a^2c^5x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-20a^2c^5x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15a^2c^5x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-c^5*a^2*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f
*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))-3*c^5*a^2*(-8/15-1/5*sec(f*x+e)^4-4/1
5*sec(f*x+e)^2)*tan(f*x+e)-c^5*a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan
(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+5*c^5*a^2*(-2/3-1/3*sec(f*x+e)^2)*ta
n(f*x+e)+5*c^5*a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))
)+c^5*a^2*tan(f*x+e)-3*c^5*a^2*ln(sec(f*x+e)+tan(f*x+e))+c^5*a^2*(f*x+e))
```

Maxima [A]
time = 0.32, size = 361, normalized size = 1.84

$$\frac{96\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^5-800\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^5+480\left(fx+e\right)a^2c^5+5a^2c^5\left(\frac{2\left(15\sin\left(fx+e\right)^5-40\sin\left(fx+e\right)^3+33\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^6-3\sin\left(fx+e\right)^4+3\sin\left(fx+e\right)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)+30a^2c^5\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-600a^2c^5\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-1440a^2c^5\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+480a^2c^5\tan\left(fx+e\right)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/480*(96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^5
- 800*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 480*(f*x + e)*a^2*c^5 + 5
*a^2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(
f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e)
 + 1) + 15*log(sin(f*x + e) - 1)) + 30*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin
(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1
) + 3*log(sin(f*x + e) - 1)) - 600*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2
- 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 1440*a^2*c^5*log(se
c(f*x + e) + tan(f*x + e)) + 480*a^2*c^5*tan(f*x + e))/f
```

Fricas [A]
time = 3.41, size = 191, normalized size = 0.97

$$\frac{480\,a^2c^5fx\cos\left(fx+e\right)^6-285\,a^2c^5\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)+285\,a^2c^5\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(176\,a^2c^5\cos\left(fx+e\right)^5-435\,a^2c^5\cos\left(fx+e\right)^4+208\,a^2c^5\cos\left(fx+e\right)^3+110\,a^2c^5\cos\left(fx+e\right)^2-144\,a^2c^5\cos\left(fx+e\right)+40\,a^2c^5\right)\sin\left(fx+e\right)}{480\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^2*c^5*f*x*cos(f*x + e)^6 - 285*a^2*c^5*cos(f*x + e)^6*log(sin(
f*x + e) + 1) + 285*a^2*c^5*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(176*
a^2*c^5*cos(f*x + e)^5 - 435*a^2*c^5*cos(f*x + e)^4 + 208*a^2*c^5*cos(f*x +
 e)^3 + 110*a^2*c^5*cos(f*x + e)^2 - 144*a^2*c^5*cos(f*x + e) + 40*a^2*c^5)
*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^5\left(\int(-1)\,dx+\int 3\sec(e+fx)\,dx+\int\left(-\sec^2(e+fx)\right)\,dx+\int\left(-5\sec^3(e+fx)\right)\,dx+\int 5\sec^4(e+fx)\,dx+\int\sec^5(e+fx)\,dx+\int\left(-3\sec^6(e+fx)\right)\,dx+\int\sec^7(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-1, x) + Integral(3*sec(e + f*x), x) + Integral(-sec(e
 + f*x)**2, x) + Integral(-5*sec(e + f*x)**3, x) + Integral(5*sec(e + f*x)*
*4, x) + Integral(sec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + I
ntegral(sec(e + f*x)**7, x))
```

Giac [A]
time = 0.57, size = 191, normalized size = 0.97

$$\frac{240\,(fx+e)a^2c^5-285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(525\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-3135\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+1746\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-366\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-95\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+45\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^6}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)*a^2*c^5 - 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(525*a^2*c^5*tan(1/
2*f*x + 1/2*e)^11 - 3135*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 1746*a^2*c^5*tan(
1/2*f*x + 1/2*e)^7 - 366*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 - 95*a^2*c^5*tan(1/
2*f*x + 1/2*e)^3 + 45*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2
 - 1)^6)/f
```

Mupad [B]
time = 2.53, size = 228, normalized size = 1.16

$$a^2\,c^5\,x-\frac{-\frac{35\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{209\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{8}-\frac{291\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{20}+\frac{61\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20}+\frac{19\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24}-\frac{3\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-20\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}-\frac{19\,a^2\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5,x)
```

```
[Out] a^2*c^5*x - ((19*a^2*c^5*tan(e/2 + (f*x)/2)^3)/24 + (61*a^2*c^5*tan(e/2 + (
f*x)/2)^5)/20 - (291*a^2*c^5*tan(e/2 + (f*x)/2)^7)/20 + (209*a^2*c^5*tan(e/
2 + (f*x)/2)^9)/8 - (35*a^2*c^5*tan(e/2 + (f*x)/2)^11)/8 - (3*a^2*c^5*tan(e
/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20
*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 +
 tan(e/2 + (f*x)/2)^12 + 1)) - (19*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

3.2 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=140

$$a^2c^4x-\frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}+\frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2 \cdots}{}$$

```
[Out] a^2*c^4*x-3/4*a^2*c^4*arctanh(sin(f*x+e))/f-a^2*c^4*tan(f*x+e)/f+3/4*a^2*c^
4*sec(f*x+e)*tan(f*x+e)/f+1/3*a^2*c^4*tan(f*x+e)^3/f-1/2*a^2*c^4*sec(f*x+e)
*tan(f*x+e)^3/f+1/5*a^2*c^4*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\frac{a^2c^4\tan^5(e+fx)}{5f}+\frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\tan(e+fx)}{f}-\frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan^3(e+fx)\sec(e+fx)}{2f}+\frac{3a^2c^4\tan(e+fx)\sec(e+fx)}{4f}+a^2c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] a^2*c^4*x - (3*a^2*c^4*ArcTanh[Sin[e + f*x]])/(4*f) - (a^2*c^4*Tan[e + f*x]
)/f + (3*a^2*c^4*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (a^2*c^4*Tan[e + f*x]^3
)/(3*f) - (a^2*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (a^2*c^4*Tan[e + f*
x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= \left(a^2c^2\right)\int (c-c\sec(e+fx))^2\tan^4(e+fx)\,dx\\
&= \left(a^2c^2\right)\int \left(c^2\tan^4(e+fx)-2c^2\sec(e+fx)\tan^4(e+fx)\right.\\
&= \left(a^2c^4\right)\int \tan^4(e+fx)\,dx+\left(a^2c^4\right)\int \sec^2(e+fx)\tan^4(e\\
&= \frac{a^2c^4\tan^3(e+fx)}{3f}-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}-\left(a^2c\right.\\
&= -\frac{a^2c^4\tan(e+fx)}{f}+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}+\frac{a^2c}{}\\
&= a^2c^4x-\frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}+\end{aligned}$$

Mathematica [A]
time = 1.19, size = 146, normalized size = 1.04

$$\frac{a^2c^4\sec^5(e+fx)\left(600(e+fx)\cos(e+fx)-720\tanh^{-1}(\sin(e+fx))\cos^5(e+fx)+300e\cos(3(e+fx))+300fx\cos(3(e+fx))+60e\cos(5(e+fx))+60fx\cos(5(e+fx))+40\sin(e+fx)+60\sin(2(e+fx))-220\sin(3(e+fx))+150\sin(4(e+fx))-68\sin(5(e+fx))\right)}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*Sec[e + f*x]^5*(600*(e + f*x)*Cos[e + f*x] - 720*ArcTanh[Sin[e + f
*x]]*Cos[e + f*x]^5 + 300*e*Cos[3*(e + f*x)] + 300*f*x*Cos[3*(e + f*x)] + 6
0*e*Cos[5*(e + f*x)] + 60*f*x*Cos[5*(e + f*x)] + 40*Sin[e + f*x] + 60*Sin[2
*(e + f*x)] - 220*Sin[3*(e + f*x)] + 150*Sin[4*(e + f*x)] - 68*Sin[5*(e + f
*x)]))/(960*f)
```

Maple [A]
time = 0.10, size = 206, normalized size = 1.47

method	result
risch	$a^2c^4x-\frac{ic^4a^2\left(75\,e^{9i(fx+e)}+60\,e^{8i(fx+e)}+30\,e^{7i(fx+e)}+360\,e^{6i(fx+e)}+320\,e^{4i(fx+e)}-30\,e^{3i(fx+e)}+280\,e^{2i(fx+e)}-7\right.}{30f\left(e^{2i(fx+e)}+1\right)^5}$
derivativedivides	$\frac{-c^4a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-2c^4a^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(}{}\right.}{}$
default	$\frac{-c^4a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-2c^4a^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(}{}\right.}{}$
norman	$\frac{a^2c^4x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^2c^4x+5a^2c^4x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-10a^2c^4x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+10a^2c^4x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-5a^2c^4x\left(\right.}{\left(\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-c^4*a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*c^4*a
^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*
x+e)))+c^4*a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+4*c^4*a^2*(1/2*sec(f*x+e)
*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-c^4*a^2*tan(f*x+e)-2*c^4*a^2*ln(
sec(f*x+e)+tan(f*x+e))+c^4*a^2*(f*x+e))
```

Maxima [A]
time = 0.30, size = 259, normalized size = 1.85

$$\frac{8\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^4-40\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^4+120\left(fx+e\right)a^2c^4+15a^2c^4\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-120a^2c^4\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-240a^2c^4\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-120a^2c^4\tan\left(fx+e\right)}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4 -
 40*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 120*(f*x + e)*a^2*c^4 + 15*
a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
+ e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*
c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) - 240*a^2*c^4*log(sec(f*x + e) + tan(f*x + e)) - 120*a^2*c^4
*tan(f*x + e))/f
```

Fricas [**A**]
time = 2.65, size = 174, normalized size = 1.24

$$\frac{120\,a^2c^4fx\cos\left(fx+e\right)^5-45\,a^2c^4\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)+45\,a^2c^4\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\,\left(68\,a^2c^4\cos\left(fx+e\right)^4-75\,a^2c^4\cos\left(fx+e\right)^3+4\,a^2c^4\cos\left(fx+e\right)^2+30\,a^2c^4\cos\left(fx+e\right)-12\,a^2c^4\right)\sin\left(fx+e\right)}{120\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*c^4*f*x*cos(f*x + e)^5 - 45*a^2*c^4*cos(f*x + e)^5*log(sin(f
*x + e) + 1) + 45*a^2*c^4*cos(f*x + e)^5*log(-sin(f*x + e) + 1) - 2*(68*a^2
*c^4*cos(f*x + e)^4 - 75*a^2*c^4*cos(f*x + e)^3 + 4*a^2*c^4*cos(f*x + e)^2
+ 30*a^2*c^4*cos(f*x + e) - 12*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2c^4\left(\int 1\,dx+\int\left(-2\sec\left(e+fx\right)\right)\,dx+\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int 4\sec^3\left(e+fx\right)dx+\int\left(-\sec^4\left(e+fx\right)\right)\,dx+\int\left(-2\sec^5\left(e+fx\right)\right)\,dx+\int\sec^6\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(1, x) + Integral(-2*sec(e + f*x), x) + Integral(-sec(e
+ f*x)**2, x) + Integral(4*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4,
 x) + Integral(-2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [**A**]
time = 0.52, size = 172, normalized size = 1.23

$$\frac{60\,(fx+e)a^2c^4-45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\,\left(105\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-530\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+328\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-110\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(f*x + e)*a^2*c^4 - 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))
+ 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(105*a^2*c^4*tan(1/2*f*
x + 1/2*e)^9 - 530*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 328*a^2*c^4*tan(1/2*f*x
 + 1/2*e)^5 - 110*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^4*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

Mupad [B]
time = 2.30, size = 195, normalized size = 1.39

$$a^2\,c^4\,x+\frac{\frac{7\,a^2\,c^4\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{2}-\frac{53\,a^2\,c^4\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{3}+\frac{164\,a^2\,c^4\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{15}-\frac{11\,a^2\,c^4\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3}+\frac{a^2\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}-\frac{3\,a^2\,c^4\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4,x)
```

```
[Out] a^2*c^4*x + ((164*a^2*c^4*tan(e/2 + (f*x)/2)^5)/15 - (11*a^2*c^4*tan(e/2 +
(f*x)/2)^3)/3 - (53*a^2*c^4*tan(e/2 + (f*x)/2)^7)/3 + (7*a^2*c^4*tan(e/2 +
(f*x)/2)^9)/2 + (a^2*c^4*tan(e/2 + (f*x)/2))/2)/(f*(5*tan(e/2 + (f*x)/2)^2
- 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 - 1)) - (3*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(2*
f)
```

3.3 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=97

$$a^2c^3x-\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}$$

```
[Out] a^2*c^3*x-3/8*a^2*c^3*arctanh(sin(f*x+e))/f-1/8*a^2*(8*c^3-3*c^3*sec(f*x+e)
)*tan(f*x+e)/f+1/12*a^2*(4*c^3-3*c^3*sec(f*x+e))*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3966, 3855}

$$-\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2\tan^3(e+fx)\,(4c^3-3c^3\sec(e+fx))}{12f}-\frac{a^2\tan(e+fx)\,(8c^3-3c^3\sec(e+fx))}{8f}+a^2c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a^2*c^3*x - (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (a^2*(8*c^3 - 3*c^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (a^2*(4*c^3 - 3*c^3*Sec[e + f*x])*Tan[e
 + f*x]^3)/(12*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = (a^2c^2)\int (c-c\sec(e+fx))\tan^4(e+fx)\,dx$$
$$= \frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f} - \frac{1}{4}(a^2c^2)\int (4c-3$$
$$= -\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f} + \frac{a^2(4c^3-3c^3\sec($$
$$= a^2c^3x - \frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f} + \frac{a^2(4c^3-3$$
$$= a^2c^3x - \frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f} - \frac{a^2(8c^3-3c^3\sec(e+f}{8f}$$

Mathematica [A]
time = 0.79, size = 122, normalized size = 1.26

$$\frac{a^2c^3\sec^4(e+fx)\left(72e+72fx-72\tanh^{-1}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24e\cos(4(e+fx))+24fx\cos(4(e+fx))-18\sin(e+fx)-32\sin(2(e+fx))+30\sin(3(e+fx))-32\sin(4(e+fx))\right)}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*Sec[e + f*x]^4*(72*e + 72*f*x - 72*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[
4*(e + f*x)] - 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] + 30*Sin[3*(e + f*x)]
- 32*Sin[4*(e + f*x)]))/(192*f)
```

Maple [A]
time = 0.08, size = 171, normalized size = 1.76

method	result
risch	$a^2c^3x - \frac{ia^2c^3\left(15\,e^{7i(fx+e)}+48\,e^{6i(fx+e)}-9\,e^{5i(fx+e)}+96\,e^{4i(fx+e)}+9\,e^{3i(fx+e)}+80\,e^{2i(fx+e)}-15\,e^{i(fx+e)}+32\right)}{12f\left(e^{2i(fx+e)}+1\right)^4} -$
derivativedivides	$\frac{-c^3a^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^3a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e}{}$
default	$\frac{-c^3a^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^3a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e}{}$
norman	$\frac{a^2c^3x+a^2c^3x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-4a^2c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+6a^2c^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-4a^2c^3x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{5c^3a^2\tan\left(\frac{f}{}\right.}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-c^3*a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*
x+e)+tan(f*x+e)))-c^3*a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*c^3*a^2*(1/2
*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2*c^3*a^2*tan(f*x+e)-
c^3*a^2*ln(sec(f*x+e)+tan(f*x+e))+c^3*a^2*(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(96) = 192$.
time = 0.29, size = 219, normalized size = 2.26

$$\frac{16\,(\tan(fx+e)^3+3\,\tan(fx+e))a^2c^3+48\,(fx+e)a^2c^3+3\,a^2c^3\left(\frac{2\left(3\,\sin(fx+e)^3-5\,\sin(fx+e)\right)}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}-3\,\log(\sin(fx+e)+1)+3\,\log(\sin(fx+e)-1)\right)-24\,a^2c^3\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-48\,a^2c^3\log(\sec(fx+e)+\tan(fx+e))-96\,a^2c^3\tan(fx+e)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 48*(f*x + e)*a^2*c^3 +
 3*a^2*c^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 24*a^
2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) - 48*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) - 96*a^2*c^3
*tan(f*x + e))/f
```

Fricas [A]
time = 2.38, size = 157, normalized size = 1.62

$$\frac{48\,a^2c^3fx\cos(fx+e)^4-9\,a^2c^3\cos(fx+e)^4\log(\sin(fx+e)+1)+9\,a^2c^3\cos(fx+e)^4\log(-\sin(fx+e)+1)-2\left(32\,a^2c^3\cos(fx+e)^3-15\,a^2c^3\cos(fx+e)^2-8\,a^2c^3\cos(fx+e)+6\,a^2c^3\right)\sin(fx+e)}{48\,f\cos(fx+e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^2*c^3*f*x*cos(f*x + e)^4 - 9*a^2*c^3*cos(f*x + e)^4*log(sin(f*x
+ e) + 1) + 9*a^2*c^3*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^2*c^3
*cos(f*x + e)^3 - 15*a^2*c^3*cos(f*x + e)^2 - 8*a^2*c^3*cos(f*x + e) + 6*a^
2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^3\left(\int(-1)\,dx+\int\sec(e+fx)\,dx+\int 2\sec^2(e+fx)\,dx+\int\left(-2\sec^3(e+fx)\right)dx+\int\left(-\sec^4(e+fx)\right)dx+\int\sec^5(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*c**3*(Integral(-1, x) + Integral(sec(e + f*x), x) + Integral(2*sec(e
+ f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4
, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A]
time = 0.53, size = 153, normalized size = 1.58

$$\frac{24\,(fx+e)a^2c^3 - 9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(33\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-137\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+71\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)*a^2*c^3 - 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) +
 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(33*a^2*c^3*tan(1/2*f*x +
 1/2*e)^7 - 137*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 71*a^2*c^3*tan(1/2*f*x + 1
/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/
f
```

Mupad [B]
time = 2.19, size = 163, normalized size = 1.68

$$\frac{\frac{11\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{137\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{71\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}-\frac{5\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}+a^2\,c^3\,x-\frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3,x)
```

```
[Out] ((71*a^2*c^3*tan(e/2 + (f*x)/2)^3)/12 - (137*a^2*c^3*tan(e/2 + (f*x)/2)^5)/
12 + (11*a^2*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^2*c^3*tan(e/2 + (f*x)/2))/4
)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^2*c^3*x - (3*a^2*c^3*atanh(tan(e/2 + (
f*x)/2)))/(4*f)
```

3.4 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=47

$$a^2c^2x-\frac{a^2c^2\tan(e+fx)}{f}+\frac{a^2c^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*c^2*x-a^2*c^2*tan(f*x+e)/f+1/3*a^2*c^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$\frac{a^2c^2\tan^3(e+fx)}{3f}-\frac{a^2c^2\tan(e+fx)}{f}+a^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^2*c^2*x - (a^2*c^2*Tan[e + f*x])/f + (a^2*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = (a^2c^2)\int \tan^4(e+fx)\,dx$$
$$= \frac{a^2c^2\tan^3(e+fx)}{3f} - (a^2c^2)\int \tan^2(e+fx)\,dx$$
$$= -\frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f} + (a^2c^2)\int 1\,dx$$
$$= a^2c^2x - \frac{a^2c^2\tan(e+fx)}{f} + \frac{a^2c^2\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.96

$$a^2c^2\left(\frac{\text{ArcTan}(\tan(e+fx))}{f} - \frac{\tan(e+fx)}{f} + \frac{\tan^3(e+fx)}{3f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
[Out] a^2*c^2*(ArcTan[Tan[e + f*x]]/f - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

Maple [A]
time = 0.05, size = 58, normalized size = 1.23

method	result
derivativedivides	$\frac{-a^2c^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-2a^2c^2\tan(fx+e)+a^2c^2(fx+e)}{f}$
default	$\frac{-a^2c^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-2a^2c^2\tan(fx+e)+a^2c^2(fx+e)}{f}$
risch	$a^2c^2x-\frac{4ia^2c^2\left(3\,\mathrm{e}^{4i(fx+e)}+3\,\mathrm{e}^{2i(fx+e)}+2\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2c^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^2c^2x+\frac{2a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{20a^2c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+\frac{2a^2c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+3a^2c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(-a^2*c^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^2*c^2*tan(f*x+e)+a^2*c
^2*(f*x+e))
```

Maxima [A]
time = 0.30, size = 61, normalized size = 1.30

$$\frac{\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{2}c^{2}+3\left(fx+e\right)a^{2}c^{2}-6\,a^{2}c^{2}\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2 + 3*(f*x + e)*a^2*c^2 - 6*a^
2*c^2*tan(f*x + e))/f
```

Fricas [A]
time = 2.50, size = 69, normalized size = 1.47

$$\frac{3\,a^{2}c^{2}fx\cos\left(fx+e\right)^{3}-\left(4\,a^{2}c^{2}\cos\left(fx+e\right)^{2}-a^{2}c^{2}\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*c^2*f*x*cos(f*x + e)^3 - (4*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*si
n(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}c^{2}\left(\int 1\,dx+\int\left(-2\sec^{2}\left(e+fx\right)\right)\,dx+\int\sec^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*c**2*(Integral(1, x) + Integral(-2*sec(e + f*x)**2, x) + Integral(sec(
e + f*x)**4, x))
```

Giac [A]
time = 0.48, size = 48, normalized size = 1.02

$$\frac{a^{2}c^{2}\tan\left(fx+e\right)^{3}+3\left(fx+e\right)a^{2}c^{2}-3\,a^{2}c^{2}\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*c^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2*c^2 - 3*a^2*c^2*tan(f*x + e))
/f
```

Mupad [B]
time = 3.71, size = 84, normalized size = 1.79

$$a^2\,c^2\,x+\frac{2\,a^2\,c^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{20\,a^2\,c^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^2\,c^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2,x)
```

```
[Out] a^2*c^2*x + (2*a^2*c^2*tan(e/2 + (f*x)/2)^5 - (20*a^2*c^2*tan(e/2 + (f*x)/2
)^3)/3 + 2*a^2*c^2*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

3.5 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=55

$$a^2cx+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}$$

```
[Out] a^2*c*x+1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*c*(2*a^2+a^2*sec(f*x+e))*tan(f*
x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3989, 3966, 3855}

$$\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{c\tan(e+fx)\left(a^2\sec(e+fx)+2a^2\right)}{2f}+a^2cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] a^2*c*x + (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (c*(2*a^2 + a^2*Sec[e + f*x
])*Tan[e + f*x])/(2*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int (a+a\sec(e+fx))\tan^2(e+fx)\,dx\right)$$
$$= -\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}+\frac{1}{2}(ac)\int (2a+a\sec($$
$$= a^2cx-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}+\frac{1}{2}(a^2c)\int \sec($$
$$= a^2cx+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{c(2a^2+a^2\sec(e+fx))\tan}{2f}$$

Mathematica [A]
time = 0.32, size = 72, normalized size = 1.31

$$\frac{a^2c\sec^2(e+fx)\left(e+fx+\tanh^{-1}(\sin(e+fx))\cos^2(e+fx)+(e+fx)\cos(2(e+fx))-\sin(e+fx)-\sin(2(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
[Out] (a^2*c*Sec[e + f*x]^2*(e + f*x + ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^2 + (e
+ f*x)*Cos[2*(e + f*x)] - Sin[e + f*x] - Sin[2*(e + f*x)]))/(2*f)
```

Maple [A]
time = 0.04, size = 84, normalized size = 1.53

method	result
derivativedivides	$\frac{-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2c\tan(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c(fx+e)}{f}$
default	$\frac{-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2c\tan(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c(fx+e)}{f}$
risch	$a^2cx+\frac{ia^2c(e^{3i(fx+e)}-2e^{2i(fx+e)}-e^{i(fx+e)}-2)}{f(e^{2i(fx+e)}+1)^2}+\frac{a^2c\ln(e^{i(fx+e)}+i)}{2f}-\frac{a^2c\ln(e^{i(fx+e)}-i)}{2f}$
norman	$\frac{a^2cx+\frac{a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+a^2cx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{3a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-2a^2cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(-a^2*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a^2*c
*tan(f*x+e)+a^2*c*ln(sec(f*x+e)+tan(f*x+e))+a^2*c*(f*x+e))
```

Maxima [A]
time = 0.28, size = 103, normalized size = 1.87

$$\frac{4(fx+e)a^2c+a^2c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+4a^2c\log(\sec(fx+e)+\tan(fx+e))-4a^2c\tan(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^2*c + a^2*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(s
in(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 4*a^2*c*log(sec(f*x + e) + tan(
f*x + e)) - 4*a^2*c*tan(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(54) = 108.
time = 2.73, size = 111, normalized size = 2.02

$$\frac{4a^2cfx\cos(fx+e)^2+a^2c\cos(fx+e)^2\log(\sin(fx+e)+1)-a^2c\cos(fx+e)^2\log(-\sin(fx+e)+1)-2(2a^2c\cos(fx+e)+a^2c)\sin(fx+e)}{4f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*c*f*x*cos(f*x + e)^2 + a^2*c*cos(f*x + e)^2*log(sin(f*x + e) + 1
) - a^2*c*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*(2*a^2*c*cos(f*x + e) +
 a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c\left(\int(-1)\,dx+\int(-\sec(e+fx))\,dx+\int\sec^2(e+fx)\,dx+\int\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-1, x) + Integral(-sec(e + f*x), x) + Integral(sec(e + f*
x)**2, x) + Integral(sec(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(51) = 102.
time = 0.48, size = 103, normalized size = 1.87

$$\frac{2(fx+e)a^2c+a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)+\frac{2\left(a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)*a^2*c + a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - a^2*c*l
og(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2
*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

Mupad [B]
time = 1.51, size = 91, normalized size = 1.65

$$a^2\,c\,x-\frac{3\,a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)),x)
```

```
[Out] a^2*c*x - (3*a^2*c*tan(e/2 + (f*x)/2) - a^2*c*tan(e/2 + (f*x)/2)^3)/(f*(tan
(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) + (a^2*c*atanh(tan(e/2 + (
f*x)/2)))/f
```

3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2x}{c}-\frac{a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}$$

```
[Out] a^2*x/c-a^2*arctanh(sin(f*x+e))/c/f-4*a^2*tan(f*x+e)/c/f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.12, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3988, 3862, 8, 3879, 3874, 3855}

$$-\frac{a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}+\frac{a^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*x)/c - (a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (4*a^2*Tan[e + f*x])/(c*f*(
1 - Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx &= \frac{\int \left(\frac{a^2}{1-\sec(e+fx)}+\frac{2a^2\sec(e+fx)}{1-\sec(e+fx)}+\frac{a^2\sec^2(e+fx)}{1-\sec(e+fx)}\right)dx}{c} \\
&= \frac{a^2\int \frac{1}{1-\sec(e+fx)}\,dx}{c}+\frac{a^2\int \frac{\sec^2(e+fx)}{1-\sec(e+fx)}\,dx}{c}+\frac{(2a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c} \\
&= -\frac{3a^2\tan(e+fx)}{cf(1-\sec(e+fx))}-\frac{a^2\int -1\,dx}{c}-\frac{a^2\int \sec(e+fx)\,dx}{c}+\frac{a^2\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{c} \\
&= \frac{a^2x}{c}-\frac{a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(56) = 112.
time = 0.31, size = 169, normalized size = 3.02

$$\frac{a^2\csc\left(\frac{e}{2}\right)\left(-\cos\left(\frac{fx}{2}\right)\left(fx+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\cos\left(e+\frac{fx}{2}\right)\left(fx+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+8\sin\left(\frac{fx}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{cf(-1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*Csc[e/2]*(-(Cos[(f*x)/2]*(f*x + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + Cos[e + (f*x)/2]*(f*x + L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]) + 8*Sin[(f*x)/2]*Sin[(e + f*x)/2]))/(c*f*(-1 + Cos[e + f*x]))
```

Maple [A]
time = 0.12, size = 64, normalized size = 1.14

method	result
derivativedivides	$\frac{4a^2\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$
default	$\frac{4a^2\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$
risch	$\frac{a^2x}{c}+\frac{8ia^2}{fc(e^{i(fx+e)}-1)}+\frac{a^2\ln(e^{i(fx+e)}-i)}{cf}-\frac{a^2\ln(e^{i(fx+e)}+i)}{cf}$
norman	$\frac{\frac{a^2x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4a^2}{cf}+\frac{4a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a^2/c*(1/4*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1/2*f*x+1/2*e)+1/2*arctan(tan
(1/2*f*x+1/2*e))-1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(57) = 114$.
time = 0.51, size = 165, normalized size = 2.95

$$\frac{a^2\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}+\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)+\frac{2a^2(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] (a^2*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*s
in(f*x + e))) - a^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) +
2*a^2*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Fricas [A]
time = 3.15, size = 94, normalized size = 1.68

$$\frac{2\,a^2fx\sin(fx+e)-a^2\log(\sin(fx+e)+1)\sin(fx+e)+a^2\log(-\sin(fx+e)+1)\sin(fx+e)+8\,a^2\cos(fx+e)+8\,a^2}{2\,cf\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*f*x*sin(f*x + e) - a^2*log(sin(f*x + e) + 1)*sin(f*x + e) + a^2*
log(-sin(f*x + e) + 1)*sin(f*x + e) + 8*a^2*cos(f*x + e) + 8*a^2)/(c*f*sin(
f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Giac [A]
time = 0.50, size = 77, normalized size = 1.38

$$\frac{\frac{(fx+e)a^2}{c}-\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c}+\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c}+\frac{4a^2}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/c - a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + a^2*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/c + 4*a^2/(c*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B]
time = 1.48, size = 46, normalized size = 0.82

$$\frac{a^2\,x}{c}-\frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4}{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x)),x)
```

```
[Out] (a^2*x)/c - (a^2*(2*atanh(tan(e/2 + (f*x)/2)) - 4/tan(e/2 + (f*x)/2)))/(c*f
)
```

3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

```
[Out] a^2*x/c^2-4/3*a^2*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2-4/3*a^2*tan(f*x+e)/c^2/
f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3988, 3862, 4004, 3879, 3881, 3882}

$$-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^2x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 - (4*a^2*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x])^2) - (4*a^2*
Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx &= \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^2}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^2}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^2}\right)dx}{c^2}\\
&= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}\\
&= -\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{a^2\int \frac{-3-\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}\\
&= \frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{(4a^2)\int \frac{\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}\\
&= \frac{a^2x}{c^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 53, normalized size = 0.75

$$
-\frac{2a^2\cot^3\left(\frac{e}{2}+\frac{fx}{2}\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3c^2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*Cot[e/2 + (f*x)/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e/2 + (f
*x)/2]^2])/(3*c^2*f)
```

Maple [A]
time = 0.13, size = 47, normalized size = 0.66

method	result	size
derivativedivides	$\frac{2a^2\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^2}$	47
default	$\frac{2a^2\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^2}$	47
risch	$\frac{a^2x}{c^2}+\frac{8ia^2\left(3\,e^{2i(fx+e)}-3\,e^{i(fx+e)}+2\right)}{3f\,c^2\left(e^{i(fx+e)}-1\right)^3}$	59
norman	$\frac{\frac{a^2x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{2a^2}{3cf}-\frac{8a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{a^2x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^2/c^2*(arctan(tan(1/2*f*x+1/2*e))-1/3/tan(1/2*f*x+1/2*e)^3+1/tan(1/2*
f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(67) = 134.
time = 0.52, size = 188, normalized size = 2.65

$$\frac{a^2\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}+\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right)-\frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}+\frac{2a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - a^
2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin
(f*x + e)^3) + 2*a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x +
 e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Fricas [A]
time = 1.82, size = 94, normalized size = 1.32

$$\frac{8\,a^2\cos\left(fx+e\right)^2+4\,a^2\cos\left(fx+e\right)-4\,a^2+3\left(a^2fx\cos\left(fx+e\right)-a^2fx\right)\sin\left(fx+e\right)}{3\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(8*a^2*cos(f*x + e)^2 + 4*a^2*cos(f*x + e) - 4*a^2 + 3*(a^2*f*x*cos(f*x
 + e) - a^2*f*x)*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\frac{2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{1}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A]
time = 0.48, size = 57, normalized size = 0.80

$$\frac{\frac{3\,(fx+e)a^2}{c^2}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^2/c^2 + 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2)/(c^2*tan(
1/2*f*x + 1/2*e)^3))/f
```

Mupad [B]
time = 1.39, size = 40, normalized size = 0.56

$$\frac{a^2\left(-2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+6\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)+3\,f\,x\right)}{3\,c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^2,x)
```

```
[Out] (a^2*(6*cot(e/2 + (f*x)/2) - 2*cot(e/2 + (f*x)/2)^3 + 3*f*x))/(3*c^2*f)
```

3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{a^2x}{c^3}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

```
[Out] a^2*x/c^3-4/5*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3-8/15*a^2*tan(f*x+e)/c^3
/f/(1-sec(f*x+e))^2-23/15*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.27, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$-\frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^2x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*x)/c^3 - (4*a^2*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) - (8*a^2*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (23*a^2*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx &= \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^3}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^3}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^3}\right)dx}{c^3} \\
&= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3} \\
&= -\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{a^2\int \frac{-5-2\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}-\frac{(3a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}+ \\
&= -\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}+\frac{a^2\int \frac{15+7\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}- \\
&= \frac{a^2x}{c^3}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{a^2\tan(e+f}{15c^3f(1-\sec(e} \\
&= \frac{a^2x}{c^3}-\frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{23a^2\tan(e+}{15c^3f(1-\sec(e}
\end{aligned}
$$

Mathematica [A]

time = 0.65, size = 171, normalized size = 1.68

$$
\frac{a^2\csc\left(\frac{e}{2}\right)\csc^5\left(\frac{1}{2}(e+fx)\right)\left(150fx\cos\left(\frac{fx}{2}\right)-150fx\cos\left(e+\frac{fx}{2}\right)-75fx\cos\left(e+\frac{3fx}{2}\right)+75fx\cos\left(2e+\frac{3fx}{2}\right)+15fx\cos\left(2e+\frac{5fx}{2}\right)-15fx\cos\left(3e+\frac{5fx}{2}\right)-500\sin\left(\frac{fx}{2}\right)-360\sin\left(e+\frac{fx}{2}\right)+280\sin\left(e+\frac{3fx}{2}\right)+150\sin\left(2e+\frac{3fx}{2}\right)-86\sin\left(2e+\frac{5fx}{2}\right)\right)}{480c^3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^5*(150*f*x*Cos[(f*x)/2] - 150*f*x*Cos[e + (f
*x)/2] - 75*f*x*Cos[e + (3*f*x)/2] + 75*f*x*Cos[2*e + (3*f*x)/2] + 15*f*x*C
os[2*e + (5*f*x)/2] - 15*f*x*Cos[3*e + (5*f*x)/2] - 500*Sin[(f*x)/2] - 360*
Sin[e + (f*x)/2] + 280*Sin[e + (3*f*x)/2] + 150*Sin[2*e + (3*f*x)/2] - 86*S
in[2*e + (5*f*x)/2]))/(480*c^3*f)
```

Maple [A]

time = 0.14, size = 63, normalized size = 0.62

method	result	size
derivativedivides	$\frac{a^2\left(2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$	63
default	$\frac{a^2\left(2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$	63
risch	$\frac{a^2x}{c^3}+\frac{2ia^2\left(75\,e^{4i(fx+e)}-180\,e^{3i(fx+e)}+250\,e^{2i(fx+e)}-140\,e^{i(fx+e)}+43\right)}{15f\,c^3\left(e^{i(fx+e)}-1\right)^5}$	81

norman	$\frac{\frac{a^2x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{a^2}{5cf}+\frac{13a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{8a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{a^2x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2/c^3*(2*arctan(tan(1/2*f*x+1/2*e))+1/5/tan(1/2*f*x+1/2*e)^5-2/3/tan(
1/2*f*x+1/2*e)^3+2/tan(1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(96) = 192$.
time = 0.50, size = 233, normalized size = 2.28

$$\frac{a^2\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)-\frac{2a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}-\frac{3a^2\left(\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(a^2*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(c
os(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) - 2*a^2*(10*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) +
 1)^5/(c^3*sin(f*x + e)^5) - 3*a^2*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Fricas [A]
time = 2.68, size = 137, normalized size = 1.34

$$\frac{43\,a^2\cos\left(fx+e\right)^3-11\,a^2\cos\left(fx+e\right)^2-31\,a^2\cos\left(fx+e\right)+23\,a^2+15\,\left(a^2fx\cos\left(fx+e\right)^2-2\,a^2fx\cos\left(fx+e\right)+a^2fx\right)\sin\left(fx+e\right)}{15\,\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(43*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 31*a^2*cos(f*x + e) +
 23*a^2 + 15*(a^2*f*x*cos(f*x + e)^2 - 2*a^2*f*x*cos(f*x + e) + a^2*f*x)*si
n(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int\frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{1}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec
(e + f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**3 - 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A]
time = 0.52, size = 72, normalized size = 0.71

$$\frac{\frac{15\,(fx+e)a^2}{c^3}+\frac{30\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^2}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*a^2/c^3 + (30*a^2*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*tan(1/
2*f*x + 1/2*e)^2 + 3*a^2)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B]
time = 1.45, size = 96, normalized size = 0.94

$$\frac{a^2\,x}{c^3}+\frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^3,x)
```

```
[Out] (a^2*x)/c^3 + ((a^2*cos(e/2 + (f*x)/2)^5)/5 + 2*a^2*cos(e/2 + (f*x)/2)*sin(
e/2 + (f*x)/2)^4 - (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)/(c^
3*f*sin(e/2 + (f*x)/2)^5)
```

3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=133

$$\frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx$$

```
[Out] a^2*x/c^4-4/7*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4-12/35*a^2*tan(f*x+e)/c^
4/f/(1-sec(f*x+e))^3-59/105*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-164/105*a
^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.33, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$-\frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^2x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*x)/c^4 - (4*a^2*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) - (12*a^2
*Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (59*a^2*Tan[e + f*x])/(105
*c^4*f*(1 - Sec[e + f*x])^2) - (164*a^2*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
 + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^4}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^4}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}$$

$$= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{a^2\int \frac{-7-3\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}-\frac{(4a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}+$$

$$= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}+\frac{a^2\int \frac{35+20\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}-$$

$$= -\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx}$$

$$= \frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+}{105c^4f(1-\sec(e}$$

$$= \frac{a^2x}{c^4}-\frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{59a^2\tan(e+}{105c^4f(1-\sec(e}$$

Mathematica [A]
time = 0.68, size = 227, normalized size = 1.71

$$\frac{a^2\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\left(3675fx\cos\left(\frac{fx}{2}\right)-3675fx\cos\left(e+\frac{fx}{2}\right)-2205fx\cos\left(e+\frac{3fx}{2}\right)+2205fx\cos\left(2e+\frac{3fx}{2}\right)+735fx\cos\left(2e+\frac{5fx}{2}\right)-735fx\cos\left(3e+\frac{5fx}{2}\right)-105fx\cos\left(3e+\frac{7fx}{2}\right)+105fx\cos\left(4e+\frac{7fx}{2}\right)-11900\sin\left(\frac{fx}{2}\right)-10430\sin\left(e+\frac{fx}{2}\right)+8568\sin\left(e+\frac{3fx}{2}\right)+4830\sin\left(2e+\frac{3fx}{2}\right)-3206\sin\left(2e+\frac{5fx}{2}\right)-1260\sin\left(3e+\frac{5fx}{2}\right)+638\sin\left(3e+\frac{7fx}{2}\right)\right)}{13440c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^7*(3675*f*x*Cos[(f*x)/2] - 3675*f*x*Cos[e +
(f*x)/2] - 2205*f*x*Cos[e + (3*f*x)/2] + 2205*f*x*Cos[2*e + (3*f*x)/2] + 73
5*f*x*Cos[2*e + (5*f*x)/2] - 735*f*x*Cos[3*e + (5*f*x)/2] - 105*f*x*Cos[3*e
 + (7*f*x)/2] + 105*f*x*Cos[4*e + (7*f*x)/2] - 11900*Sin[(f*x)/2] - 10430*S
in[e + (f*x)/2] + 8568*Sin[e + (3*f*x)/2] + 4830*Sin[2*e + (3*f*x)/2] - 320
6*Sin[2*e + (5*f*x)/2] - 1260*Sin[3*e + (5*f*x)/2] + 638*Sin[3*e + (7*f*x)/
2]))/(13440*c^4*f)
```

Maple [A]
time = 0.16, size = 77, normalized size = 0.58

method	result
derivativedivides	$\frac{a^2\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^4}$

default	$\frac{a^2\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2f\,c^4}$
risch	$\frac{a^2x}{c^4}+\frac{2ia^2\left(630\,\mathrm{e}^{6i(fx+e)}-2415\,\mathrm{e}^{5i(fx+e)}+5215\,\mathrm{e}^{4i(fx+e)}-5950\,\mathrm{e}^{3i(fx+e)}+4284\,\mathrm{e}^{2i(fx+e)}-1603\,\mathrm{e}^{i(fx+e)}+319\right)}{105f\,c^4\left(\mathrm{e}^{i(fx+e)}-1\right)^7}$
norman	$\frac{\frac{a^2x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{a^2}{14cf}-\frac{13a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35cf}+\frac{29a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{30cf}-\frac{8a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{a^2x(\mathrm{ta}}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*a^2/c^4*(4*arctan(tan(1/2*f*x+1/2*e))-1/7/tan(1/2*f*x+1/2*e)^7+3/5/ta
n(1/2*f*x+1/2*e)^5-4/3/tan(1/2*f*x+1/2*e)^3+4/tan(1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(125) = 250$.
time = 0.50, size = 320, normalized size = 2.41

$$\frac{5\,a^2\left(\frac{336\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{77\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right)+\frac{a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{6\,a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(5*a^2*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
 + e)^7)) + a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(
f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 6*a^2*(21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Fricas [A]
time = 3.96, size = 184, normalized size = 1.38

$$\frac{319\,a^2\cos(fx+e)^4-327\,a^2\cos(fx+e)^3-95\,a^2\cos(fx+e)^2+387\,a^2\cos(fx+e)-164\,a^2+105\left(a^2fx\cos(fx+e)^3-3\,a^2fx\cos(fx+e)^2+3\,a^2fx\cos(fx+e)-a^2fx\right)\sin(fx+e)}{105\left(c^4f\cos(fx+e)^3-3\,c^4f\cos(fx+e)^2+3\,c^4f\cos(fx+e)-c^4f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(319*a^2*cos(f*x + e)^4 - 327*a^2*cos(f*x + e)^3 - 95*a^2*cos(f*x + e
)^2 + 387*a^2*cos(f*x + e) - 164*a^2 + 105*(a^2*f*x*cos(f*x + e)^3 - 3*a^2*
f*x*cos(f*x + e)^2 + 3*a^2*f*x*cos(f*x + e) - a^2*f*x)*sin(f*x + e))/((c^4*
```

```
f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*s
in(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx+\int \frac{1}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(1/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec
(e + f*x) + 1), x))/c**4
```

Giac [A]
time = 0.52, size = 88, normalized size = 0.66

$$\frac{\frac{210\,(fx+e)a^2}{c^4}+\frac{420\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-140\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,a^2}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{210\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/210*(210*(f*x + e)*a^2/c^4 + (420*a^2*tan(1/2*f*x + 1/2*e)^6 - 140*a^2*ta
n(1/2*f*x + 1/2*e)^4 + 63*a^2*tan(1/2*f*x + 1/2*e)^2 - 15*a^2)/(c^4*tan(1/2
*f*x + 1/2*e)^7))/f
```

Mupad [B]
time = 1.50, size = 124, normalized size = 0.93

$$\frac{a^2\,x}{c^4}-\frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14}-\frac{3\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10}+\frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}-2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{c^4\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^4,x)
```

```
[Out] (a^2*x)/c^4 - ((a^2*cos(e/2 + (f*x)/2)^7)/14 - 2*a^2*cos(e/2 + (f*x)/2)*sin
(e/2 + (f*x)/2)^6 + (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4)/3 - (
3*a^2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/10)/(c^4*f*sin(e/2 + (f*x)
/2)^7)
```

3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=164

$$\frac{a^2x}{c^5}-\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+f}$$

```
[Out] a^2*x/c^5-4/9*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5-16/63*a^2*tan(f*x+e)/c^
5/f/(1-sec(f*x+e))^4-37/105*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-179/315*a
^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-494/315*a^2*tan(f*x+e)/c^5/f/(1-sec(f*
x+e))
```

Rubi [A]
time = 0.42, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$-\frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))}-\frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{a^2x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*x)/c^5 - (4*a^2*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) - (16*a^2
*Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105
*c^5*f*(1 - Sec[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
 + f*x])^2) - (494*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{\int \left(\frac{a^2}{(1-\sec(e+fx))^5}+\frac{2a^2\sec(e+fx)}{(1-\sec(e+fx))^5}+\frac{a^2\sec^2(e+fx)}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$= \frac{a^2\int \frac{1}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{a^2\int \frac{\sec^2(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{(2a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{a^2\int \frac{-9-4\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}-\frac{(5a^2)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}+$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}+\frac{a^2\int \frac{63+39\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{63c^5}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+f}$$

$$= -\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+f}$$

$$= \frac{a^2x}{c^5}-\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+}{105c^5f(1-\sec(}$$

$$= \frac{a^2x}{c^5}-\frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{37a^2\tan(e+}{105c^5f(1-\sec(}$$

Mathematica [A]
time = 0.93, size = 283, normalized size = 1.73

$$\frac{a^2\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(39690fx\cos\left(\frac{fx}{2}\right)-39690fx\cos\left(e+\frac{fx}{2}\right)-26460fx\cos\left(e+\frac{3fx}{2}\right)+26460fx\cos\left(2e+\frac{3fx}{2}\right)+11340fx\cos\left(2e+\frac{5fx}{2}\right)-11340fx\cos\left(3e+\frac{5fx}{2}\right)-2835fx\cos\left(3e+\frac{7fx}{2}\right)+2835fx\cos\left(4e+\frac{7fx}{2}\right)+315fx\cos\left(4e+\frac{9fx}{2}\right)-315fx\cos\left(5e+\frac{9fx}{2}\right)-135198\sin\left(\frac{fx}{2}\right)-117810\sin\left(e+\frac{fx}{2}\right)+100002\sin\left(e+\frac{3fx}{2}\right)+68670\sin\left(2e+\frac{3fx}{2}\right)-48978\sin\left(2e+\frac{5fx}{2}\right)-23310\sin\left(3e+\frac{5fx}{2}\right)+13662\sin\left(3e+\frac{7fx}{2}\right)+4410\sin\left(4e+\frac{7fx}{2}\right)-2008\sin\left(4e+\frac{9fx}{2}\right)\right)}{161280c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^9*(39690*f*x*Cos[(f*x)/2] - 39690*f*x*Cos[e
+ (f*x)/2] - 26460*f*x*Cos[e + (3*f*x)/2] + 26460*f*x*Cos[2*e + (3*f*x)/2]
+ 11340*f*x*Cos[2*e + (5*f*x)/2] - 11340*f*x*Cos[3*e + (5*f*x)/2] - 2835*f*
x*Cos[3*e + (7*f*x)/2] + 2835*f*x*Cos[4*e + (7*f*x)/2] + 315*f*x*Cos[4*e +
(9*f*x)/2] - 315*f*x*Cos[5*e + (9*f*x)/2] - 135198*Sin[(f*x)/2] - 117810*Si
n[e + (f*x)/2] + 100002*Sin[e + (3*f*x)/2] + 68670*Sin[2*e + (3*f*x)/2] - 4
8978*Sin[2*e + (5*f*x)/2] - 23310*Sin[3*e + (5*f*x)/2] + 13662*Sin[3*e + (7
*f*x)/2] + 4410*Sin[4*e + (7*f*x)/2] - 2008*Sin[4*e + (9*f*x)/2]))/(161280*
c^5*f)
```

Maple [A]
time = 0.17, size = 90, normalized size = 0.55

method	result
derivativedivides	$\frac{a^2\left(8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{4}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{8}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4f\,c^5}$
default	$\frac{a^2\left(8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{4}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{8}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4f\,c^5}$
risch	$\frac{a^2x}{c^5}+\frac{2ia^2\left(2205\,\mathrm{e}^{8i(fx+e)}-11655\,\mathrm{e}^{7i(fx+e)}+34335\,\mathrm{e}^{6i(fx+e)}-58905\,\mathrm{e}^{5i(fx+e)}+67599\,\mathrm{e}^{4i(fx+e)}-50001\,\mathrm{e}^{3i(fx+e)}+244\right.}{315f\,c^5\left(\mathrm{e}^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^2x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{a^2}{36cf}+\frac{43a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{252cf}-\frac{69a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{140cf}+\frac{61a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{60cf}-\frac{8a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2a^2\left(\mathrm{ta}\right.}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*a^2/c^5*(8*arctan(tan(1/2*f*x+1/2*e))+1/9/tan(1/2*f*x+1/2*e)^9-4/7/ta
n(1/2*f*x+1/2*e)^7+7/5/tan(1/2*f*x+1/2*e)^5-8/3/tan(1/2*f*x+1/2*e)^3+8/tan(
1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(154) = 308$.
time = 0.52, size = 365, normalized size = 2.23

$$\frac{a^2\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5}-\frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right)-\frac{2a^2\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}-\frac{5a^2\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}}{5040f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(a^2*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 2*a^2*(180*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e
) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^2*(18*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x
 + e)^9))/f
```

Fricas [A]
time = 3.22, size = 227, normalized size = 1.38

$$\frac{1004\,a^2\cos\left(fx+e\right)^5-1811\,a^2\cos\left(fx+e\right)^4+797\,a^2\cos\left(fx+e\right)^3+1457\,a^2\cos\left(fx+e\right)^2-1661\,a^2\cos\left(fx+e\right)+494\,a^2+315\left(a^2fx\cos\left(fx+e\right)^4-4\,a^2fx\cos\left(fx+e\right)^3+6\,a^2fx\cos\left(fx+e\right)^2-4\,a^2fx\cos\left(fx+e\right)+a^2fx\right)\sin\left(fx+e\right)}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(1004*a^2*cos(f*x + e)^5 - 1811*a^2*cos(f*x + e)^4 + 797*a^2*cos(f*x
+ e)^3 + 1457*a^2*cos(f*x + e)^2 - 1661*a^2*cos(f*x + e) + 494*a^2 + 315*(a
^2*f*x*cos(f*x + e)^4 - 4*a^2*f*x*cos(f*x + e)^3 + 6*a^2*f*x*cos(f*x + e)^2
 - 4*a^2*f*x*cos(f*x + e) + a^2*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 -
 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c
^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{1}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*se
c(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 -
5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x
) - 1), x))/c**5
```

Giac [A]
time = 0.55, size = 104, normalized size = 0.63

$$\frac{\frac{1260\,(fx+e)a^2}{c^5}+\frac{2520\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-840\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+441\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-180\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{1260\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/1260*(1260*(f*x + e)*a^2/c^5 + (2520*a^2*tan(1/2*f*x + 1/2*e)^8 - 840*a^2
*tan(1/2*f*x + 1/2*e)^6 + 441*a^2*tan(1/2*f*x + 1/2*e)^4 - 180*a^2*tan(1/2*
f*x + 1/2*e)^2 + 35*a^2)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

Mupad [B]
time = 1.54, size = 146, normalized size = 0.89

$$\frac{a^2\left(\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{36}-\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{7}+\frac{7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{20}-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3}+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\left(e+f\,x\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\right)}{c^5\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^5,x)
```

```
[Out] (a^2*(cos(e/2 + (f*x)/2)^9/36 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8 +
 sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2
)^6)/3 + (7*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/20 - (cos(e/2 + (f*x
)/2)^7*sin(e/2 + (f*x)/2)^2)/7))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.11 $\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^5\, dx$

Optimal. Leaf size=188

$$a^3c^5x - \frac{5a^3c^5\tanh^{-1}(\sin(e + fx))}{8f} - \frac{a^3c^5\tan(e + fx)}{f} + \frac{5a^3c^5\sec(e + fx)\tan(e + fx)}{8f} + \frac{a^3c^5\tan^3(e + fx)}{3f} - $$

```
[Out] a^3*c^5*x-5/8*a^3*c^5*arctanh(sin(f*x+e))/f-a^3*c^5*tan(f*x+e)/f+5/8*a^3*c^
5*sec(f*x+e)*tan(f*x+e)/f+1/3*a^3*c^5*tan(f*x+e)^3/f-5/12*a^3*c^5*sec(f*x+e
)*tan(f*x+e)^3/f-1/5*a^3*c^5*tan(f*x+e)^5/f+1/3*a^3*c^5*sec(f*x+e)*tan(f*x+
e)^5/f-1/7*a^3*c^5*tan(f*x+e)^7/f
```

Rubi [A]

time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{a^3c^5\tan^7(e+fx)}{7f}-\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{a^3c^5\tan(e+fx)}{f}-\frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^5\tan^5(e+fx)\sec(e+fx)}{3f}-\frac{5a^3c^5\tan^3(e+fx)\sec(e+fx)}{12f}+\frac{5a^3c^5\tan(e+fx)\sec(e+fx)}{8f}+a^3c^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] a^3*c^5*x - (5*a^3*c^5*ArcTanh[Sin[e + f*x]])/(8*f) - (a^3*c^5*Tan[e + f*x]
)/f + (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (a^3*c^5*Tan[e + f*x]^3
)/(3*f) - (5*a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(12*f) - (a^3*c^5*Tan[e +
 f*x]^5)/(5*f) + (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(3*f) - (a^3*c^5*Tan
[e + f*x]^7)/(7*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -\left(\left(a^3c^3\right)\int (c-c\sec(e+fx))^2\tan^6(e+fx)\,dx\right)$$

$$= -\left(\left(a^3c^3\right)\int \left(c^2\tan^6(e+fx)-2c^2\sec(e+fx)\tan^6(e+f\right.\right.$$

$$= -\left(\left(a^3c^5\right)\int \tan^6(e+fx)\,dx\right)-\left(a^3c^5\right)\int \sec^2(e+fx)\tan$$

$$= -\frac{a^3c^5\tan^5(e+fx)}{5f}+\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f}+\left(a^3\right.$$

$$= \frac{a^3c^5\tan^3(e+fx)}{3f}-\frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f}-\frac{a^3c^5}{}$$

$$= -\frac{a^3c^5\tan(e+fx)}{f}+\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^5}{}$$

$$= a^3c^5x-\frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c^5\tan(e+fx)}{f}+\frac{5}{}$$

Mathematica [A]

time = 2.33, size = 189, normalized size = 1.01

$$\frac{a^3c^5\sec^7(e+fx)\left(14700(e+fx)\cos(e+fx)-16800\tanh^{-1}(\sin(e+fx))\cos^7(e+fx)+8820e\cos(3(e+fx))+8820fx\cos(3(e+fx))+2940e\cos(5(e+fx))+2940fx\cos(5(e+fx))+420e\cos(7(e+fx))+420fx\cos(7(e+fx))-4200\sin(e+fx)+2975\sin(2(e+fx))-2184\sin(3(e+fx))+980\sin(4(e+fx))-2408\sin(5(e+fx))+1155\sin(6(e+fx))-584\sin(7(e+fx))\right)}{26880f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*Sec[e + f*x]^7*(14700*(e + f*x)*Cos[e + f*x] - 16800*ArcTanh[Sin[e
 + f*x]]*Cos[e + f*x]^7 + 8820*e*Cos[3*(e + f*x)] + 8820*f*x*Cos[3*(e + f*x
)] + 2940*e*Cos[5*(e + f*x)] + 2940*f*x*Cos[5*(e + f*x)] + 420*e*Cos[7*(e +
 f*x)] + 420*f*x*Cos[7*(e + f*x)] - 4200*Sin[e + f*x] + 2975*Sin[2*(e + f*x
)] - 2184*Sin[3*(e + f*x)] + 980*Sin[4*(e + f*x)] - 2408*Sin[5*(e + f*x)] +
 1155*Sin[6*(e + f*x)] - 584*Sin[7*(e + f*x)]))/(26880*f)
```

Maple [A]

time = 0.14, size = 288, normalized size = 1.53

method	result
risch	$a^3c^5x-\frac{ic^5a^3\left(1155\,\mathrm{e}^{13i(fx+e)}+1680\,\mathrm{e}^{12i(fx+e)}+980\,\mathrm{e}^{11i(fx+e)}+10080\,\mathrm{e}^{10i(fx+e)}+2975\,\mathrm{e}^{9i(fx+e)}+16240\,\mathrm{e}^{8i(fx+e)}\right.}{420f\left(\mathrm{e}^{2i(f\right.}}$
derivativedivides	$\frac{c^5a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+2c^5a^3\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5}{}\right.\right.}{}$

default	$\frac{c^5 a^3 \left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+2c^5a^3\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\,\mathrm{se}}{}\right.\right.}{}$
norman	$\frac{a^3c^5x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^3c^5x+7a^3c^5x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-21a^3c^5x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+35a^3c^5x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-35a^3c^5x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^5*a^3*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*
tan(f*x+e)+2*c^5*a^3*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e)
)*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))-2*c^5*a^3*(-8/15-1/5*sec(f*x+e
)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-6*c^5*a^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*
x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+6*c^5*a^3*(1/2*sec(f*x+e)*t
an(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2*c^5*a^3*tan(f*x+e)-2*c^5*a^3*ln(
sec(f*x+e)+tan(f*x+e))+c^5*a^3*(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(185) = 370$.
time = 0.29, size = 385, normalized size = 2.05

$$-\frac{48\left(5\tan\left(fx+e\right)^7+21\tan\left(fx+e\right)^5+35\tan\left(fx+e\right)^3+35\tan\left(fx+e\right)\right)a^3c^5-224\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^3c^5-1680\left(fx+e\right)a^3c^5+35\,a^3c^5\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-630\,a^3c^5\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)+2520\,a^3c^5\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+3360\,a^3c^5\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+3360\,a^3c^5\tan\left(fx+e\right)}{1680\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/1680*(48*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^3*c^5 - 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^3*c^5 - 1680*(f*x + e)*a^3*c^5 + 35*a^3*c^5*(2*(15*sin(f*x + e)
^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^
4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
- 1)) - 630*a^3*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) + 2520*a^3*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) +
1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^5*log(sec(f*x + e) + tan(f*x + e))
 + 3360*a^3*c^5*tan(f*x + e))/f
```

Fricas [A]
time = 2.93, size = 208, normalized size = 1.11

$$\frac{1680\,a^3c^5fx\cos\left(fx+e\right)^7-525\,a^3c^5\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)+525\,a^3c^5\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(1168\,a^3c^5\cos\left(fx+e\right)^6-1155\,a^3c^5\cos\left(fx+e\right)^5-256\,a^3c^5\cos\left(fx+e\right)^4+910\,a^3c^5\cos\left(fx+e\right)^3-192\,a^3c^5\cos\left(fx+e\right)^2-280\,a^3c^5\cos\left(fx+e\right)+120\,a^3c^5\right)\sin\left(fx+e\right)}{1680\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*a^3*c^5*f*x*cos(f*x + e)^7 - 525*a^3*c^5*cos(f*x + e)^7*log(si
n(f*x + e) + 1) + 525*a^3*c^5*cos(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(11
68*a^3*c^5*cos(f*x + e)^6 - 1155*a^3*c^5*cos(f*x + e)^5 - 256*a^3*c^5*cos(f
*x + e)^4 + 910*a^3*c^5*cos(f*x + e)^3 - 192*a^3*c^5*cos(f*x + e)^2 - 280*a
^3*c^5*cos(f*x + e) + 120*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3c^5\left(\int(-1)\,dx+\int 2\sec(e+fx)\,dx+\int 2\sec^2(e+fx)\,dx+\int\left(-6\sec^3(e+fx)\right)\,dx+\int 6\sec^5(e+fx)\,dx+\int\left(-2\sec^6(e+fx)\right)\,dx+\int\left(-2\sec^7(e+fx)\right)\,dx+\int\sec^8(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*c**5*(Integral(-1, x) + Integral(2*sec(e + f*x), x) + Integral(2*sec(
e + f*x)**2, x) + Integral(-6*sec(e + f*x)**3, x) + Integral(6*sec(e + f*x)
**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x)
 + Integral(sec(e + f*x)**8, x))
```

Giac [A]
time = 0.58, size = 210, normalized size = 1.12

$$\frac{840\,(fx+e)a^3c^5-525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(1365\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-9660\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+29673\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}-21216\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+9863\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-2660\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+315\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^7}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/840*(840*(f*x + e)*a^3*c^5 - 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(1365*a^3*c^5*tan(1
/2*f*x + 1/2*e)^13 - 9660*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 29673*a^3*c^5*t
an(1/2*f*x + 1/2*e)^9 - 21216*a^3*c^5*tan(1/2*f*x + 1/2*e)^7 + 9863*a^3*c^5
*tan(1/2*f*x + 1/2*e)^5 - 2660*a^3*c^5*tan(1/2*f*x + 1/2*e)^3 + 315*a^3*c^5
*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B]
time = 2.62, size = 259, normalized size = 1.38

$$\frac{\frac{13\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{4}-23\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\frac{1413\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{20}-\frac{1768\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{1409\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{60}-\frac{19\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{3}+\frac{3\,a^3\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}-7\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+21\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-35\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{8}+35\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{6}-21\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{4}+7\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{2}-1\right)}+a^3\,c^5\,x-\frac{5\,a^3\,c^5\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5,x)
```

```
[Out] ((1409*a^3*c^5*tan(e/2 + (f*x)/2)^5)/60 - (19*a^3*c^5*tan(e/2 + (f*x)/2)^3)
/3 - (1768*a^3*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1413*a^3*c^5*tan(e/2 + (f*x)
/2)^9)/20 - 23*a^3*c^5*tan(e/2 + (f*x)/2)^11 + (13*a^3*c^5*tan(e/2 + (f*x)/
2)^13)/4 + (3*a^3*c^5*tan(e/2 + (f*x)/2))/4)/(f*(7*tan(e/2 + (f*x)/2)^2 - 2
1*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8
 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^1
4 - 1)) + a^3*c^5*x - (5*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.12 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=132

$$a^3c^4x-\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3(16c^4-5c^4\sec(e+fx))\tan(e+fx)}{16f}+\frac{a^3(8c^4-5c^4\sec(e+fx))\tan^3(e}{24f}$$

```
[Out] a^3*c^4*x-5/16*a^3*c^4*arctanh(sin(f*x+e))/f-1/16*a^3*(16*c^4-5*c^4*sec(f*x
+e))*tan(f*x+e)/f+1/24*a^3*(8*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^3/f-1/30*a^3
*(6*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3966, 3855}

$$-\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3\tan^5(e+fx)\,(6c^4-5c^4\sec(e+fx))}{30f}+\frac{a^3\tan^3(e+fx)\,(8c^4-5c^4\sec(e+fx))}{24f}-\frac{a^3\tan(e+fx)\,(16c^4-5c^4\sec(e+fx))}{16f}+a^3c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] a^3*c^4*x - (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (a^3*(16*c^4 - 5*c^4
*Sec[e + f*x])*Tan[e + f*x])/(16*f) + (a^3*(8*c^4 - 5*c^4*Sec[e + f*x])*Tan
[e + f*x]^3)/(24*f) - (a^3*(6*c^4 - 5*c^4*Sec[e + f*x])*Tan[e + f*x]^5)/(30
*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\left(\left(a^3c^3\right)\int (c-c\sec(e+fx))\tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3(6c^4-5c^4\sec(e+fx))\tan^5(e+fx)}{30f} + \frac{1}{6}\left(a^3c^3\right)\int (6c \cdots$$
$$= \frac{a^3(8c^4-5c^4\sec(e+fx))\tan^3(e+fx)}{24f} - \frac{a^3(6c^4-5c^4\sec(\cdots}{}$$
$$= -\frac{a^3(16c^4-5c^4\sec(e+fx))\tan(e+fx)}{16f} + \frac{a^3(8c^4-5c^4\,\mathrm{se} \cdots}{}$$
$$= a^3c^4x - \frac{a^3(16c^4-5c^4\sec(e+fx))\tan(e+fx)}{16f} + \frac{a^3(8c^4 - \cdots}{}$$
$$= a^3c^4x - \frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f} - \frac{a^3(16c^4-5c^4\sec(e+ \cdots}{16 \cdots}$$

Mathematica [A]
time = 1.97, size = 165, normalized size = 1.25

$$\frac{a^3c^4\sec^6(e+fx)\left(1200e+1200fx-1200\tanh^{-1}(\sin(e+fx))\cos^6(e+fx)+1800(e+fx)\cos(2(e+fx))+720e\cos(4(e+fx))+720fx\cos(4(e+fx))+120e\cos(6(e+fx))+120fx\cos(6(e+fx))+450\sin(e+fx)-600\sin(2(e+fx))-25\sin(3(e+fx))-384\sin(4(e+fx))+165\sin(5(e+fx))-184\sin(6(e+fx))\right)}{3840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*Sec[e + f*x]^6*(1200*e + 1200*f*x - 1200*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 72
0*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)]
+ 450*Sin[e + f*x] - 600*Sin[2*(e + f*x)] - 25*Sin[3*(e + f*x)] - 384*Sin[4
*(e + f*x)] + 165*Sin[5*(e + f*x)] - 184*Sin[6*(e + f*x)]))/(3840*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(124) = 248$.
time = 0.11, size = 267, normalized size = 2.02

method	result
risch	$a^3c^4x - \frac{ic^4a^3\left(165\,\mathrm{e}^{11i(fx+e)}+720\,\mathrm{e}^{10i(fx+e)}-25\,\mathrm{e}^{9i(fx+e)}+2160\,\mathrm{e}^{8i(fx+e)}+450\,\mathrm{e}^{7i(fx+e)}+3680\,\mathrm{e}^{6i(fx+e)}-450\,\mathrm{e}^{5i \cdots}\right.}{120f\left(\mathrm{e}^{2i(fx+e)}+1\right)^6}$
derivativedivides	$\frac{c^4a^3\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+c^4a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(f \cdots\right.}{5}\right.}{}$
default	$\frac{c^4a^3\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+c^4a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(f \cdots\right.}{5}\right.}{}$

norman	$\frac{a^3c^4x+a^3c^4x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-6a^3c^4x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15a^3c^4x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-20a^3c^4x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15a^3c^4x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^4*a^3*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*
x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+c^4*a^3*(-8/15-1/5*sec(f*x+e)^4-4/15*s
ec(f*x+e)^2)*tan(f*x+e)-3*c^4*a^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(
f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-3*c^4*a^3*(-2/3-1/3*sec(f*x+e)^2)*tan
(f*x+e)+3*c^4*a^3*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))
-3*c^4*a^3*tan(f*x+e)-c^4*a^3*ln(sec(f*x+e)+tan(f*x+e))+c^4*a^3*(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(131) = 262$.
time = 0.28, size = 361, normalized size = 2.73

$$-\frac{32\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^3c^4-480\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3c^4-480\left(fx+e\right)a^3c^4+5a^3c^4\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-90a^3c^4\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)+360a^3c^4\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+480a^3c^4\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+1440a^3c^4\tan\left(fx+e\right)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^4
 - 480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 - 480*(f*x + e)*a^3*c^4 +
5*a^3*c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin
(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e
) + 1) + 15*log(sin(f*x + e) - 1)) - 90*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*si
n(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
1) + 3*log(sin(f*x + e) - 1)) + 360*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^3*c^4*log(se
c(f*x + e) + tan(f*x + e)) + 1440*a^3*c^4*tan(f*x + e))/f
```

Fricas [A]
time = 2.35, size = 191, normalized size = 1.45

$$\frac{480a^3c^4fx\cos\left(fx+e\right)^6-75a^3c^4\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)+75a^3c^4\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(368a^3c^4\cos\left(fx+e\right)^5-165a^3c^4\cos\left(fx+e\right)^4-176a^3c^4\cos\left(fx+e\right)^3+130a^3c^4\cos\left(fx+e\right)^2+48a^3c^4\cos\left(fx+e\right)-40a^3c^4\right)\sin\left(fx+e\right)}{480f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^3*c^4*f*x*cos(f*x + e)^6 - 75*a^3*c^4*cos(f*x + e)^6*log(sin(f
*x + e) + 1) + 75*a^3*c^4*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(368*a^
3*c^4*cos(f*x + e)^5 - 165*a^3*c^4*cos(f*x + e)^4 - 176*a^3*c^4*cos(f*x + e
)^3 + 130*a^3*c^4*cos(f*x + e)^2 + 48*a^3*c^4*cos(f*x + e) - 40*a^3*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3c^4\left(\int 1\,dx+\int\left(-\sec\left(e+fx\right)\right)dx+\int\left(-3\sec^2\left(e+fx\right)\right)dx+\int 3\sec^3\left(e+fx\right)dx+\int 3\sec^4\left(e+fx\right)dx+\int\left(-3\sec^5\left(e+fx\right)\right)dx+\int\left(-\sec^6\left(e+fx\right)\right)dx+\int\sec^7\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(1, x) + Integral(-sec(e + f*x), x) + Integral(-3*sec(e
+ f*x)**2, x) + Integral(3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4
, x) + Integral(-3*sec(e + f*x)**5, x) + Integral(-sec(e + f*x)**6, x) + In
tegral(sec(e + f*x)**7, x))
```

Giac [A]
time = 0.58, size = 191, normalized size = 1.45

$$\frac{240\left(fx+e\right)a^3c^4-75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(315\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-1945\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+5118\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-3138\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+1095\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-165\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^6}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)*a^3*c^4 - 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)
) + 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(315*a^3*c^4*tan(1/2*
f*x + 1/2*e)^11 - 1945*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 5118*a^3*c^4*tan(1/
2*f*x + 1/2*e)^7 - 3138*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 1095*a^3*c^4*tan(1
/2*f*x + 1/2*e)^3 - 165*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)
^2 - 1)^6)/f
```

Mupad [B]
time = 2.59, size = 227, normalized size = 1.72

$$a^3\,c^4\,x+\frac{\frac{21\,a^3\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{389\,a^3\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{24}+\frac{853\,a^3\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{20}-\frac{523\,a^3\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20}+\frac{73\,a^3\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{8}-\frac{11\,a^3\,c^4\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}-6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+15\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-20\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+15\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}-\frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4,x)
```

```
[Out] a^3*c^4*x + ((73*a^3*c^4*tan(e/2 + (f*x)/2)^3)/8 - (523*a^3*c^4*tan(e/2 + (
f*x)/2)^5)/20 + (853*a^3*c^4*tan(e/2 + (f*x)/2)^7)/20 - (389*a^3*c^4*tan(e/
2 + (f*x)/2)^9)/24 + (21*a^3*c^4*tan(e/2 + (f*x)/2)^11)/8 - (11*a^3*c^4*tan
(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 -
20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10
 + tan(e/2 + (f*x)/2)^12 + 1)) - (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f
)
```

3.13 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=68

$$a^3c^3x-\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}$$

```
[Out] a^3*c^3*x-a^3*c^3*tan(f*x+e)/f+1/3*a^3*c^3*tan(f*x+e)^3/f-1/5*a^3*c^3*tan(f
*x+e)^5/f
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$-\frac{a^3c^3\tan^5(e+fx)}{5f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan(e+fx)}{f}+a^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a^3*c^3*x - (a^3*c^3*Tan[e + f*x])/f + (a^3*c^3*Tan[e + f*x]^3)/(3*f) - (a^
3*c^3*Tan[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\left((a^3c^3)\int \tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3c^3\tan^5(e+fx)}{5f} + (a^3c^3)\int \tan^4(e+fx)\,dx$$
$$= \frac{a^3c^3\tan^3(e+fx)}{3f} - \frac{a^3c^3\tan^5(e+fx)}{5f} - (a^3c^3)\int \tan^2(e$$
$$= -\frac{a^3c^3\tan(e+fx)}{f} + \frac{a^3c^3\tan^3(e+fx)}{3f} - \frac{a^3c^3\tan^5(e+f}{5f}$$
$$= a^3c^3x - \frac{a^3c^3\tan(e+fx)}{f} + \frac{a^3c^3\tan^3(e+fx)}{3f} - \frac{a^3c^3\tan^5}{5}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.90

$$-a^3c^3\left(-\frac{\text{ArcTan}(\tan(e+fx))}{f} + \frac{\tan(e+fx)}{f} - \frac{\tan^3(e+fx)}{3f} + \frac{\tan^5(e+fx)}{5f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
[Out] -(a^3*c^3*(-(ArcTan[Tan[e + f*x]]/f) + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*f
) + Tan[e + f*x]^5/(5*f)))
```

Maple [A]
time = 0.08, size = 93, normalized size = 1.37

method	result
risch	$a^3c^3x - \frac{2ic^3a^3\left(45\,e^{8i(fx+e)}+90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}+70\,e^{2i(fx+e)}+23\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$
derivativedivides	$\frac{c^3a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-3c^3a^3\tan(fx+e)+c^3}{f}$
default	$\frac{c^3a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-3c^3a^3\tan(fx+e)+c^3}{f}$
norman	$\frac{a^3c^3x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^3c^3x+5a^3c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-10a^3c^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+10a^3c^3x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-5a^3c^3}{\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^3*a^3*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-3*c^3*a^
3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-3*c^3*a^3*tan(f*x+e)+c^3*a^3*(f*x+e))
```

Maxima [A]
time = 0.27, size = 101, normalized size = 1.49

$$-\frac{\left(3\,\tan\left(fx+e\right)^{5}+10\,\tan\left(fx+e\right)^{3}+15\,\tan\left(fx+e\right)\right)a^{3}c^{3}-15\,\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c^{3}-15\,(fx+e)a^{3}c^{3}+45\,a^{3}c^{3}\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^3 - 1
5*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 - 15*(f*x + e)*a^3*c^3 + 45*a^3
*c^3*tan(f*x + e))/f
```

Fricas [A]
time = 2.93, size = 86, normalized size = 1.26

$$\frac{15\,a^{3}c^{3}fx\cos\left(fx+e\right)^{5}-\left(23\,a^{3}c^{3}\cos\left(fx+e\right)^{4}-11\,a^{3}c^{3}\cos\left(fx+e\right)^{2}+3\,a^{3}c^{3}\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*c^3*f*x*cos(f*x + e)^5 - (23*a^3*c^3*cos(f*x + e)^4 - 11*a^3*c
^3*cos(f*x + e)^2 + 3*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{3}c^{3}\left(\int\left(-1\right)\,dx+\int 3\sec^{2}\left(e+fx\right)dx+\int\left(-3\sec^{4}\left(e+fx\right)\right)\,dx+\int\sec^{6}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-1, x) + Integral(3*sec(e + f*x)**2, x) + Integral(-3*
sec(e + f*x)**4, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A]
time = 0.54, size = 65, normalized size = 0.96

$$-\frac{3\,a^{3}c^{3}\tan\left(fx+e\right)^{5}-5\,a^{3}c^{3}\tan\left(fx+e\right)^{3}-15\,(fx+e)a^{3}c^{3}+15\,a^{3}c^{3}\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*c^3*tan(f*x + e)^5 - 5*a^3*c^3*tan(f*x + e)^3 - 15*(f*x + e)*a
^3*c^3 + 15*a^3*c^3*tan(f*x + e))/f
```

Mupad [B]
time = 4.96, size = 122, normalized size = 1.79

$$a^3\,c^3\,x+\frac{2\,a^3\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9-\frac{32\,a^3\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}+\frac{356\,a^3\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}-\frac{32\,a^3\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^3\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3,x)
```

```
[Out] a^3*c^3*x + ((356*a^3*c^3*tan(e/2 + (f*x)/2)^5)/15 - (32*a^3*c^3*tan(e/2 +
(f*x)/2)^3)/3 - (32*a^3*c^3*tan(e/2 + (f*x)/2)^7)/3 + 2*a^3*c^3*tan(e/2 + (
f*x)/2)^9 + 2*a^3*c^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^5)
```

3.14 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=97

$$a^3c^2x+\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}-\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f}+\frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e}{12f}$$

```
[Out] a^3*c^2*x+3/8*a^3*c^2*arctanh(sin(f*x+e))/f-1/8*c^2*(8*a^3+3*a^3*sec(f*x+e)
)*tan(f*x+e)/f+1/12*c^2*(4*a^3+3*a^3*sec(f*x+e))*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3966, 3855}

$$\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{c^2\tan^3(e+fx)\,(3a^3\sec(e+fx)+4a^3)}{12f}-\frac{c^2\tan(e+fx)\,(3a^3\sec(e+fx)+8a^3)}{8f}+a^3c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] a^3*c^2*x + (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (c^2*(8*a^3 + 3*a^3*S
ec[e + f*x])*Tan[e + f*x])/(8*f) + (c^2*(4*a^3 + 3*a^3*Sec[e + f*x])*Tan[e
+ f*x]^3)/(12*f)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int (a+a\sec(e+fx))\tan^4(e+fx)\,dx$$

$$= \frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f} - \frac{1}{4}\left(a^2c^2\right)\int (4a+$$

$$= -\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f} + \frac{c^2(4a^3+3a^3\sec$$

$$= a^3c^2x - \frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f} + \frac{c^2(4a^3+$$

$$= a^3c^2x + \frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{c^2(8a^3+3a^3\sec(e+}{8f}$$

Mathematica [A]
time = 0.86, size = 122, normalized size = 1.26

$$\frac{a^3c^2\sec^4(e+fx)\left(72e+72fx+72\tanh^{-1}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24e\cos(4(e+fx))+24fx\cos(4(e+fx))+18\sin(e+fx)-32\sin(2(e+fx))-30\sin(3(e+fx))-32\sin(4(e+fx))\right)}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]^4*(72*e + 72*f*x + 72*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[
4*(e + f*x)] + 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 30*Sin[3*(e + f*x)]
- 32*Sin[4*(e + f*x)]))/(192*f)
```

Maple [A]
time = 0.08, size = 169, normalized size = 1.74

method	result
risch	$a^3c^2x + \frac{ic^2a^3\left(15\,e^{7i(fx+e)}-48\,e^{6i(fx+e)}-9\,e^{5i(fx+e)}-96\,e^{4i(fx+e)}+9\,e^{3i(fx+e)}-80\,e^{2i(fx+e)}-15\,e^{i(fx+e)}-32\right)}{12f\left(e^{2i(fx+e)}+1\right)^4} +$
derivativedivides	$c^2a^3\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^2a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e$
default	$c^2a^3\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-c^2a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e$
norman	$\frac{a^3c^2x+a^3c^2x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-4a^3c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+6a^3c^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-4a^3c^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{11c^2a^3\tan}{4}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c^2*a^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x
+e)+tan(f*x+e)))-c^2*a^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*c^2*a^3*(1/2*
sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2*c^2*a^3*tan(f*x+e)+c
^2*a^3*ln(sec(f*x+e)+tan(f*x+e))+c^2*a^3*(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(96) = 192$.
time = 0.31, size = 219, normalized size = 2.26

$$\frac{16\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c^{2}+48\left(fx+e\right)a^{3}c^{2}-3\,a^{3}c^{2}\left(\frac{2\left(3\,\sin(fx+e)^{3}-5\,\sin(fx+e)\right)}{\sin(fx+e)^{4}-2\,\sin(fx+e)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)+24\,a^{3}c^{2}\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+48\,a^{3}c^{2}\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-96\,a^{3}c^{2}\tan\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 48*(f*x + e)*a^3*c^2 -
 3*a^3*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 24*a^
3*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) + 48*a^3*c^2*log(sec(f*x + e) + tan(f*x + e)) - 96*a^3*c^2
*tan(f*x + e))/f
```

Fricas [A]
time = 3.37, size = 157, normalized size = 1.62

$$\frac{48\,a^{3}c^{2}fx\cos\left(fx+e\right)^{4}+9\,a^{3}c^{2}\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-9\,a^{3}c^{2}\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(32\,a^{3}c^{2}\cos\left(fx+e\right)^{3}+15\,a^{3}c^{2}\cos\left(fx+e\right)^{2}-8\,a^{3}c^{2}\cos\left(fx+e\right)-6\,a^{3}c^{2}\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^3*c^2*f*x*cos(f*x + e)^4 + 9*a^3*c^2*cos(f*x + e)^4*log(sin(f*x
+ e) + 1) - 9*a^3*c^2*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^3*c^2
*cos(f*x + e)^3 + 15*a^3*c^2*cos(f*x + e)^2 - 8*a^3*c^2*cos(f*x + e) - 6*a^
3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}c^{2}\left(\int 1\,dx+\int\sec\left(e+fx\right)dx+\int\left(-2\sec^{2}\left(e+fx\right)\right)\,dx+\int\left(-2\sec^{3}\left(e+fx\right)\right)\,dx+\int\sec^{4}\left(e+fx\right)dx+\int\sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*c**2*(Integral(1, x) + Integral(sec(e + f*x), x) + Integral(-2*sec(e +
 f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4,
x) + Integral(sec(e + f*x)**5, x))
```

Giac [A]
time = 0.55, size = 153, normalized size = 1.58

$$\frac{24\,(fx+e)a^3c^2+9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-71\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+137\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-33\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)*a^3*c^2 + 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) -
 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(15*a^3*c^2*tan(1/2*f*x +
 1/2*e)^7 - 71*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 137*a^3*c^2*tan(1/2*f*x + 1
/2*e)^3 - 33*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/
f
```

Mupad [B]
time = 2.13, size = 163, normalized size = 1.68

$$\frac{\frac{5\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{71\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{137\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}-\frac{11\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+a^3\,c^2\,x+\frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2,x)
```

```
[Out] ((137*a^3*c^2*tan(e/2 + (f*x)/2)^3)/12 - (71*a^3*c^2*tan(e/2 + (f*x)/2)^5)/
12 + (5*a^3*c^2*tan(e/2 + (f*x)/2)^7)/4 - (11*a^3*c^2*tan(e/2 + (f*x)/2))/4
)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^3*c^2*x + (3*a^3*c^2*atanh(tan(e/2 + (
f*x)/2)))/(4*f)
```

3.15 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=77

$$a^3cx+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\sec(e+fx)\tan(e+fx)}{f}-\frac{a^3c\tan^3(e+fx)}{3f}$$

```
[Out] a^3*c*x+a^3*c*arctanh(sin(f*x+e))/f-a^3*c*tan(f*x+e)/f-a^3*c*sec(f*x+e)*tan
(f*x+e)/f-1/3*a^3*c*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3989, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{a^3c\tan^3(e+fx)}{3f}-\frac{a^3c\tan(e+fx)}{f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)\sec(e+fx)}{f}+a^3cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] a^3*c*x + (a^3*c*ArcTanh[Sin[e + f*x]])/f - (a^3*c*Tan[e + f*x])/f - (a^3*c
*Sec[e + f*x]*Tan[e + f*x])/f - (a^3*c*Tan[e + f*x]^3)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx &= -\left((ac)\int (a+a\sec(e+fx))^2\tan^2(e+fx)\,dx\right)\\ &= -\left((ac)\int \left(a^2\tan^2(e+fx)+2a^2\sec(e+fx)\tan^2(e+fx)\right.\right.\\ &= -\left(\left(a^3c\right)\int \tan^2(e+fx)\,dx\right)-\left(a^3c\right)\int \sec^2(e+fx)\tan^2(\\ &= -\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\sec(e+fx)\tan(e+fx)}{f}+\left(a^3c\right)\int\\ &= a^3cx+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)}{f}-\frac{a^3c\,\mathrm{se}}{}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 101, normalized size = 1.31

$$\frac{a^3c\sec^3(e+fx)\left(9(e+fx)\cos(e+fx)+12\tanh^{-1}(\sin(e+fx))\cos^3(e+fx)+3e\cos(3(e+fx))+3fx\cos(3(e+fx))-6\sin(e+fx)-6\sin(2(e+fx))-2\sin(3(e+fx))\right)}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]

[Out] (a^3*c*Sec[e + f*x]^3*(9*(e + f*x)*Cos[e + f*x] + 12*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^3 + 3*e*Cos[3*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] - 6*Sin[e + f*x] - 6*Sin[2*(e + f*x)] - 2*Sin[3*(e + f*x)]))/(12*f)

Maple [A]
time = 0.06, size = 96, normalized size = 1.25

method	result
derivativedivides	$\frac{a^3 c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-2a^3 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^3 c\ln(\sec(fx+e)+\tan(fx}{f}$
default	$\frac{a^3 c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-2a^3 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^3 c\ln(\sec(fx+e)+\tan(fx}{f}$
risch	$a^3 cx+\frac{2ia^3 c(3\,e^{5i(fx+e)}-6\,e^{2i(fx+e)}-3\,e^{i(fx+e)}-2)}{3f\left(e^{2i(fx+e)}+1\right)^3}+\frac{a^3 c\ln(e^{i(fx+e)}+i)}{f}-\frac{a^3 c\ln(e^{i(fx+e)}-i)}{f}$
norman	$\frac{a^3 cx\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a^3 cx+\frac{4a^3 c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4a^3 c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}+3a^3 cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3a^3 cx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/f*(a^3*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^3*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+2*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+a^3*c*(f*x+e))

Maxima [A]
time = 0.28, size = 116, normalized size = 1.51

$$-\frac{2\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c-6(fx+e)a^3c-3a^3c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-12a^3c\log(\sec(fx+e)+\tan(fx+e))}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] -1/6*(2*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 6*(f*x + e)*a^3*c - 3*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*a^3*c*log(sec(f*x + e) + tan(f*x + e)))/f

Fricas [A]
time = 2.71, size = 127, normalized size = 1.65

$$\frac{6a^3cfx\cos(fx+e)^3+3a^3c\cos(fx+e)^3\log(\sin(fx+e)+1)-3a^3c\cos(fx+e)^3\log(-\sin(fx+e)+1)-2\left(2a^3c\cos(fx+e)^2+3a^3c\cos(fx+e)+a^3c\right)\sin(fx+e)}{6f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c*f*x*cos(f*x + e)^3 + 3*a^3*c*cos(f*x + e)^3*log(sin(f*x + e) +
 1) - 3*a^3*c*cos(f*x + e)^3*log(-sin(f*x + e) + 1) - 2*(2*a^3*c*cos(f*x +
e)^2 + 3*a^3*c*cos(f*x + e) + a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3c\left(\int(-1)\,dx+\int(-2\sec(e+fx))\,dx+\int 2\sec^3(e+fx)\,dx+\int\sec^4(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-1, x) + Integral(-2*sec(e + f*x), x) + Integral(2*sec(e
+ f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Giac [A]
time = 0.54, size = 104, normalized size = 1.35

$$\frac{3(fx+e)a^3c+3a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{4\left(a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^3*c + 3*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^3
*c*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 4*(a^3*c*tan(1/2*f*x + 1/2*e)^3 - 3
*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B]
time = 1.55, size = 104, normalized size = 1.35

$$\frac{4a^3c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{4a^3c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{3}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}+a^3cx+\frac{2a^3c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)),x)
```

```
[Out] (4*a^3*c*tan(e/2 + (f*x)/2) - (4*a^3*c*tan(e/2 + (f*x)/2)^3)/3)/(f*(3*tan(e
/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + a^3
*c*x + (2*a^3*c*atanh(tan(e/2 + (f*x)/2)))/f
```

3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=78

$$\frac{a^3x}{c}-\frac{4a^3\tanh^{-1}(\sin(e+fx))}{cf}+\frac{8a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{a^3\tan(e+fx)}{cf}$$

```
[Out] a^3*x/c-4*a^3*arctanh(sin(f*x+e))/c/f+8*a^3*cot(f*x+e)/c/f+8*a^3*csc(f*x+e)
/c/f-a^3*tan(f*x+e)/c/f
```

Rubi [A]
time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.423$, Rules used = {3989, 3971, 3554, 8, 2686, 3852, 2701, 327, 213, 2700, 14}

$$-\frac{a^3\tan(e+fx)}{cf}+\frac{8a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{4a^3\tanh^{-1}(\sin(e+fx))}{cf}+\frac{a^3x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*x)/c - (4*a^3*ArcTanh[Sin[e + f*x]])/(c*f) + (8*a^3*Cot[e + f*x])/(c*f
) + (8*a^3*Csc[e + f*x])/(c*f) - (a^3*Tan[e + f*x])/(c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^4\,dx}{ac}\\
&= -\frac{\int \left(a^4\cot^2(e+fx)+4a^4\cot(e+fx)\csc(e+fx)+6a^4\csc^2(e+fx)+4a^4\csc\right.}{ac}\\
&= -\frac{a^3\int \cot^2(e+fx)\,dx}{c}-\frac{a^3\int \csc^2(e+fx)\sec^2(e+fx)\,dx}{c}-\frac{(4a^3)\int \cot(e+f}{c}\\
&= \frac{a^3\cot(e+fx)}{cf}+\frac{a^3\int 1\,dx}{c}-\frac{a^3\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(e+fx)\right)}{cf}+\frac{(4a^3)\,\text{Subs}}{}\\
&= \frac{a^3x}{c}+\frac{7a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{a^3\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\tan(e+\right.}{cf}\\
&= \frac{a^3x}{c}-\frac{4a^3\tanh^{-1}(\sin(e+fx))}{cf}+\frac{8a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{a^3\tan(}{c}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(78) = 156.
time = 2.80, size = 240, normalized size = 3.08

$$\frac{a^3\cos^2(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\tan\left(\frac{1}{2}(e+fx)\right)\left(8\csc\left(\frac{e}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{fx}{2}\right)+\left(-fx-4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\tan\left(\frac{1}{2}(e+fx)\right)\right)}{4f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^3*Tan[(e + f*x)/2
]*(8*Csc[e/2]*Sec[(e + f*x)/2]*Sin[(f*x)/2] + (-(f*x) - 4*Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] + 4*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[
f*x]/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*Tan[(e + f*x)/2]))/(4*
f*(c - c*Sec[e + f*x]))
```

Maple [A]
time = 0.12, size = 94, normalized size = 1.21

method	result
derivativedivides	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$

default	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
risch	$\frac{a^3x}{c}+\frac{2ia^3\left(8\,e^{2i(fx+e)}-e^{i(fx+e)}+9\right)}{fc\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}+\frac{4a^3\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{4a^3\ln\left(e^{i(fx+e)}+i\right)}{cf}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}+\frac{a^3x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{8a^3}{cf}-\frac{18a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{10a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2a^3x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{4a^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*a^3/c*(1/8/(tan(1/2*f*x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/4*arct
an(tan(1/2*f*x+1/2*e))+1/8/(tan(1/2*f*x+1/2*e)+1)-1/2*ln(tan(1/2*f*x+1/2*e)
+1)+1/tan(1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(82) = 164.
time = 0.51, size = 296, normalized size = 3.79

$$-\frac{a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)-a^3\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}+\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)+3a^3\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-\frac{3a^3(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x
 + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) - a^3*(
2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*sin(f*x
 + e))) + 3*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 3*a^
3*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Fricas [A]
time = 3.54, size = 137, normalized size = 1.76

$$\frac{a^3fx\cos(fx+e)\sin(fx+e)-2a^3\cos(fx+e)\log(\sin(fx+e)+1)\sin(fx+e)+2a^3\cos(fx+e)\log(-\sin(fx+e)+1)\sin(fx+e)+9a^3\cos(fx+e)^2+8a^3\cos(fx+e)-a^3}{cf\cos(fx+e)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] (a^3*f*x*cos(f*x + e)*sin(f*x + e) - 2*a^3*cos(f*x + e)*log(sin(f*x + e) +
1)*sin(f*x + e) + 2*a^3*cos(f*x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) +
9*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) - a^3)/(c*f*cos(f*x + e)*sin(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{\sec^3(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e +
f*x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1
), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Giac [A]
time = 0.51, size = 111, normalized size = 1.42

$$\frac{\frac{(fx+e)a^3}{c} - \frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c} + \frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c} + \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/c - 4*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + 4*a^3*log(a
bs(tan(1/2*f*x + 1/2*e) - 1))/c + 2*(5*a^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^3)/
((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

Mupad [B]
time = 1.48, size = 85, normalized size = 1.09

$$\frac{a^3\,x}{c} - \frac{10\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - 8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) - c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)} - \frac{8\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x)),x)
```

```
[Out] (a^3*x)/c - (10*a^3*tan(e/2 + (f*x)/2)^2 - 8*a^3)/(f*(c*tan(e/2 + (f*x)/2)
- c*tan(e/2 + (f*x)/2)^3)) - (8*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{a^3x}{c^2}+\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^2f}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

```
[Out] a^3*x/c^2+a^3*arctanh(sin(f*x+e))/c^2/f-8/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x
+e))^2+4/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.27, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3988, 3862, 4004, 3879, 3881, 3882, 3884, 4083, 3855}

$$\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*x)/c^2 + (a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) - (8*a^3*Tan[e + f*x])/(3
*c^2*f*(1 - Sec[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^2}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^2}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^2}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{c^2}+\frac{(3a^3$$

$$= -\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}-\frac{a^3\int \frac{-3-\sec(e+fx)}{1-\sec(e+fx)}\,dx}{3c^2}+\frac{a^3\int \frac{(-2-3\sec(e+fx))\sec(e+fx)}{1-\sec(e+fx)}}{3c^2}$$

$$= \frac{a^3x}{c^2}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{a^3\tan(e+fx)}{c^2f(1-\sec(e+fx))}+\frac{a^3\int \sec(e+fx)\,dx}{c^2}$$

$$= \frac{a^3x}{c^2}+\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^2f}-\frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx}{3c^2f(1-\sec(e+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.

time = 1.28, size = 177, normalized size = 2.01

$$\frac{a^3(1+\cos(e+fx))^3\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)\left(4\csc\left(\frac{e}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{fx}{2}\right)-4\cot\left(\frac{e}{2}\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)+3\left(fx-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^3\left(\frac{1}{2}(e+fx)\right)\right)}{6c^2f(-1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(4*Csc[e/2]*S
ec[(e + f*x)/2]*Sin[(f*x)/2] - 4*Cot[e/2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2] + 3*(f*x - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^3))/(6*c^2*f*(-1 + Cos[e + f*x])^2
)
```

Maple [A]
time = 0.14, size = 66, normalized size = 0.75

method	result
derivativedivides	$\frac{4a^3\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,c^2}$
default	$\frac{4a^3\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,c^2}$
risch	$\frac{a^3x}{c^2}+\frac{8ia^3\left(3\,e^{2i(fx+e)}+1\right)}{3f\,c^2\left(e^{i(fx+e)}-1\right)^3}+\frac{a^3\ln\left(e^{i(fx+e)}+i\right)}{c^2f}-\frac{a^3\ln\left(e^{i(fx+e)}-i\right)}{c^2f}$

norman	$\frac{\frac{a^{3}x\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{a^{3}x\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4a^{3}}{3cf}+\frac{8a^{3}\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{4a^{3}\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{2a^{3}x\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{a^{3}\ln\left(\mathrm{t}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a^3/c^2*(-1/3/tan(1/2*f*x+1/2*e)^3-1/4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*arc
tan(tan(1/2*f*x+1/2*e))+1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(85) = 170$.
time = 0.51, size = 296, normalized size = 3.36

$$\frac{a^{3}\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{2}}+\frac{\left(\frac{9\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}\right)+a^{3}\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{2}}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{2}}-\frac{\left(\frac{9\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}\right)-\frac{3a^{3}\left(\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}+\frac{3a^{3}\left(\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^3*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) + a^
3*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(
f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 3*a^3*(3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + 3*a^3*(3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)
)/f
```

Fricas [A]
time = 2.88, size = 168, normalized size = 1.91

$$\frac{8a^{3}\cos(fx+e)^{2}+16a^{3}\cos(fx+e)+8a^{3}+3\left(a^{3}\cos(fx+e)-a^{3}\right)\log\left(\sin(fx+e)+1\right)\sin(fx+e)-3\left(a^{3}\cos(fx+e)-a^{3}\right)\log\left(-\sin(fx+e)+1\right)\sin(fx+e)+6\left(a^{3}fx\cos(fx+e)-a^{3}fx\right)\sin(fx+e)}{6\left(c^{2}f\cos(fx+e)-c^{2}f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(8*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 8*a^3 + 3*(a^3*cos(f*x +
e) - a^3)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*(a^3*cos(f*x + e) - a^3)*l
og(-sin(f*x + e) + 1)*sin(f*x + e) + 6*(a^3*f*x*cos(f*x + e) - a^3*f*x)*sin
(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{3}\left(\int\frac{3\sec(e+fx)}{\sec^{2}(e+fx)-2\sec(e+fx)+1}\,dx+\int\frac{3\sec^{2}(e+fx)}{\sec^{2}(e+fx)-2\sec(e+fx)+1}\,dx+\int\frac{\sec^{3}(e+fx)}{\sec^{2}(e+fx)-2\sec(e+fx)+1}\,dx+\int\frac{1}{\sec^{2}(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A]
time = 0.51, size = 80, normalized size = 0.91

$$\frac{\frac{3\,(fx+e)a^3}{c^2}+\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{4\,a^3}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^3/c^2 + 3*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3
*a^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/c^2 - 4*a^3/(c^2*tan(1/2*f*x + 1/2*
e)^3))/f
```

Mupad [B]
time = 1.44, size = 45, normalized size = 0.51

$$\frac{a^3\,x}{c^2}+\frac{a^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\,{\mathrm{cot}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3}\right)}{c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^2,x)
```

```
[Out] (a^3*x)/c^2 + (a^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*cot(e/2 + (f*x)/2)^3)/
3))/(c^2*f)
```

3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{a^3x}{c^3}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

```
[Out] a^3*x/c^3-8/5*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3+4/15*a^3*tan(f*x+e)/c^3
/f/(1-sec(f*x+e))^2-26/15*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.34, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$-\frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{a^3x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*x)/c^3 - (8*a^3*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) + (4*a^3*
Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*c
^3*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^3}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^3}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^3}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{c^3}+\frac{(3a^3)}{}$$

$$= -\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}-\frac{a^3\int \frac{-5-2\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{5c^3}+\frac{a^3\int \frac{(-3-5\sec(e+fx))\sec(e+fx)}{(1-\sec(e+fx))^2}}{5c^3}$$

$$= -\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}+\frac{a^3\int \frac{15+7\sec(e+fx)}{1-\sec(e+fx)}\,dx}{15c^3}+$$

$$= \frac{a^3x}{c^3}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{4a^3\tan(e+f}{15c^3f(1-\sec(e+}$$

$$= \frac{a^3x}{c^3}-\frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2}-\frac{26a^3\tan(e+f}{15c^3f(1-\sec(e+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 53, normalized size = 0.52

$$\frac{2a^3\cot^5\left(\frac{e}{2}+\frac{fx}{2}\right)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
[Out] (2*a^3*Cot[e/2 + (f*x)/2]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e/2 + (f*
x)/2]^2])/(5*c^3*f)
```

Maple [A]
time = 0.15, size = 60, normalized size = 0.59

method	result
derivativedivides	$\frac{2a^3\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$

default	$\frac{2a^3\left(\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$
risch	$\frac{a^3x}{c^3}+\frac{4ia^3\left(45\,\mathrm{e}^{4i(fx+e)}-90\,\mathrm{e}^{3i(fx+e)}+140\,\mathrm{e}^{2i(fx+e)}-70\,\mathrm{e}^{i(fx+e)}+23\right)}{15f\,c^3\left(\mathrm{e}^{i(fx+e)}-1\right)^5}$
norman	$\frac{\frac{a^3x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{a^3x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{2a^3}{5cf}-\frac{22a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}+\frac{56a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{14a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+2a^3(\mathrm{ta}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a^3/c^3*(arctan(tan(1/2*f*x+1/2*e))-1/3/tan(1/2*f*x+1/2*e)^3+1/5/tan(1/
2*f*x+1/2*e)^5+1/tan(1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(96) = 192$.

time = 0.50, size = 306, normalized size = 3.00

$$\frac{a^3\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)+\frac{a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}-\frac{3a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}-\frac{9a^3\left(\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(a^3*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(c
os(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) + a^3*(10*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1
)^5/(c^3*sin(f*x + e)^5) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f
*x + e)^5) - 9*a^3*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e
) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Fricas [A]

time = 2.11, size = 137, normalized size = 1.34

$$\frac{46\,a^3\cos(fx+e)^3-2\,a^3\cos(fx+e)^2-22\,a^3\cos(fx+e)+26\,a^3+15\left(a^3fx\cos(fx+e)^2-2\,a^3fx\cos(fx+e)+a^3fx\right)\sin(fx+e)}{15\left(c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(46*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)^2 - 22*a^3*cos(f*x + e) +
26*a^3 + 15*(a^3*f*x*cos(f*x + e)^2 - 2*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin
(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{\sec^3(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{1}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec
(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*
x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A]
time = 0.53, size = 73, normalized size = 0.72

$$\frac{\frac{15\,(fx+e)a^3}{c^3}+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*a^3/c^3 + 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 5*a^3*tan(1
/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B]
time = 1.38, size = 96, normalized size = 0.94

$$\frac{a^3\,x}{c^3}+\frac{\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^3,x)
```

```
[Out] (a^3*x)/c^3 + ((2*a^3*cos(e/2 + (f*x)/2)^5)/5 + 2*a^3*cos(e/2 + (f*x)/2)*si
n(e/2 + (f*x)/2)^4 - (2*a^3*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)/(
c^3*f*sin(e/2 + (f*x)/2)^5)
```

3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=133

$$\frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+f$$

```
[Out] a^3*x/c^4-8/7*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4+4/35*a^3*tan(f*x+e)/c^4
/f/(1-sec(f*x+e))^3-62/105*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-167/105*a^
3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

Rubi [A]
time = 0.44, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{a^3x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*x)/c^4 - (8*a^3*Tan[e + f*x])/(7*c^4*f*(1 - Sec[e + f*x])^4) + (4*a^3*
Tan[e + f*x])/(35*c^4*f*(1 - Sec[e + f*x])^3) - (62*a^3*Tan[e + f*x])/(105*
c^4*f*(1 - Sec[e + f*x])^2) - (167*a^3*Tan[e + f*x])/(105*c^4*f*(1 - Sec[e
+ f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
```

```
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^4}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^4}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^4}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{c^4}+\frac{(3a^3}{}$$

$$= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}-\frac{a^3\int \frac{-7-3\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{7c^4}+\frac{a^3\int \frac{(-4-7\sec(e+fx))\sec(e+fx}{(1-\sec(e+fx))^3}}{7c^4}$$

$$= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}+\frac{a^3\int \frac{35+20\sec(e+fx)}{(1-\sec(e+fx))^2}\,dx}{35c^4}$$

$$= -\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+f}$$

$$= \frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+}{105c^4f(1-\sec(e}$$

$$= \frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+}{105c^4f(1-\sec(e}$$

Mathematica [A]
time = 0.64, size = 227, normalized size = 1.71

$$\frac{a^3\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\left(3675fx\cos\left(\frac{fx}{2}\right)-3675fx\cos\left(e+\frac{fx}{2}\right)-2205fx\cos\left(e+\frac{3fx}{2}\right)+2205fx\cos\left(2e+\frac{3fx}{2}\right)+735fx\cos\left(2e+\frac{5fx}{2}\right)-735fx\cos\left(3e+\frac{5fx}{2}\right)-105fx\cos\left(3e+\frac{7fx}{2}\right)+105fx\cos\left(4e+\frac{7fx}{2}\right)-12320\sin\left(\frac{fx}{2}\right)-11270\sin\left(e+\frac{fx}{2}\right)+9114\sin\left(e+\frac{3fx}{2}\right)+5040\sin\left(2e+\frac{3fx}{2}\right)-3248\sin\left(2e+\frac{5fx}{2}\right)-1470\sin\left(3e+\frac{5fx}{2}\right)+674\sin\left(3e+\frac{7fx}{2}\right)\right)}{13440c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*Csc[e/2]*Csc[(e + f*x)/2]^7*(3675*f*x*Cos[(f*x)/2] - 3675*f*x*Cos[e +
(f*x)/2] - 2205*f*x*Cos[e + (3*f*x)/2] + 2205*f*x*Cos[2*e + (3*f*x)/2] + 73
5*f*x*Cos[2*e + (5*f*x)/2] - 735*f*x*Cos[3*e + (5*f*x)/2] - 105*f*x*Cos[3*e
 + (7*f*x)/2] + 105*f*x*Cos[4*e + (7*f*x)/2] - 12320*Sin[(f*x)/2] - 11270*S
in[e + (f*x)/2] + 9114*Sin[e + (3*f*x)/2] + 5040*Sin[2*e + (3*f*x)/2] - 324
8*Sin[2*e + (5*f*x)/2] - 1470*Sin[3*e + (5*f*x)/2] + 674*Sin[3*e + (7*f*x)/
2]))/(13440*c^4*f)
```

Maple [A]
time = 0.16, size = 76, normalized size = 0.57

method	result
derivativedivides	$\frac{a^3\left(2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^4}$
default	$\frac{a^3\left(2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^4}$
risch	$\frac{a^3x}{c^4}+\frac{2ia^3\left(735\,\mathrm{e}^{6i(fx+e)}-2520\,\mathrm{e}^{5i(fx+e)}+5635\,\mathrm{e}^{4i(fx+e)}-6160\,\mathrm{e}^{3i(fx+e)}+4557\,\mathrm{e}^{2i(fx+e)}-1624\,\mathrm{e}^{i(fx+e)}+337\right)}{105f\,c^4\left(\mathrm{e}^{i(fx+e)}-1\right)^7}$
norman	$\frac{\frac{a^3x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{a^3x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{a^3}{7cf}+\frac{24a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35cf}-\frac{169a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cf}+\frac{56a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{14a^3(\mathrm{t}}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3/c^4*(2*arctan(tan(1/2*f*x+1/2*e))-1/7/tan(1/2*f*x+1/2*e)^7+2/5/tan(
1/2*f*x+1/2*e)^5-2/3/tan(1/2*f*x+1/2*e)^3+2/tan(1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(125) = 250$.
time = 0.51, size = 417, normalized size = 3.14

$$\frac{5\,a^3\left(\frac{336\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{77\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right)+\frac{3\,a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{9\,a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}-\frac{a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(5*a^3*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
 + e)^7)) + 3*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(co
s(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 9*a^3*(21*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^
4*sin(f*x + e)^7))/f
```

Fricas [A]
time = 4.92, size = 184, normalized size = 1.38

$$\frac{337\,a^3\cos(fx+e)^4-276\,a^3\cos(fx+e)^3-50\,a^3\cos(fx+e)^2+396\,a^3\cos(fx+e)-167\,a^3+105\left(a^3fx\cos(fx+e)^3-3\,a^3fx\cos(fx+e)^2+3\,a^3fx\cos(fx+e)-a^3fx\right)\sin(fx+e)}{105\left(c^4f\cos(fx+e)^3-3\,c^4f\cos(fx+e)^2+3\,c^4f\cos(fx+e)-c^4f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(337*a^3*cos(f*x + e)^4 - 276*a^3*cos(f*x + e)^3 - 50*a^3*cos(f*x + e
)^2 + 396*a^3*cos(f*x + e) - 167*a^3 + 105*(a^3*f*x*cos(f*x + e)^3 - 3*a^3*
f*x*cos(f*x + e)^2 + 3*a^3*f*x*cos(f*x + e) - a^3*f*x)*sin(f*x + e))/((c^4*
f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*s
in(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx+\int \frac{\sec^3(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx+\int \frac{1}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
+ Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**4 - 4*sec(e
+ f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A]
time = 0.55, size = 88, normalized size = 0.66

$$\frac{\frac{105\,(fx+e)a^3}{c^4}+\frac{210\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-70\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+42\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,a^3}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(f*x + e)*a^3/c^4 + (210*a^3*tan(1/2*f*x + 1/2*e)^6 - 70*a^3*tan
(1/2*f*x + 1/2*e)^4 + 42*a^3*tan(1/2*f*x + 1/2*e)^2 - 15*a^3)/(c^4*tan(1/2*
f*x + 1/2*e)^7))/f
```

Mupad [B]
time = 1.44, size = 122, normalized size = 0.92

$$\frac{a^3\left(-\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{7}+\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+(e+f\,x)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\right)}{c^4\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^4,x)
```

```
[Out] (a^3*(2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 - cos(e/2 + (f*x)/2)^7/7 +
sin(e/2 + (f*x)/2)^7*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)
^4)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/5))/(c^4*f*sin(e/2 +
(f*x)/2)^7)
```

3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=164

$$\frac{a^3x}{c^5}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}-\frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+f$$

```
[Out] a^3*x/c^5-8/9*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5+4/63*a^3*tan(f*x+e)/c^5
/f/(1-sec(f*x+e))^4-38/105*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-181/315*a^
3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-496/315*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x
+e))
```

Rubi [A]
time = 0.52, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$-\frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}-\frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{a^3x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*x)/c^5 - (8*a^3*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) + (4*a^3*
Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*
c^5*f*(1 - Sec[e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e
+ f*x])^2) - (496*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
```

```
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{\int \left(\frac{a^3}{(1-\sec(e+fx))^5}+\frac{3a^3\sec(e+fx)}{(1-\sec(e+fx))^5}+\frac{3a^3\sec^2(e+fx)}{(1-\sec(e+fx))^5}+\frac{a^3\sec^3(e+fx)}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$= \frac{a^3\int \frac{1}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{a^3\int \frac{\sec^3(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{(3a^3)\int \frac{\sec(e+fx)}{(1-\sec(e+fx))^5}\,dx}{c^5}+\frac{(3a^3$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}-\frac{a^3\int \frac{-9-4\sec(e+fx)}{(1-\sec(e+fx))^4}\,dx}{9c^5}+\frac{a^3\int \frac{(-5-9\sec(e+fx))\sec(e+fx}{(1-\sec(e+fx))^4}}{9c^5}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}+\frac{a^3\int \frac{63+39\sec(e+fx)}{(1-\sec(e+fx))^3}\,dx}{63c^5}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+f}$$

$$= -\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+f}$$

$$= \frac{a^3x}{c^5}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+}{105c^5f(1-\sec(}$$

$$= \frac{a^3x}{c^5}-\frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5}+\frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4}-\frac{38a^3\tan(e+}{105c^5f(1-\sec(}$$

Mathematica [A]
time = 0.96, size = 283, normalized size = 1.73

$$\frac{a^3\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(39690fx\cos\left(\frac{fx}{2}\right)-39690fx\cos\left(e+\frac{fx}{2}\right)-26460fx\cos\left(e+\frac{3fx}{2}\right)+26460fx\cos\left(2e+\frac{3fx}{2}\right)+11340fx\cos\left(2e+\frac{5fx}{2}\right)-11340fx\cos\left(3e+\frac{5fx}{2}\right)-2835fx\cos\left(3e+\frac{7fx}{2}\right)+2835fx\cos\left(4e+\frac{7fx}{2}\right)+315fx\cos\left(4e+\frac{9fx}{2}\right)-315fx\cos\left(5e+\frac{9fx}{2}\right)-142002\sin\left(\frac{fx}{2}\right)-122850\sin\left(e+\frac{fx}{2}\right)+103278\sin\left(e+\frac{3fx}{2}\right)+73290\sin\left(2e+\frac{3fx}{2}\right)-51102\sin\left(2e+\frac{5fx}{2}\right)-24570\sin\left(3e+\frac{5fx}{2}\right)+13878\sin\left(3e+\frac{7fx}{2}\right)+5040\sin\left(4e+\frac{7fx}{2}\right)-2102\sin\left(4e+\frac{9fx}{2}\right)\right)}{161280c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*Csc[e/2]*Csc[(e + f*x)/2]^9*(39690*f*x*Cos[(f*x)/2] - 39690*f*x*Cos[e
+ (f*x)/2] - 26460*f*x*Cos[e + (3*f*x)/2] + 26460*f*x*Cos[2*e + (3*f*x)/2]
+ 11340*f*x*Cos[2*e + (5*f*x)/2] - 11340*f*x*Cos[3*e + (5*f*x)/2] - 2835*f*
x*Cos[3*e + (7*f*x)/2] + 2835*f*x*Cos[4*e + (7*f*x)/2] + 315*f*x*Cos[4*e +
(9*f*x)/2] - 315*f*x*Cos[5*e + (9*f*x)/2] - 142002*Sin[(f*x)/2] - 122850*Si
n[e + (f*x)/2] + 103278*Sin[e + (3*f*x)/2] + 73290*Sin[2*e + (3*f*x)/2] - 5
```

```
1102*Sin[2*e + (5*f*x)/2] - 24570*Sin[3*e + (5*f*x)/2] + 13878*Sin[3*e + (7
*f*x)/2] + 5040*Sin[4*e + (7*f*x)/2] - 2102*Sin[4*e + (9*f*x)/2]))/(161280*
c^5*f)
```

Maple [A]
time = 0.19, size = 90, normalized size = 0.55

method	result
derivativedivides	$\frac{a^3\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2fc^5}$
default	$\frac{a^3\left(4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{2fc^5}$
risch	$\frac{a^3x}{c^5}+\frac{2ia^3\left(2520\,e^{8i(fx+e)}-12285\,e^{7i(fx+e)}+36645\,e^{6i(fx+e)}-61425\,e^{5i(fx+e)}+71001\,e^{4i(fx+e)}-51639\,e^{3i(fx+e)}+255\right.}{315fc^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^3x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{a^3x\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{a^3}{18cf}-\frac{41a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{126cf}+\frac{557a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{630cf}-\frac{353a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{210cf}+56a^3}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*a^3/c^5*(4*arctan(tan(1/2*f*x+1/2*e))+1/9/tan(1/2*f*x+1/2*e)^9-3/7/ta
n(1/2*f*x+1/2*e)^7+4/5/tan(1/2*f*x+1/2*e)^5-4/3/tan(1/2*f*x+1/2*e)^3+4/tan(
1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(154) = 308$.
time = 0.52, size = 439, normalized size = 2.68

$$\frac{a^3\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5}-\frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right)-\frac{3a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}-\frac{15a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}-\frac{7a^3\left(\frac{18\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{45\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-5\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}}{5040f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(a^3*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 3*a^3*(180*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e
) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 15*a^3*(18*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6
3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*
```

```
x + e)^9) - 7*a^3*(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Fricas [A]
time = 3.10, size = 227, normalized size = 1.38

$$\frac{1051\,a^{3}\cos\left(fx+e\right)^{5}-1684\,a^{3}\cos\left(fx+e\right)^{4}+898\,a^{3}\cos\left(fx+e\right)^{3}+1468\,a^{3}\cos\left(fx+e\right)^{2}-1669\,a^{3}\cos\left(fx+e\right)+496\,a^{3}+315\,\left(a^{3}fx\cos\left(fx+e\right)^{4}-4\,a^{3}fx\cos\left(fx+e\right)^{3}+6\,a^{3}fx\cos\left(fx+e\right)^{2}-4\,a^{3}fx\cos\left(fx+e\right)+a^{3}fx\right)\sin\left(fx+e\right)}{315\,\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}-4\,c^{5}f\cos\left(fx+e\right)+c^{5}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(1051*a^3*cos(f*x + e)^5 - 1684*a^3*cos(f*x + e)^4 + 898*a^3*cos(f*x
+ e)^3 + 1468*a^3*cos(f*x + e)^2 - 1669*a^3*cos(f*x + e) + 496*a^3 + 315*(a
^3*f*x*cos(f*x + e)^4 - 4*a^3*f*x*cos(f*x + e)^3 + 6*a^3*f*x*cos(f*x + e)^2
 - 4*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 -
 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c
^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{3}\left(\int\frac{3\sec\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-5\sec^{4}\left(e+fx\right)+10\sec^{3}\left(e+fx\right)-10\sec^{2}\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^{2}\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-5\sec^{4}\left(e+fx\right)+10\sec^{3}\left(e+fx\right)-10\sec^{2}\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{3}\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-5\sec^{4}\left(e+fx\right)+10\sec^{3}\left(e+fx\right)-10\sec^{2}\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{1}{\sec^{5}\left(e+fx\right)-5\sec^{4}\left(e+fx\right)+10\sec^{3}\left(e+fx\right)-10\sec^{2}\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx\right)}{c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*se
c(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*s
ec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 -
10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(se
c(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2
 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 - 5*sec(e + f*x)**
4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Giac [A]
time = 0.58, size = 104, normalized size = 0.63

$$\frac{\frac{630\,(fx+e)a^{3}}{c^{5}}+\frac{1260\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-420\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+252\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-135\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+35\,a^{3}}{c^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}}{630\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/630*(630*(f*x + e)*a^3/c^5 + (1260*a^3*tan(1/2*f*x + 1/2*e)^8 - 420*a^3*t
an(1/2*f*x + 1/2*e)^6 + 252*a^3*tan(1/2*f*x + 1/2*e)^4 - 135*a^3*tan(1/2*f*
x + 1/2*e)^2 + 35*a^3)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

Mupad [B]
time = 1.46, size = 146, normalized size = 0.89

$$\frac{a^3\left(\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{18}-\frac{3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{14}+\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{3}+2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8+(e+fx)\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9\right)}{c^5 f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^5,x)
```

```
[Out] (a^3*(cos(e/2 + (f*x)/2)^9/18 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8 +
 sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2
)^6)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/5 - (3*cos(e/2 + (f*
x)/2)^7*sin(e/2 + (f*x)/2)^2)/14))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{c^5x}{a^2}-\frac{47c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{13c^5\tan(e+fx)}{2a^2f}+\frac{112c^5\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{32c^5\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c^5-$$

```
[Out] c^5*x/a^2-47/2*c^5*arctanh(sin(f*x+e))/a^2/f+13/2*c^5*tan(f*x+e)/a^2/f+112/
3*c^5*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-32/3*c^5*tan(f*x+e)/f/(a+a*sec(f*x+e)
)^2+1/2*(c^5-c^5*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A]
time = 0.28, antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 26, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3989, 3971, 3554, 8, 2686, 2687, 30, 3852, 2701, 308, 213, 2700, 276, 294}

$$\frac{7c^5\tan(e+fx)}{a^2f}-\frac{64c^5\cot^3(e+fx)}{3a^2f}-\frac{48c^5\cot(e+fx)}{a^2f}+\frac{131c^5\csc^3(e+fx)}{6a^2f}+\frac{33c^5\csc(e+fx)}{2a^2f}-\frac{47c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{c^5\csc^3(e+fx)\sec^2(e+fx)}{2a^2f}+\frac{c^5x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^5*x)/a^2 - (47*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (48*c^5*Cot[e + f*
x])/(a^2*f) - (64*c^5*Cot[e + f*x]^3)/(3*a^2*f) + (33*c^5*Csc[e + f*x])/(2*
a^2*f) + (131*c^5*Csc[e + f*x]^3)/(6*a^2*f) - (c^5*Csc[e + f*x]^3*Sec[e + f
*x]^2)/(2*a^2*f) + (7*c^5*Tan[e + f*x])/(a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx &= \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^7\,dx}{a^2c^2} \\
&= \frac{\int \left(c^7\cot^4(e+fx)-7c^7\cot^3(e+fx)\csc(e+fx)+21c^7\cot^2(e+fx)\csc^2(e+\right.}{} \\
&= \frac{c^5\int \cot^4(e+fx)\,dx}{a^2}-\frac{c^5\int \csc^4(e+fx)\sec^3(e+fx)\,dx}{a^2}-\frac{(7c^5)\int \cot^3(e+f}{a} \\
&= -\frac{c^5\cot^3(e+fx)}{3a^2f}-\frac{c^5\int \cot^2(e+fx)\,dx}{a^2}+\frac{c^5\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx,x,\csc(e+\right.}{a^2f} \\
&= -\frac{34c^5\cot(e+fx)}{a^2f}-\frac{19c^5\cot^3(e+fx)}{a^2f}-\frac{7c^5\csc(e+fx)}{a^2f}+\frac{14c^5\csc^3(e+fx}{a^2f} \\
&= \frac{c^5x}{a^2}-\frac{48c^5\cot(e+fx)}{a^2f}-\frac{64c^5\cot^3(e+fx)}{3a^2f}+\frac{14c^5\csc(e+fx)}{a^2f}+\frac{21c^5\csc^3(e}{a^2f} \\
&= \frac{c^5x}{a^2}-\frac{21c^5\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{48c^5\cot(e+fx)}{a^2f}-\frac{64c^5\cot^3(e+fx)}{3a^2f}+\frac{33}{} \\
&= \frac{c^5x}{a^2}-\frac{47c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{48c^5\cot(e+fx)}{a^2f}-\frac{64c^5\cot^3(e+fx)}{3a^2f}+\frac{33}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(136) = 272.

time = 3.44, size = 384, normalized size = 2.82

$$\frac{\cos^3(e+fx)\cot\left(\frac{1}{2}(e+fx)\right)\csc^6\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(-\frac{320\cot^2\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}-\frac{64\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}+3\cot^3\left(\frac{1}{2}(e+fx)\right)\left(-4x-\frac{94\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{94\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{1}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{1}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{28\sin(fx)}{f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)-\frac{64\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)}{f}\right)}{96a^2(1+\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^6*(c - c*Sec[e + f*x])^5*
((-320*Cot[(e + f*x)/2]^2*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2])/f - (64*C
sc[(e + f*x)/2]^3*Sec[e/2]*Sin[(f*x)/2])/f + 3*Cot[(e + f*x)/2]^3*(-4*x - (
94*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f + (94*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]])/f + 1/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) - 1/(
f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) - (28*Sin[f*x])/(f*(Cos[e/2] - S
in[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))) - (64*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2
*Tan[e/2])/f))/(96*a^2*(1 + Sec[e + f*x])^2)
```

Maple [A]
time = 0.17, size = 137, normalized size = 1.01

method	result
derivativedivides	$\frac{16c^5\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}+\frac{\arctan\left(\tan\left(\frac{fx}{2}\right.\right.}{8}\right.}{f\,a^2}$
default	$\frac{16c^5\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}+\frac{\arctan\left(\tan\left(\frac{fx}{2}\right.\right.}{8}\right.}{f\,a^2}$
risch	$\frac{c^5x}{a^2}+\frac{ic^5\left(99\,e^{6i(fx+e)}+435\,e^{5i(fx+e)}+484\,e^{4i(fx+e)}+930\,e^{3i(fx+e)}+575\,e^{2i(fx+e)}+507\,e^{i(fx+e)}+202\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}+1\right)^2}+\frac{47c^5\ln\left(e^{i(\right.}}{2a^2}$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4c^5x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{6c^5x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4c^5x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{45c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{491c^5\left(\tan^3\right.}{3a}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/f*c^5/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)-1/32/(tan(1/2*f
*x+1/2*e)-1)^2-15/32/(tan(1/2*f*x+1/2*e)-1)+47/32*ln(tan(1/2*f*x+1/2*e)-1)+
1/8*arctan(tan(1/2*f*x+1/2*e))+1/32/(tan(1/2*f*x+1/2*e)+1)^2-15/32/(tan(1/2
*f*x+1/2*e)+1)-47/32*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(135) = 270.
time = 0.49, size = 653, normalized size = 4.80

$$\frac{c^5\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+5c^5\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+10c^5\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-c^5\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{10c^5\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{5c^5\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f
*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 5*c^5*((15*
sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
- 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
- c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 10*c^5*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 5
*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)
^3)/a^2)/f
```

Fricas [A]
time = 2.31, size = 260, normalized size = 1.91

$$\frac{12\,c^5fx\cos\left(fx+e\right)^4+24\,c^5fx\cos\left(fx+e\right)^3+12\,c^5fx\cos\left(fx+e\right)^2-141\,\left(c^5\cos\left(fx+e\right)^4+2\,c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)+141\,\left(c^5\cos\left(fx+e\right)^4+2\,c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\,\left(202\,c^5\cos\left(fx+e\right)^3+305\,c^5\cos\left(fx+e\right)^2+36\,c^5\cos\left(fx+e\right)-3\,c^5\right)\sin\left(fx+e\right)}{12\,\left(a^2f\cos\left(fx+e\right)^4+2\,a^2f\cos\left(fx+e\right)^3+a^2f\cos\left(fx+e\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^5*f*x*cos(f*x + e)^4 + 24*c^5*f*x*cos(f*x + e)^3 + 12*c^5*f*x*co
s(f*x + e)^2 - 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f*x
 + e)^2)*log(sin(f*x + e) + 1) + 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x +
e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(202*c^5*cos(f*x + e)
^3 + 305*c^5*cos(f*x + e)^2 + 36*c^5*cos(f*x + e) - 3*c^5)*sin(f*x + e))/(a
^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^5\left(\int\frac{5\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{10\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{10\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{5\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^5\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{1}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
```

```
Integral(10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-5*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.57, size = 153, normalized size = 1.12

$$\frac{\frac{6(fx+e)c^5}{a^2}-\frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}+\frac{141\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{6\left(15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-13\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2}+\frac{32\left(a^4c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(f*x + e)*c^5/a^2 - 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*(15*c^5*tan(1/2*f*x + 1
/2*e)^3 - 13*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2)
 + 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^5*tan(1/2*f*x + 1/2*e))/a^6
)/f
```

Mupad [B]
time = 1.50, size = 145, normalized size = 1.07

$$\frac{c^5\,x}{a^2}-\frac{15\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-13\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)}+\frac{32\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}+\frac{16\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}-\frac{47\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c^5*x)/a^2 - (15*c^5*tan(e/2 + (f*x)/2)^3 - 13*c^5*tan(e/2 + (f*x)/2))/(f*
(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) + (32*c^5*ta
n(e/2 + (f*x)/2))/(a^2*f) + (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) - (47*c
^5*atanh(tan(e/2 + (f*x)/2)))/(a^2*f)
```

3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{c^4x}{a^2}-\frac{6c^4\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{16c^4\cot(e+fx)}{a^2f}-\frac{32c^4\cot^3(e+fx)}{3a^2f}+\frac{32c^4\csc^3(e+fx)}{3a^2f}+\frac{c^4\tan(e+fx)}{a^2f}$$

```
[Out] c^4*x/a^2-6*c^4*arctanh(sin(f*x+e))/a^2/f-16*c^4*cot(f*x+e)/a^2/f-32/3*c^4*
cot(f*x+e)^3/a^2/f+32/3*c^4*csc(f*x+e)^3/a^2/f+c^4*tan(f*x+e)/a^2/f
```

Rubi [A]
time = 0.22, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3989, 3971, 3554, 8, 2686, 2687, 30, 3852, 2701, 308, 213, 2700, 276}

$$\frac{c^4\tan(e+fx)}{a^2f}-\frac{32c^4\cot^3(e+fx)}{3a^2f}-\frac{16c^4\cot(e+fx)}{a^2f}+\frac{32c^4\csc^3(e+fx)}{3a^2f}-\frac{6c^4\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{c^4x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^4*x)/a^2 - (6*c^4*ArcTanh[Sin[e + f*x]])/(a^2*f) - (16*c^4*Cot[e + f*x])
/(a^2*f) - (32*c^4*Cot[e + f*x]^3)/(3*a^2*f) + (32*c^4*Csc[e + f*x]^3)/(3*a
^2*f) + (c^4*Tan[e + f*x])/(a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^6\,dx}{a^2c^2}$$
$$= \frac{\int \left(c^6\cot^4(e+fx) - 6c^6\cot^3(e+fx)\csc(e+fx) + 15c^6\cot^2(e+fx)\csc^2(e+\right.}{}$$
$$= \frac{c^4\int \cot^4(e+fx)\,dx}{a^2} + \frac{c^4\int \csc^4(e+fx)\sec^2(e+fx)\,dx}{a^2} - \frac{(6c^4)\int \cot^3(e+f}{a}$$
$$= -\frac{c^4\cot^3(e+fx)}{3a^2f} - \frac{c^4\int \cot^2(e+fx)\,dx}{a^2} + \frac{c^4\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(e+f\right.}{a^2f}$$
$$= -\frac{14c^4\cot(e+fx)}{a^2f} - \frac{31c^4\cot^3(e+fx)}{3a^2f} - \frac{6c^4\csc(e+fx)}{a^2f} + \frac{26c^4\csc^3(e+fx}{3a^2f}$$
$$= \frac{c^4x}{a^2} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\tan(e+}{a^2f}$$
$$= \frac{c^4x}{a^2} - \frac{6c^4\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 753 vs. $2(102) = 204$.
time = 6.28, size = 753, normalized size = 7.38

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (x*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e +
f*x])^4)/(4*(a + a*Sec[e + f*x])^2) + (3*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^
```

```
4*Csc[e/2 + (f*x)/2]^4*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*
Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x])^2) - (3*Cos[e + f*x]^2*Cot[e/2 +
 (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*Log[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2
]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x])^2) + (4*Cos[e + f*x]^2
*Cot[e/2 + (f*x)/2]^3*Csc[e/2 + (f*x)/2]^5*Sec[e/2]*(c - c*Sec[e + f*x])^4*
Sin[(f*x)/2])/(3*f*(a + a*Sec[e + f*x])^2) + (2*Cos[e + f*x]^2*Cot[e/2 + (f
*x)/2]*Csc[e/2 + (f*x)/2]^7*Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(
3*f*(a + a*Sec[e + f*x])^2) + (Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2
+ (f*x)/2]^4*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(4*f*(a + a*Sec[e + f*x])
^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) + (Cos[
e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4
*Sin[(f*x)/2])/(4*f*(a + a*Sec[e + f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
 (f*x)/2] + Sin[e/2 + (f*x)/2])) + (2*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^2*C
sc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*Tan[e/2])/(3*f*(a + a*Sec[e + f*
x])^2)
```

Maple [A]
time = 0.14, size = 105, normalized size = 1.03

method	result
derivativedivides	$\frac{8c^4\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\right.}{f\,a^2}$
default	$\frac{8c^4\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\right.}{f\,a^2}$
risch	$\frac{c^4x}{a^2}+\frac{2ic^4\left(51\,e^{3i(fx+e)}+25\,e^{2i(fx+e)}+57\,e^{i(fx+e)}+19\right)}{3f\,a^2\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}+1\right)^3}+\frac{6c^4\ln\left(e^{i(fx+e)}-i\right)}{a^2f}-\frac{6c^4\ln\left(e^{i(fx+e)}+i\right)}{a^2f}$
norman	$\frac{\frac{c^4x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{c^4x}{a}-\frac{10c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{76c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{18c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{8c^4\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{3c^4x\left(\tan^2\left(\ldots\right.\right.}{a}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*c^4/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x+1/2*e)-1/8/(tan(1/2*f*x+1
/2*e)-1)+3/4*ln(tan(1/2*f*x+1/2*e)-1)+1/4*arctan(tan(1/2*f*x+1/2*e))-1/8/(t
an(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(103) = 206$.
time = 0.50, size = 447, normalized size = 4.38

$$\frac{c^4\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+4\,c^4\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-c^4\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{6\,c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{4\,c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^4*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 4*c^4*((9*sin(f*x + e
)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/a^2) - c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) +
6*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2 - 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(103) = 206$.
time = 2.28, size = 237, normalized size = 2.32

$$\frac{3\,c^4 fx\cos(fx+e)^3+6\,c^4 fx\cos(fx+e)^2+3\,c^4 fx\cos(fx+e)-9\left(c^4\cos(fx+e)^3+2\,c^4\cos(fx+e)^2+c^4\cos(fx+e)\right)\log(\sin(fx+e)+1)+9\left(c^4\cos(fx+e)^3+2\,c^4\cos(fx+e)^2+c^4\cos(fx+e)\right)\log(-\sin(fx+e)+1)+\left(19\,c^4\cos(fx+e)^2+38\,c^4\cos(fx+e)+3\,c^4\right)\sin(fx+e)}{3\left(a^2 f\cos(fx+e)^3+2\,a^2 f\cos(fx+e)^2+a^2 f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^4*f*x*cos(f*x + e)^3 + 6*c^4*f*x*cos(f*x + e)^2 + 3*c^4*f*x*cos(f*
x + e) - 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*l
og(sin(f*x + e) + 1) + 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*c
os(f*x + e))*log(-sin(f*x + e) + 1) + (19*c^4*cos(f*x + e)^2 + 38*c^4*cos(f
*x + e) + 3*c^4)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{6\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{4\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(6*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.51, size = 134, normalized size = 1.31

$$\frac{\frac{3\,(fx+e)c^4}{a^2}-\frac{18\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}+\frac{18\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{6\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2}+\frac{8\left(a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^4/a^2 - 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
18*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*c^4*tan(1/2*f*x + 1/2*e)/
((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) + 8*(a^4*c^4*tan(1/2*f*x + 1/2*e)^3 + 3*
a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.47, size = 112, normalized size = 1.10

$$\frac{c^4\,x}{a^2}+\frac{8\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}+\frac{8\,c^4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3\,a^2\,f}-\frac{12\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}-\frac{2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c^4*x)/a^2 + (8*c^4*tan(e/2 + (f*x)/2))/(a^2*f) + (8*c^4*tan(e/2 + (f*x)/2
)^3)/(3*a^2*f) - (12*c^4*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (2*c^4*tan(e/
2 + (f*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{c^3x}{a^2}-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}+\frac{4c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

```
[Out] c^3*x/a^2-c^3*arctanh(sin(f*x+e))/a^2/f-8/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*x
+e))^2+4/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))
```

Rubi [A]
time = 0.25, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3988, 3862, 4004, 3879, 3881, 3882, 3884, 4083, 3855}

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{4c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{8c^3\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^3*x)/a^2 - (c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (8*c^3*Tan[e + f*x])/(3
*a^2*f*(1 + Sec[e + f*x])^2) + (4*c^3*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \left(\frac{c^3}{(1+\sec(e+fx))^2} - \frac{3c^3\sec(e+fx)}{(1+\sec(e+fx))^2} + \frac{3c^3\sec^2(e+fx)}{(1+\sec(e+fx))^2} - \frac{c^3\sec^3(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}$$

$$= \frac{c^3\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2} - \frac{c^3\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} - \frac{(3c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2} + \frac{(3c^3)}{}$$

$$= -\frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} - \frac{c^3\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2} - \frac{c^3\int \frac{\sec(e+fx)(-2+3\sec(e+fx))}{1+\sec(e+fx)}}{3a^2}$$

$$= \frac{c^3x}{a^2} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{c^3\tan(e+fx)}{a^2f(1+\sec(e+fx))} - \frac{c^3\int \sec(e+fx)\,dx}{a^2}$$

$$= \frac{c^3x}{a^2} - \frac{c^3\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{4c^3\tan(e+fx}{3a^2f(1+\sec(e+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 216 vs. $2(85) = 170$.
time = 1.17, size = 216, normalized size = 2.54

$$-\frac{c^3(-1+\cos(e+fx))^3\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(3\cot^3\left(\frac{1}{2}(e+fx)\right)\left(fx+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-4\cot^2\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+4\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+4\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)}{6a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/6*(c^3*(-1 + Cos[e + f*x])^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(3*Cot[
(e + f*x)/2]^3*(f*x + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]) - 4*Cot[(e + f*x)/2]^2*Csc[(e + f*x)/2]*Sec
[e/2]*Sin[(f*x)/2] + 4*Csc[(e + f*x)/2]^3*Sec[e/2]*Sin[(f*x)/2] + 4*Cot[(e
+ f*x)/2]*Csc[(e + f*x)/2]^2*Tan[e/2]))/(a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 0.14, size = 66, normalized size = 0.78

method	result
derivativedivides	$\frac{4c^3\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,a^2}$
default	$\frac{4c^3\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,a^2}$
risch	$\frac{c^3x}{a^2}-\frac{8ic^3\left(3\,e^{2i(fx+e)}+1\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3}+\frac{c^3\ln\left(e^{i(fx+e)}-i\right)}{a^2f}-\frac{c^3\ln\left(e^{i(fx+e)}+i\right)}{a^2f}$

norman	$\frac{\frac{c^3x}{a}+\frac{c^3x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{4c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{8c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{4c^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{2c^3x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2a}+\frac{c^3\ln\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*c^3/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+1/4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*arct
an(tan(1/2*f*x+1/2*e))-1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(86) = 172$.
time = 0.51, size = 290, normalized size = 3.41

$$\frac{c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{3c^3\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{3c^3\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - c^3*((9*sin(f*x + e)/(cos(f*x + e) +
 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/a^2 - 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1)
 - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(86) = 172$.
time = 2.17, size = 185, normalized size = 2.18

$$\frac{6\,c^3fx\cos(fx+e)^2+12\,c^3fx\cos(fx+e)+6\,c^3fx-3\,(c^3\cos(fx+e)^2+2\,c^3\cos(fx+e)+c^3)\log(\sin(fx+e)+1)+3\,(c^3\cos(fx+e)^2+2\,c^3\cos(fx+e)+c^3)\log(-\sin(fx+e)+1)-8\,(c^3\cos(fx+e)-c^3)\sin(fx+e)}{6\,(a^2f\cos(fx+e)^2+2\,a^2f\cos(fx+e)+a^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*c^3*f*x*cos(f*x + e)^2 + 12*c^3*f*x*cos(f*x + e) + 6*c^3*f*x - 3*(c^
3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(sin(f*x + e) + 1) + 3*(c^3
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 8*(c^3
*cos(f*x + e) - c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x
+ e) + a^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3\left(\int\frac{3\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integr
al(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.51, size = 80, normalized size = 0.94

$$\frac{\frac{4\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{a^2}+\frac{3\,(fx+e)c^3}{a^2}-\frac{3\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}+\frac{3\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*c^3*tan(1/2*f*x + 1/2*e)^3/a^2 + 3*(f*x + e)*c^3/a^2 - 3*c^3*log(abs
(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 3*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/
a^2)/f
```

Mupad [B]
time = 1.41, size = 46, normalized size = 0.54

$$\frac{c^3\,x}{a^2}-\frac{c^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c^3*x)/a^2 - (c^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*tan(e/2 + (f*x)/2)^3)/
3))/(a^2*f)
```

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=67

$$\frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

```
[Out] c^2*x/a^2-4/3*c^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-4/3*c^2*tan(f*x+e)/a^2/
f/(1+sec(f*x+e))
```

Rubi [A]
time = 0.17, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3988, 3862, 4004, 3879, 3881, 3882}

$$-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{4c^2\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{c^2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*x)/a^2 - (4*c^2*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (4*c^2*
Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx &= \frac{\int \left(\frac{c^2}{(1+\sec(e+fx))^2}-\frac{2c^2\sec(e+fx)}{(1+\sec(e+fx))^2}+\frac{c^2\sec^2(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}\\
&= \frac{c^2\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2}+\frac{c^2\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}-\frac{(2c^2)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}\\
&= -\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c^2\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{(4c^2)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 1.00

$$
\frac{c^2\left(\frac{2\text{ArcTan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*((2*ArcTan[Tan[e/2 + (f*x)/2]])/f - (2*Tan[e/2 + (f*x)/2])/f + (2*Tan[
e/2 + (f*x)/2]^3)/(3*f)))/a^2
```

Maple [A]
time = 0.13, size = 47, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2c^2\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	47
default	$\frac{2c^2\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	47
risch	$\frac{c^2x}{a^2}-\frac{8ic^2\left(3\,\mathrm{e}^{2i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}+2\right)}{3f\,a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}$	59
norman	$\frac{\frac{c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{c^2x}{a}+\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{8c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{2c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c^2/a^2*(1/3*tan(1/2*f*x+1/2*e)^3-tan(1/2*f*x+1/2*e)+arctan(tan(1/2*f*x
+1/2*e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(67) = 134$.
time = 0.53, size = 184, normalized size = 2.75

$$-\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{2\,c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*si
n(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 +
2*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2)/f
```

Fricas [A]
time = 2.86, size = 100, normalized size = 1.49

$$\frac{3\,c^2fx\cos\left(fx+e\right)^2+6\,c^2fx\cos\left(fx+e\right)+3\,c^2fx-4\,(2\,c^2\cos\left(fx+e\right)+c^2)\sin\left(fx+e\right)}{3\,\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
[Out] 1/3*(3*c^2*f*x*cos(f*x + e)^2 + 6*c^2*f*x*cos(f*x + e) + 3*c^2*f*x - 4*(2*c
^2*cos(f*x + e) + c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*
x + e) + a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}dx+\int\frac{1}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.47, size = 60, normalized size = 0.90

$$\frac{\frac{3\,(fx+e)c^2}{a^2}+\frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")
[Out] 1/3*(3*(f*x + e)*c^2/a^2 + 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*ta
n(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.38, size = 38, normalized size = 0.57

$$\frac{2\,c^2\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{3\,f\,x}{2}\right)}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^2,x)
[Out] (2*c^2*(tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) + (3*f*x)/2))/(3*a^2*f)
```

3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

```
[Out] c*x/a^2-2/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-5/3*c*tan(f*x+e)/a^2/f/(1+s
ec(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {3988, 3862, 4004, 3879, 3881}

$$-\frac{5c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)^2}+\frac{cx}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*x)/a^2 - (2*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e
+ f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx &= \frac{\int \left(\frac{c}{(1+\sec(e+fx))^2}-\frac{c\sec(e+fx)}{(1+\sec(e+fx))^2}\right)dx}{a^2}\\
&= \frac{c\int \frac{1}{(1+\sec(e+fx))^2}\,dx}{a^2}-\frac{c\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{a^2}\\
&= -\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c\int \frac{-3+\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}-\frac{c\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{(4c)\int \frac{\sec(e+fx)}{1+\sec(e+fx)}\,dx}{3a^2}\\
&= \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 113, normalized size = 1.85

$$\frac{c\sec\left(\frac{e}{2}\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\left(9fx\cos\left(\frac{fx}{2}\right)+9fx\cos\left(e+\frac{fx}{2}\right)+3fx\cos\left(e+\frac{3fx}{2}\right)+3fx\cos\left(2e+\frac{3fx}{2}\right)-24\sin\left(\frac{fx}{2}\right)+18\sin\left(e+\frac{fx}{2}\right)-14\sin\left(e+\frac{3fx}{2}\right)\right)}{24a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^3*(9*f*x*Cos[(f*x)/2] + 9*f*x*Cos[e + (f*x)/2]
 + 3*f*x*Cos[e + (3*f*x)/2] + 3*f*x*Cos[2*e + (3*f*x)/2] - 24*Sin[(f*x)/2]
+ 18*Sin[e + (f*x)/2] - 14*Sin[e + (3*f*x)/2]))/(24*a^2*f)
```

Maple [A]
time = 0.11, size = 46, normalized size = 0.75

method	result	size
derivativedivides	$\frac{c\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	46
default	$\frac{c\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$	46
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{a}$	50
risch	$\frac{cx}{a^2}-\frac{2ic\left(9\,e^{2i(fx+e)}+12\,e^{i(fx+e)}+7\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c/a^2*(1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f*x+1/2*e)+2*arctan(tan(1/2*f
*x+1/2*e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(61) = 122$.
time = 0.50, size = 129, normalized size = 2.11

$$-\frac{c\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{c\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c*(3*sin(f*
x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 3.32, size = 92, normalized size = 1.51

$$\frac{3\,cfx\cos\left(fx+e\right)^2+6\,cfx\cos\left(fx+e\right)+3\,cfx-\left(7\,c\cos\left(fx+e\right)+5\,c\right)\sin\left(fx+e\right)}{3\,\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c*f*x*cos(f*x + e)^2 + 6*c*f*x*cos(f*x + e) + 3*c*f*x - (7*c*cos(f*x
 + e) + 5*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a
^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.49, size = 53, normalized size = 0.87

$$\frac{\frac{3(fx+e)c}{a^2}+\frac{a^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6a^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c/a^2 + (a^4*c*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*tan(1/2*f*
x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.34, size = 41, normalized size = 0.67

$$\frac{c\,x}{a^2}-\frac{c\left(6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^2,x)
```

```
[Out] (c*x)/a^2 - (c*(6*tan(e/2 + (f*x)/2) - tan(e/2 + (f*x)/2)^3))/(3*a^2*f)
```

3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=69

$$\frac{x}{a^2c}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}$$

```
[Out] x/a^2/c+1/3*cot(f*x+e)*(3-2*sec(f*x+e))/a^2/c/f-1/3*cot(f*x+e)^3*(1-sec(f*x
+e))/a^2/c/f
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3967, 8}

$$-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}+\frac{x}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] x/(a^2*c) + (Cot[e + f*x]*(3 - 2*Sec[e + f*x]))/(3*a^2*c*f) - (Cot[e + f*x]
^3*(1 - Sec[e + f*x]))/(3*a^2*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\int \cot^4(e+fx)(c-c\sec(e+fx))\,dx}{a^2c^2}$$
$$= -\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\int \cot^2(e+fx)(-3c+2}{3a^2c^2}$$
$$= \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+}{3a^2cf}$$
$$= \frac{x}{a^2c} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-}{3a^2cf}$$

Mathematica [A]
time = 0.58, size = 135, normalized size = 1.96

$$\frac{\csc\left(\frac{e}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(6fx\cos(fx)-6fx\cos(2e+fx)+3fx\cos(e+2fx)-3fx\cos(3e+2fx)-10\sin(fx)+10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)-8\sin(e+2fx))}{96a^2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]*Sec[e/2]*Sec[(e + f*x)/2]^3*(6*f*x*Cos[f*x] - 6*
f*x*Cos[2*e + f*x] + 3*f*x*Cos[e + 2*f*x] - 3*f*x*Cos[3*e + 2*f*x] - 10*Sin
[f*x] + 10*Sin[e + f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] - 8*Sin[e +
 2*f*x]))/(96*a^2*c*f)
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.87

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f\,a^2c}$	60
default	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f\,a^2c}$	60
risch	$\frac{x}{a^2c}-\frac{2i\left(3\,e^{3i(fx+e)}-5\,e^{i(fx+e)}-4\right)}{3f\,a^2c\left(e^{i(fx+e)}+1\right)^3\left(e^{i(fx+e)}-1\right)}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{4acf}-\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)}{12acf}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a^2/c*(1/3*tan(1/2*f*x+1/2*e)^3-4*tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+1/
2*e)+8*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [A]
time = 0.50, size = 110, normalized size = 1.59

$$-\frac{\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}-\frac{24\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c}-\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*((12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/(a^2*c) - 24*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c) - 3*(co
s(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Fricas [A]
time = 2.40, size = 76, normalized size = 1.10

$$\frac{4\cos(fx+e)^2+3(fx\cos(fx+e)+fx)\sin(fx+e)+\cos(fx+e)-2}{3(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(f*x + e)^2 + 3*(f*x*cos(f*x + e) + f*x)*sin(f*x + e) + cos(f*x +
 e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1}\,dx}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1), x)/(a**
2*c)
```

Giac [A]
time = 0.49, size = 81, normalized size = 1.17

$$\frac{\frac{12(fx+e)}{a^2c}+\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(f*x + e)/(a^2*c) + 3/(a^2*c*tan(1/2*f*x + 1/2*e)) + (a^4*c^2*tan(
1/2*f*x + 1/2*e)^3 - 12*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

Mupad [B]
time = 1.42, size = 69, normalized size = 1.00

$$\frac{x}{a^2\,c}+\frac{\frac{4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}-\frac{7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{6}+\frac{1}{12}}{a^2\,c\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

```
[Out] x/(a^2*c) + ((4*cos(e/2 + (f*x)/2)^4)/3 - (7*cos(e/2 + (f*x)/2)^2)/6 + 1/12
)/(a^2*c*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a^2c^2}+\frac{\cot(e+fx)}{a^2c^2f}-\frac{\cot^3(e+fx)}{3a^2c^2f}$$

```
[Out] x/a^2/c^2+cot(f*x+e)/a^2/c^2/f-1/3*cot(f*x+e)^3/a^2/c^2/f
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3989, 3554, 8}

$$-\frac{\cot^3(e+fx)}{3a^2c^2f}+\frac{\cot(e+fx)}{a^2c^2f}+\frac{x}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] x/(a^2*c^2) + Cot[e + f*x]/(a^2*c^2*f) - Cot[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)\,dx}{a^2c^2}$$
$$= -\frac{\cot^3(e+fx)}{3a^2c^2f} - \frac{\int \cot^2(e+fx)\,dx}{a^2c^2}$$
$$= \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f} + \frac{\int 1\,dx}{a^2c^2}$$
$$= \frac{x}{a^2c^2} + \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 39, normalized size = 0.85

$$-\frac{\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(a^
2*c^2*f)
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.70

method	result	size
default	$\frac{-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e}{a^2c^2f}$	32
risch	$\frac{x}{a^2c^2}+\frac{4i\left(3\,e^{4i(fx+e)}-3\,e^{2i(fx+e)}+2\right)}{3f\,a^2c^2\left(e^{i(fx+e)}+1\right)^3\left(e^{i(fx+e)}-1\right)^3}$	72
norman	$\frac{\frac{x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}-\frac{1}{24acf}+\frac{5\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}-\frac{5\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}+\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{24acf}}{ca\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	116
derivativedivides	`error in RationalFunction: argument is not a rational function\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/c^2/f*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)
```

Maxima [A]
time = 0.51, size = 49, normalized size = 1.07

$$\frac{\frac{3\,(fx+e)}{a^2c^2}+\frac{3\,\tan(fx+e)^2-1}{a^2c^2\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^2 - 1)/(a^2*c^2*tan(f*x + e)^3
))/f
```

Fricas [A]
time = 2.14, size = 87, normalized size = 1.89

$$\frac{4\,\cos\left(fx+e\right)^3+3\left(fx\cos\left(fx+e\right)^2-fx\right)\sin\left(fx+e\right)-3\,\cos\left(fx+e\right)}{3\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(f*x + e)^3 + 3*(f*x*cos(f*x + e)^2 - f*x)*sin(f*x + e) - 3*cos(f
*x + e))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.
time = 0.50, size = 95, normalized size = 2.07

$$\frac{\frac{24\,(fx+e)}{a^2c^2}+\frac{15\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}{a^2c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}+\frac{a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6c^6}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(f*x + e)/(a^2*c^2) + (15*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^2*c^2*tan
(1/2*f*x + 1/2*e)^3) + (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 15*a^4*c^4*tan(1/2
*f*x + 1/2*e))/(a^6*c^6))/f
```

Mupad [B]
time = 1.47, size = 58, normalized size = 1.26

$$-\frac{\cos\left(3\,e+3\,f\,x\right)+\frac{3\,\sin\left(3\,e+3\,f\,x\right)\,\left(e+f\,x\right)}{4}-\frac{9\,\sin\left(e+f\,x\right)\,\left(e+f\,x\right)}{4}}{3\,a^2\,c^2\,f\,{\sin\left(e+f\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

```
[Out] -(cos(3*e + 3*f*x) + (3*sin(3*e + 3*f*x)*(e + f*x))/4 - (9*sin(e + f*x)*(e
+ f*x))/4)/(3*a^2*c^2*f*sin(e + f*x)^3)
```

3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3} \, dx$

Optimal. Leaf size=98

$$\frac{x}{a^2c^3}+\frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f}-\frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f}+\frac{\cot(e+fx)(15+8\sec(e+fx))}{15a^2c^3f}$$

```
[Out] x/a^2/c^3+1/5*cot(f*x+e)^5*(1+sec(f*x+e))/a^2/c^3/f-1/15*cot(f*x+e)^3*(5+4*
sec(f*x+e))/a^2/c^3/f+1/15*cot(f*x+e)*(15+8*sec(f*x+e))/a^2/c^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3967, 8}

$$\frac{\cot^5(e+fx)(\sec(e+fx)+1)}{5a^2c^3f}-\frac{\cot^3(e+fx)(4\sec(e+fx)+5)}{15a^2c^3f}+\frac{\cot(e+fx)(8\sec(e+fx)+15)}{15a^2c^3f}+\frac{x}{a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] x/(a^2*c^3) + (Cot[e + f*x]^5*(1 + Sec[e + f*x]))/(5*a^2*c^3*f) - (Cot[e +
f*x]^3*(5 + 4*Sec[e + f*x]))/(15*a^2*c^3*f) + (Cot[e + f*x]*(15 + 8*Sec[e +
 f*x]))/(15*a^2*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))\,dx}{a^3c^3}$$

$$= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\int \cot^4(e+fx)(-5a-4$$

$$= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e}{15a^2c^3f}$$

$$= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e}{15a^2c^3f}$$

$$= \frac{x}{a^2c^3} + \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+}{15a^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(98) = 196.

time = 1.35, size = 257, normalized size = 2.62

$$\frac{\csc\left(\frac{e}{2}\right)\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(360fx\cos(fx)-360fx\cos(2e+fx)-120fx\cos(e+2fx)+120fx\cos(3e+2fx)-120fx\cos(2e+3fx)+120fx\cos(4e+3fx)+60fx\cos(3e+4fx)-60fx\cos(5e+4fx)+200\sin(e)-584\sin(fx)-534\sin(e+fx)+178\sin(2(e+fx))+178\sin(3(e+fx))-89\sin(4(e+fx))-520\sin(2e+fx)+248\sin(e+2fx)+120\sin(3e+2fx)+248\sin(2e+3fx)+120\sin(4e+3fx)-184\sin(3e+4fx))}{30720a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^5*Sec[e/2]*Sec[(e + f*x)/2]^3*(360*f*x*Cos[f*x]
- 360*f*x*Cos[2*e + f*x] - 120*f*x*Cos[e + 2*f*x] + 120*f*x*Cos[3*e + 2*f*x
] - 120*f*x*Cos[2*e + 3*f*x] + 120*f*x*Cos[4*e + 3*f*x] + 60*f*x*Cos[3*e +
4*f*x] - 60*f*x*Cos[5*e + 4*f*x] + 200*Sin[e] - 584*Sin[f*x] - 534*Sin[e +
f*x] + 178*Sin[2*(e + f*x)] + 178*Sin[3*(e + f*x)] - 89*Sin[4*(e + f*x)] -
520*Sin[2*e + f*x] + 248*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 248*Sin[2*
e + 3*f*x] + 120*Sin[4*e + 3*f*x] - 184*Sin[3*e + 4*f*x]))/(30720*a^2*c^3*f
)
```

Maple [A]
time = 0.13, size = 88, normalized size = 0.90

method	result	si
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^3a^2}$	88
default	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^3a^2}$	88
risch	$\frac{x}{a^2c^3}+\frac{2i\left(15\,e^{7i(fx+e)}+15\,e^{6i(fx+e)}-65\,e^{5i(fx+e)}+25\,e^{4i(fx+e)}+73\,e^{3i(fx+e)}-31\,e^{2i(fx+e)}-31\,e^{i(fx+e)}+23\right)}{15f\,c^3a^2\left(e^{i(fx+e)}-1\right)^5\left(e^{i(fx+e)}+1\right)^3}$	12

norman	$\frac{\frac{\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}+\frac{1}{80acf}-\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}-\frac{3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}+\frac{\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)}{48acf}}{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/c^3/a^2*(1/3*tan(1/2*f*x+1/2*e)^3-6*tan(1/2*f*x+1/2*e)+32*arctan(tan
(1/2*f*x+1/2*e))+1/5/tan(1/2*f*x+1/2*e)^5-2/tan(1/2*f*x+1/2*e)^3+16/tan(1/2
*f*x+1/2*e))
```

Maxima [A]
time = 0.52, size = 159, normalized size = 1.62

$$-\frac{\frac{5\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^3}+\frac{3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{80\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(5*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/(a^2*c^3) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^3)
 + 3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 80*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^5))/f
```

Fricas [A]
time = 3.03, size = 166, normalized size = 1.69

$$\frac{23\cos(fx+e)^4-8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3-fx\cos(fx+e)^2-fx\cos(fx+e)+fx\right)\sin(fx+e)+7\cos(fx+e)+8}{15\left(a^2c^3f\cos(fx+e)^3-a^2c^3f\cos(fx+e)^2-a^2c^3f\cos(fx+e)+a^2c^3f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^4 - 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*co
s(f*x + e)^3 - f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) + f*x)*sin(f*x + e) +
7*cos(f*x + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 -
 a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{\sec^5(e+fx)-\sec^4(e+fx)-2\sec^3(e+fx)+2\sec^2(e+fx)+\sec(e+fx)-1}\,dx}{a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(
e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Giac [A]
time = 0.50, size = 110, normalized size = 1.12

$$\frac{\frac{240\,(fx+e)}{a^2c^3}+\frac{3\left(80\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}+\frac{5\left(a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-18\,a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^9}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)/(a^2*c^3) + 3*(80*tan(1/2*f*x + 1/2*e)^4 - 10*tan(1/2*
f*x + 1/2*e)^2 + 1)/(a^2*c^3*tan(1/2*f*x + 1/2*e)^5) + 5*(a^4*c^6*tan(1/2*f
*x + 1/2*e)^3 - 18*a^4*c^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

Mupad [B]
time = 1.54, size = 161, normalized size = 1.64

$$\frac{3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-90\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-30\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+240\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5(e+f\,x)}{240\,a^2\,c^3\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (3*cos(e/2 + (f*x)/2)^8 + 5*sin(e/2 + (f*x)/2)^8 - 90*cos(e/2 + (f*x)/2)^2*
sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 30*c
os(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 + 240*cos(e/2 + (f*x)/2)^3*sin(e/2
 + (f*x)/2)^5*(e + f*x))/(240*a^2*c^3*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x
)/2)^5)
```

3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=166

$$\frac{x}{a^2c^4}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+$$

```
[Out] x/a^2/c^4+cot(f*x+e)/a^2/c^4/f-1/3*cot(f*x+e)^3/a^2/c^4/f+1/5*cot(f*x+e)^5/
a^2/c^4/f-2/7*cot(f*x+e)^7/a^2/c^4/f+2*csc(f*x+e)/a^2/c^4/f-2*csc(f*x+e)^3/
a^2/c^4/f+6/5*csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Rubi [A]
time = 0.16, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30}

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}+\frac{x}{a^2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] x/(a^2*c^4) + Cot[e + f*x]/(a^2*c^4*f) - Cot[e + f*x]^3/(3*a^2*c^4*f) + Cot
[e + f*x]^5/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + (2*Csc[e + f
*x])/(a^2*c^4*f) - (2*Csc[e + f*x]^3)/(a^2*c^4*f) + (6*Csc[e + f*x]^5)/(5*a
^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a^2*c^4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^2\,dx}{a^4c^4}$$
$$= \frac{\int \left(a^2\cot^8(e+fx)+2a^2\cot^7(e+fx)\csc(e+fx)+a^2\cot^6\right.}{a^4c^4}$$
$$= \frac{\int \cot^8(e+fx)\,dx}{a^2c^4}+\frac{\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^2c^4}+\frac{2\int}{}$$
$$= -\frac{\cot^7(e+fx)}{7a^2c^4f}-\frac{\int \cot^6(e+fx)\,dx}{a^2c^4}+\frac{\text{Subst}\left(\int x^6\,dx,x,-\right.}{a^2c^4f}$$
$$= \frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\int \cot^4(e+fx)\,dx}{a^2c^4}-\frac{2\text{Sul}}{}$$
$$= -\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e}{a^2c}$$
$$= \frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+f}{7a^2c^4f}$$
$$= \frac{x}{a^2c^4}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot}{7}$$

Mathematica [A]
time = 1.36, size = 315, normalized size = 1.90

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^7*Sec[e/2]*Sec[(e + f*x)/2]^3*(5880*f*x*Cos[f*x]
 - 5880*f*x*Cos[2*e + f*x] - 3360*f*x*Cos[e + 2*f*x] + 3360*f*x*Cos[3*e + 2
*f*x] - 1260*f*x*Cos[2*e + 3*f*x] + 1260*f*x*Cos[4*e + 3*f*x] + 1680*f*x*Co
s[3*e + 4*f*x] - 1680*f*x*Cos[5*e + 4*f*x] - 420*f*x*Cos[4*e + 5*f*x] + 420
*f*x*Cos[6*e + 5*f*x] + 4032*Sin[e] - 9632*Sin[f*x] - 16002*Sin[e + f*x] +
9144*Sin[2*(e + f*x)] + 3429*Sin[3*(e + f*x)] - 4572*Sin[4*(e + f*x)] + 114
3*Sin[5*(e + f*x)] - 11760*Sin[2*e + f*x] + 8864*Sin[e + 2*f*x] + 3360*Sin[
3*e + 2*f*x] + 2064*Sin[2*e + 3*f*x] + 2520*Sin[4*e + 3*f*x] - 4432*Sin[3*e
 + 4*f*x] - 1680*Sin[5*e + 4*f*x] + 1528*Sin[4*e + 5*f*x]))/(860160*a^2*c^4
*f)
```

Maple [A]
time = 0.13, size = 101, normalized size = 0.61

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{42}{\tan\left(\frac{fx}{2}+\right.}}{32f\,c^4a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{42}{\tan\left(\frac{fx}{2}+\right.}}{32f\,c^4a^2}$
risch	$\frac{x}{a^2c^4}+\frac{2i\left(210\,e^{9i(fx+e)}-315\,e^{8i(fx+e)}-420\,e^{7i(fx+e)}+1470\,e^{6i(fx+e)}-504\,e^{5i(fx+e)}-1204\,e^{4i(fx+e)}+1108\,e^{3i(fx+e}\right.}{105f\,c^4a^2\left(e^{i(fx+e)}-1\right)^7\left(e^{i(fx+e)}+1\right)^3}$
norman	$\frac{\frac{x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}-\frac{1}{224acf}+\frac{7\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{160acf}-\frac{11\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{48acf}+\frac{21\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}-\frac{7\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32acf}+\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{96acf}}{a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/f/c^4/a^2*(1/3*tan(1/2*f*x+1/2*e)^3-7*tan(1/2*f*x+1/2*e)+64*arctan(tan
(1/2*f*x+1/2*e))-1/7/tan(1/2*f*x+1/2*e)^7+7/5/tan(1/2*f*x+1/2*e)^5-22/3/tan
(1/2*f*x+1/2*e)^3+42/tan(1/2*f*x+1/2*e))
```

Maxima [A]

time = 0.49, size = 181, normalized size = 1.09

$$-\frac{\frac{35\left(\frac{21\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}-\frac{6720\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^4}-\frac{\left(\frac{147\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{770\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4410\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{a^2c^4\sin(fx+e)^7}}{3360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/3360*(35*(21*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2*c^4) - 6720*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c
^4) - (147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 770*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 4410*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Fricas [A]

time = 2.03, size = 179, normalized size = 1.08

$$\frac{191\cos(fx+e)^5-172\cos(fx+e)^4-253\cos(fx+e)^3+258\cos(fx+e)^2+105\left(fx\cos(fx+e)^4-2\,fx\cos(fx+e)^3+2\,fx\cos(fx+e)-fx\right)\sin(fx+e)+87\cos(fx+e)-96}{105\left(a^2c^4f\cos(fx+e)^4-2\,a^2c^4f\cos(fx+e)^3+2\,a^2c^4f\cos(fx+e)-a^2c^4f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(191*cos(f*x + e)^5 - 172*cos(f*x + e)^4 - 253*cos(f*x + e)^3 + 258*c
os(f*x + e)^2 + 105*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(
```

```
f*x + e) - f*x)*sin(f*x + e) + 87*cos(f*x + e) - 96)/((a^2*c^4*f*cos(f*x +
e)^4 - 2*a^2*c^4*f*cos(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*s
in(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{6}\left(e+fx\right)-2\sec^{5}\left(e+fx\right)-\sec^{4}\left(e+fx\right)+4\sec^{3}\left(e+fx\right)-\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{a^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(1/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**4 + 4*sec(e
 + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**4)
```

Giac [A]
time = 0.51, size = 122, normalized size = 0.73

$$\frac{\frac{3360\,(fx+e)}{a^{2}c^{4}}+\frac{4410\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+147\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-15}{a^{2}c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}}+\frac{35\left(a^{4}c^{8}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-21\,a^{4}c^{8}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{6}c^{12}}}{3360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(f*x + e)/(a^2*c^4) + (4410*tan(1/2*f*x + 1/2*e)^6 - 770*tan(1
/2*f*x + 1/2*e)^4 + 147*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^2*c^4*tan(1/2*f*x +
 1/2*e)^7) + 35*(a^4*c^8*tan(1/2*f*x + 1/2*e)^3 - 21*a^4*c^8*tan(1/2*f*x +
1/2*e))/(a^6*c^12))/f
```

Mupad [B]
time = 1.64, size = 185, normalized size = 1.11

$$\frac{35\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-735\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+4410\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}-770\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+147\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+3360\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}(e+fx)}{3360\,a^{2}c^{4}f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (35*sin(e/2 + (f*x)/2)^10 - 15*cos(e/2 + (f*x)/2)^10 - 735*cos(e/2 + (f*x)/
2)^2*sin(e/2 + (f*x)/2)^8 + 4410*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^6
- 770*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^4 + 147*cos(e/2 + (f*x)/2)^8*
sin(e/2 + (f*x)/2)^2 + 3360*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7*(e +
f*x))/(3360*a^2*c^4*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7)
```

3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=210

$$\frac{x}{a^2c^5}+\frac{\cot(e+fx)}{a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{3\csc(e+fx)}{a^2c^5f}-\frac{13\csc^3(e}{3a^2c^5}$$

```
[Out] x/a^2/c^5+cot(f*x+e)/a^2/c^5/f-1/3*cot(f*x+e)^3/a^2/c^5/f+1/5*cot(f*x+e)^5/
a^2/c^5/f-1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+3*csc(f*x+e
)/a^2/c^5/f-13/3*csc(f*x+e)^3/a^2/c^5/f+21/5*csc(f*x+e)^5/a^2/c^5/f-15/7*cs
c(f*x+e)^7/a^2/c^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Rubi [A]
time = 0.22, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot(e+fx)}{a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{21\csc^5(e+fx)}{5a^2c^5f}-\frac{13\csc^3(e+fx)}{3a^2c^5f}+\frac{3\csc(e+fx)}{a^2c^5f}+\frac{x}{a^2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] x/(a^2*c^5) + Cot[e + f*x]/(a^2*c^5*f) - Cot[e + f*x]^3/(3*a^2*c^5*f) + Cot
[e + f*x]^5/(5*a^2*c^5*f) - Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^
9)/(9*a^2*c^5*f) + (3*Csc[e + f*x])/(a^2*c^5*f) - (13*Csc[e + f*x]^3)/(3*a^
2*c^5*f) + (21*Csc[e + f*x]^5)/(5*a^2*c^5*f) - (15*Csc[e + f*x]^7)/(7*a^2*c
^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx &= -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^3\,dx}{a^5c^5} \\
&= -\frac{\int \left(a^3\cot^{10}(e+fx)+3a^3\cot^9(e+fx)\csc(e+fx)+3a\right.}{} \\
&= -\frac{\int \cot^{10}(e+fx)\,dx}{a^2c^5} - \frac{\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^2c^5} - \\
&= \frac{\cot^9(e+fx)}{9a^2c^5f} + \frac{\int \cot^8(e+fx)\,dx}{a^2c^5} + \frac{\mathrm{Subst}\left(\int x^2(-1+x^2\right.}{a^2} \\
&= -\frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} - \frac{\int \cot^6(e+fx)\,dx}{a^2c^5} + \frac{\mathrm{S}}{} \\
&= \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{3\csc(e-}{a^2c^5} \\
&= -\frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e}{9a^2c} \\
&= \frac{\cot(e+fx)}{a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^7(e+f}{7a^2c^5f} \\
&= \frac{x}{a^2c^5} + \frac{\cot(e+fx)}{a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \mathrm{cot}
\end{aligned}
$$

Mathematica [A]
time = 1.31, size = 383, normalized size = 1.82

$$
\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\sec^6(e+fx)(181440fx\cos(fx)-181440fx\cos(2e+fx)-136080fx\cos(e+2fx)+136080fx\cos(3e+2fx)-10080fx\cos(2e+3fx)+10080fx\cos(4e+3fx)+60480fx\cos(3e+4fx)-60480fx\cos(5e+4fx)-30240fx\cos(4e+5fx)+30240fx\cos(6e+5fx)+5040fx\cos(5e+6fx)-5040fx\cos(7e+6fx)+169344\sin(e)-338112\sin(fx)-675036\sin(e+fx)+\cdots)}{645120a^2c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^6*(181440*f*x*Cos[f*x] - 181440*f*x*Cos[2*e
 + f*x] - 136080*f*x*Cos[e + 2*f*x] + 136080*f*x*Cos[3*e + 2*f*x] - 10080*f
*x*Cos[2*e + 3*f*x] + 10080*f*x*Cos[4*e + 3*f*x] + 60480*f*x*Cos[3*e + 4*f*
x] - 60480*f*x*Cos[5*e + 4*f*x] - 30240*f*x*Cos[4*e + 5*f*x] + 30240*f*x*Co
s[6*e + 5*f*x] + 5040*f*x*Cos[5*e + 6*f*x] - 5040*f*x*Cos[7*e + 6*f*x] + 16
9344*Sin[e] - 338112*Sin[f*x] - 675036*Sin[e + f*x] + 506277*Sin[2*(e + f*x
)] + 37502*Sin[3*(e + f*x)] - 225012*Sin[4*(e + f*x)] + 112506*Sin[5*(e + f
*x)] - 18751*Sin[6*(e + f*x)] - 431424*Sin[2*e + f*x] + 375552*Sin[e + 2*f*
x] + 201600*Sin[3*e + 2*f*x] - 41248*Sin[2*e + 3*f*x] + 84000*Sin[4*e + 3*f
*x] - 155712*Sin[3*e + 4*f*x] - 100800*Sin[5*e + 4*f*x] + 98016*Sin[4*e + 5
*f*x] + 30240*Sin[6*e + 5*f*x] - 21376*Sin[5*e + 6*f*x])*Tan[e + f*x])/(645
120*a^2*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^2)
```

Maple [A]
time = 0.14, size = 114, normalized size = 0.54

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{64}{3\tan\left(\frac{fx}{2}+\cdots\right.}\cdots}{64fc^5a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{64}{3\tan\left(\frac{fx}{2}+\cdots\right.}\cdots}{64fc^5a^2}$
risch	$\frac{x}{a^2c^5}+\frac{2i\left(945\,e^{11i(fx+e)}-3150\,e^{10i(fx+e)}+2625\,e^{9i(fx+e)}+6300\,e^{8i(fx+e)}-13482\,e^{7i(fx+e)}+5292\,e^{6i(fx+e)}+10566\,e^{5\cdots}\right.}{315fc^5a^2\left(e^{i(fx+e)}-1\right)^9\left(e^{i(fx+e)}+\cdots\right.}$
norman	$\frac{\frac{x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}+\frac{1}{576acf}-\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{56acf}+\frac{29\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{320acf}-\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}+\frac{99\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64acf}-\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}+\frac{\tan^{12}\left(\cdots\right.}{192a\cdots}}{ac^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/f/c^5/a^2*(1/3*tan(1/2*f*x+1/2*e)^3-8*tan(1/2*f*x+1/2*e)+128*arctan(ta
n(1/2*f*x+1/2*e))+1/9/tan(1/2*f*x+1/2*e)^9-8/7/tan(1/2*f*x+1/2*e)^7+29/5/ta
n(1/2*f*x+1/2*e)^5-64/3/tan(1/2*f*x+1/2*e)^3+99/tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.53, size = 202, normalized size = 0.96

$$-\frac{\frac{105\left(\frac{24\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5}-\frac{40320\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^5}+\frac{\left(\frac{360\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1827\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{31185\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{a^2c^5\sin(fx+e)^9}}{20160f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/20160*(105*(24*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2*c^5) - 40320*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^
2*c^5) + (360*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1827*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 6720*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 31185*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(f*x +
 e)^9))/f
```

Fricas [A]
time = 2.78, size = 250, normalized size = 1.19

$$\frac{668\cos(fx+e)^6-1059\cos(fx+e)^5-573\cos(fx+e)^4+1813\cos(fx+e)^3-393\cos(fx+e)^2+315\left(fx\cos(fx+e)^5-3fx\cos(fx+e)^4+2fx\cos(fx+e)^3+2fx\cos(fx+e)^2-3fx\cos(fx+e)+fx\right)\sin(fx+e)-789\cos(fx+e)+368}{315\left(a^2c^5f\cos(fx+e)^5-3a^2c^5f\cos(fx+e)^4+2a^2c^5f\cos(fx+e)^3+2a^2c^5f\cos(fx+e)^2-3a^2c^5f\cos(fx+e)+a^2c^5f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(668*cos(f*x + e)^6 - 1059*cos(f*x + e)^5 - 573*cos(f*x + e)^4 + 1813
*cos(f*x + e)^3 - 393*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^5 - 3*f*x*cos(
f*x + e)^4 + 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 - 3*f*x*cos(f*x +
e) + f*x)*sin(f*x + e) - 789*cos(f*x + e) + 368)/((a^2*c^5*f*cos(f*x + e)^5
 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f*co
s(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sec^{7}{\left(e+fx \right)}-3\sec^{6}{\left(e+fx \right)}+\sec^{5}{\left(e+fx \right)}+5\sec^{4}{\left(e+fx \right)}-5\sec^{3}{\left(e+fx \right)}-\sec^{2}{\left(e+fx \right)}+3\sec{\left(e+fx \right)}-1}\, dx}{a^{2}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec(
e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)
/(a**2*c**5)
```

Giac [A]

time = 0.55, size = 135, normalized size = 0.64

$$\frac{\frac{20160\,(fx+e)}{a^2c^5}+\frac{31185\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-6720\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+1827\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-360\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35}{a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}+\frac{105\left(a^4c^{10}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-24\,a^4c^{10}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^{15}}}{20160\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/20160*(20160*(f*x + e)/(a^2*c^5) + (31185*tan(1/2*f*x + 1/2*e)^8 - 6720*t
an(1/2*f*x + 1/2*e)^6 + 1827*tan(1/2*f*x + 1/2*e)^4 - 360*tan(1/2*f*x + 1/2
*e)^2 + 35)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) + 105*(a^4*c^10*tan(1/2*f*x +
1/2*e)^3 - 24*a^4*c^10*tan(1/2*f*x + 1/2*e))/(a^6*c^15))/f
```

Mupad [B]

time = 1.78, size = 209, normalized size = 1.00

$$\frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+105\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-2520\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+31185\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-6720\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+1827\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-360\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+20160\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9(e+fx)}{20160\,a^2\,c^5\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (35*cos(e/2 + (f*x)/2)^12 + 105*sin(e/2 + (f*x)/2)^12 - 2520*cos(e/2 + (f*x
)/2)^2*sin(e/2 + (f*x)/2)^10 + 31185*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2
)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1827*cos(e/2 + (f*x)
/2)^8*sin(e/2 + (f*x)/2)^4 - 360*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^2
 + 20160*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9*(e + f*x))/(20160*a^2*c^
5*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9)
```

3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3} \, dx$

Optimal. Leaf size=162

$$\frac{c^5x}{a^3}+\frac{8c^5\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}-\frac{16c^5\csc(e+fx)}{a^3f}$$

```
[Out] c^5*x/a^3+8*c^5*arctanh(sin(f*x+e))/a^3/f+32*c^5*cot(f*x+e)/a^3/f+128/3*c^5
*cot(f*x+e)^3/a^3/f+128/5*c^5*cot(f*x+e)^5/a^3/f-16*c^5*csc(f*x+e)/a^3/f+64
/3*c^5*csc(f*x+e)^3/a^3/f-128/5*c^5*csc(f*x+e)^5/a^3/f-c^5*tan(f*x+e)/a^3/f
```

Rubi [A]
time = 0.32, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30, 14, 3852, 2701, 308, 213, 2700, 276}

$$-\frac{c^5\tan(e+fx)}{a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}-\frac{128c^5\csc^5(e+fx)}{5a^3f}+\frac{64c^5\csc^3(e+fx)}{3a^3f}-\frac{16c^5\csc(e+fx)}{a^3f}+\frac{8c^5\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{c^5x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^5*x)/a^3 + (8*c^5*ArcTanh[Sin[e + f*x]])/(a^3*f) + (32*c^5*Cot[e + f*x])
/(a^3*f) + (128*c^5*Cot[e + f*x]^3)/(3*a^3*f) + (128*c^5*Cot[e + f*x]^5)/(5
*a^3*f) - (16*c^5*Csc[e + f*x])/(a^3*f) + (64*c^5*Csc[e + f*x]^3)/(3*a^3*f)
 - (128*c^5*Csc[e + f*x]^5)/(5*a^3*f) - (c^5*Tan[e + f*x])/(a^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^8\,dx}{a^3c^3}$$

$$= -\frac{\int \left(c^8\cot^6(e+fx) - 8c^8\cot^5(e+fx)\csc(e+fx) + 28c^8\cot^4(e+fx)\csc^2(e\right.}{}$$

$$= -\frac{c^5\int \cot^6(e+fx)\,dx}{a^3} - \frac{c^5\int \csc^6(e+fx)\sec^2(e+fx)\,dx}{a^3} + \frac{(8c^5)\int \cot^5(e+f}{a}$$

$$= \frac{c^5\cot^5(e+fx)}{5a^3f} + \frac{c^5\int \cot^4(e+fx)\,dx}{a^3} - \frac{c^5\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx, x, \tan(e+fx)\right)}{a^3f}$$

$$= \frac{28c^5\cot(e+fx)}{a^3f} + \frac{55c^5\cot^3(e+fx)}{3a^3f} + \frac{57c^5\cot^5(e+fx)}{5a^3f} - \frac{56c^5\csc^5(e+fx)}{5a^3f}$$

$$= \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128c^5\cot^5(e+fx)}{5a^3f} - \frac{16c^5\csc(e+fx}{a^3f}$$

$$= \frac{c^5x}{a^3} + \frac{8c^5\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{32c^5\cot(e+fx)}{a^3f} + \frac{128c^5\cot^3(e+fx)}{3a^3f} + \frac{128}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 557 vs. $2(162) = 324$.
time = 6.17, size = 557, normalized size = 3.44

$$-\frac{c^5(-1+\cos(e+fx))^5\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(-60\cos(e)\cot^7\left(\frac{1}{2}(e+fx)\right)\left(fx-8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec\left(\frac{e}{2}\right)+48\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{e}{2}\right)\left(\sin\left(\frac{e}{2}\right)-\sin\left(\frac{3e}{2}\right)\right)+8(-7+\cos(e+fx))\cot^3\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{e}{2}\right)\left(\sin\left(\frac{e}{2}\right)-\sin\left(\frac{3e}{2}\right)\right)+1016\cot^6\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{fx}{2}\right)+(-140+76\cos(e)+131\cos(fx)-210\cos(e+fx)-84\cos(2(e+fx))-14\cos(3(e+fx))+131\cos(2e+fx)+66\cos(e+2fx)+66\cos(3e+2fx)+21\cos(2e+3fx)+21\cos(4e+3fx))\csc^7\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+2\cot^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(30\cos(e)\left(fx-8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-(\cos(e)+15(-1+\cos(fx)+\cos(e+fx)))\csc^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)\right)}{240a^3f(1+\cos(e+fx))^3\left(-1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(-1+\tan\left(\frac{e}{2}\right)\right)\left(1+\tan\left(\frac{e}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -1/240*(c^5*(-1 + Cos[e + f*x])^5*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Sec[e
/2]*(-60*Cos[e]*Cot[(e + f*x)/2]^7*(f*x - 8*Log[Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]] + 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sec[e/2] + 48*Cot[(
e + f*x)/2]*Csc[(e + f*x)/2]^4*Sec[e/2]^2*(Sin[e/2] - Sin[(3*e)/2]) + 8*(-7
 + Cos[e + f*x])*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2]^4*Sec[e/2]^2*(Sin[e/2]
 - Sin[(3*e)/2]) + 1016*Cot[(e + f*x)/2]^6*Csc[(e + f*x)/2]*Sin[(f*x)/2] +
(-140 + 76*Cos[e] + 131*Cos[f*x] - 210*Cos[e + f*x] - 84*Cos[2*(e + f*x)] -
 14*Cos[3*(e + f*x)] + 131*Cos[2*e + f*x] + 66*Cos[e + 2*f*x] + 66*Cos[3*e
+ 2*f*x] + 21*Cos[2*e + 3*f*x] + 21*Cos[4*e + 3*f*x])*Csc[(e + f*x)/2]^7*Se
c[e/2]^2*Sin[(f*x)/2] + 2*Cot[(e + f*x)/2]^5*Sec[e/2]*(30*Cos[e]*(f*x - 8*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 8*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]) - (Cos[e] + 15*(-1 + Cos[f*x] + Cos[e + f*x]))*Csc[(e + f*x)/2]^
2*Tan[e/2])))/(a^3*f*(1 + Cos[e + f*x])^3*(-1 + Cot[(e + f*x)/2])*(1 + Cot[
(e + f*x)/2])*(-1 + Tan[e/2])*(1 + Tan[e/2]))
```

Maple [**A**]
time = 0.17, size = 118, normalized size = 0.73

method	result
derivativedivides	$\frac{8c^5\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4} \ldots}{f a^3}$
default	$\frac{8c^5\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4} \ldots}{f a^3}$
risch	$\frac{c^5 x}{a^3}-\frac{2ic^5\left(240\,e^{6i(fx+e)}+735\,e^{5i(fx+e)}+1835\,e^{4i(fx+e)}+1750\,e^{3i(fx+e)}+1894\,e^{2i(fx+e)}+955\,e^{i(fx+e)}+239\right)}{15 f a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{2i(fx+e)}+1\right)}+\frac{8c^5 \ldots}{\ldots}$
norman	$\frac{\frac{c^5 x}{a}+\frac{c^5 x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4c^5 x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{6c^5 x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4c^5 x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{18c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{202c^5\left(\tan \ldots\right)}{\ldots}}{\left(\tan^2\left(\ldots\right)\ldots\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*c^5/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f
*x+1/2*e)+1/8/(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)-1)+1/4*arctan(ta
n(1/2*f*x+1/2*e))+ln(tan(1/2*f*x+1/2*e)+1)+1/8/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(162) = 324$.
time = 0.50, size = 610, normalized size = 3.77

$$-\frac{3\,c^5\left(\frac{40\,\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+5\,c^5\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+c^5\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{10\,c^5\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{5\,c^5\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{30\,c^5\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(3*c^5*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60
*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/a^3) + 5*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/
a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^3) + c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) -
 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 10*c^5*(15*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 5*c^5*(15*sin(f*x + e)/(cos(f*x
 + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)/a^3 - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 2.62, size = 311, normalized size = 1.92

$$\frac{15\,c^5fx\cos(fx+e)^4+45\,c^5fx\cos(fx+e)^3+45\,c^5fx\cos(fx+e)^2+15\,c^5fx\cos(fx+e)+60\left(c^5\cos(fx+e)^4+3\,c^5\cos(fx+e)^3+3\,c^5\cos(fx+e)^2+c^5\cos(fx+e)\right)\log(\sin(fx+e)+1)-60\left(c^5\cos(fx+e)^4+3\,c^5\cos(fx+e)^3+3\,c^5\cos(fx+e)^2+c^5\cos(fx+e)\right)\log(-\sin(fx+e)+1)-\left(239\,c^5\cos(fx+e)^3+477\,c^5\cos(fx+e)^2+349\,c^5\cos(fx+e)+15\,c^5\right)\sin(fx+e)}{15\left(a^3f\cos(fx+e)^4+3\,a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2+a^3f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^5*f*x*cos(f*x + e)^4 + 45*c^5*f*x*cos(f*x + e)^3 + 45*c^5*f*x*co
s(f*x + e)^2 + 15*c^5*f*x*cos(f*x + e) + 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos
(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(sin(f*x + e) + 1
) - 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 +
c^5*cos(f*x + e))*log(-sin(f*x + e) + 1) - (239*c^5*cos(f*x + e)^3 + 477*c^
5*cos(f*x + e)^2 + 349*c^5*cos(f*x + e) + 15*c^5)*sin(f*x + e))/(a^3*f*cos(
f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*
x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^5\left(\int\frac{5\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{10\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{10\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{5\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{1}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**3/(sec(e
+ f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-5*sec(e
 + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x))/a**3
```

Giac [A]
time = 0.56, size = 154, normalized size = 0.95

$$\frac{\frac{15\,(fx+e)c^5}{a^3}+\frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{30\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{8\left(3\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^5/a^3 + 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3
 - 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^5*tan(1/2*f*x + 1/
2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 8*(3*a^12*c^5*tan(1/2*f*x + 1/2*e
)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1/2*f*x + 1/2*e))
/a^15)/f
```

Mupad [B]
time = 1.49, size = 134, normalized size = 0.83

$$\frac{c^5\,x}{a^3}-\frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}-\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}+\frac{16\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}+\frac{2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^5*x)/a^3 - (16*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (8*c^5*tan(e/2 + (f*x)/
2)^3)/(3*a^3*f) - (8*c^5*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) + (16*c^5*atanh(ta
n(e/2 + (f*x)/2)))/(a^3*f) + (2*c^5*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (
f*x)/2)^2 - a^3))
```

3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{c^4x}{a^3}+\frac{c^4\tanh^{-1}(\sin(e+fx))}{a^3f}-\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}$$

```
[Out] c^4*x/a^3+c^4*arctanh(sin(f*x+e))/a^3/f-3*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+e
))^3-1/5*c^4*sec(f*x+e)^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+14/5*c^4*tan(f*
x+e)/a^3/f/(1+sec(f*x+e))^2-23/5*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A]
time = 0.44, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085, 3901, 4093, 4083, 3855}

$$\frac{c^4\tanh^{-1}(\sin(e+fx))}{a^3f}-\frac{c^4\tan(e+fx)\sec^2(e+fx)}{5a^3f(\sec(e+fx)+1)^3}-\frac{23c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}+\frac{14c^4\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{3c^4\tan(e+fx)}{a^3f(\sec(e+fx)+1)^3}+\frac{c^4x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^4*x)/a^3 + (c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) - (3*c^4*Tan[e + f*x])/(a
^3*f*(1 + Sec[e + f*x])^3) - (c^4*Sec[e + f*x]^2*Tan[e + f*x])/(5*a^3*f*(1
+ Sec[e + f*x])^3) + (14*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) -
 (23*c^4*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx &= \frac{\int \left(\frac{c^4}{(1+\sec(e+fx))^3}-\frac{4c^4\sec(e+fx)}{(1+\sec(e+fx))^3}+\frac{6c^4\sec^2(e+fx)}{(1+\sec(e+fx))^3}-\frac{4c^4\sec^3(e+fx)}{(1+\sec(e+fx))^3}+\frac{c^4\sec^4(e+fx)}{(1+\sec(e+fx))^3}\right.}{a^3} \\
&= \frac{c^4\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3}+\frac{c^4\int \frac{\sec^4(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{(4c^4)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{(4c^4)}{} \\
&= -\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{c^4\int \frac{(2-5\sec(e+fx))\sec}{(1+\sec(e+fx)}}{5a^3} \\
&= -\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{14c^4\tan(e+fx}{5a^3f(1+\sec(e+f} \\
&= \frac{c^4x}{a^3}-\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{14c^4\tan(e}{5a^3f(1+\sec(e} \\
&= \frac{c^4x}{a^3}+\frac{c^4\tanh^{-1}(\sin(e+fx))}{a^3f}-\frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(}{5a^3f(1+\sec(e+}
\end{aligned}
$$

Mathematica [A]
time = 1.32, size = 231, normalized size = 1.56

$$
\frac{c^4(-1+\cos(e+fx))^4\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(5\cot^5\left(\frac{1}{2}(e+fx)\right)\left(fx-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-(9+8\cos(e+fx)+3\cos(2(e+fx)))\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+8\cot^3\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)-4\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)}{10a^3f(1+\cos(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^4*(-1 + Cos[e + f*x])^4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(5*Cot[(e +
f*x)/2]^5*(f*x - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]) - (9 + 8*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Csc[
(e + f*x)/2]^5*Sec[e/2]*Sin[(f*x)/2] + 8*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2
]^2*Tan[e/2] - 4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Tan[e/2]))/(10*a^3*f*(
1 + Cos[e + f*x])^3)
```

Maple [A]
time = 0.16, size = 77, normalized size = 0.52

method	result
derivativedivides	$\frac{4c^4\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,a^3}$
default	$\frac{4c^4\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f\,a^3}$

risch	$\frac{c^4 x}{a^3}-\frac{16 i c^4 \left(5 \,{\mathrm e}^{4 i (f x+e)}+10 \,{\mathrm e}^{3 i (f x+e)}+20 \,{\mathrm e}^{2 i (f x+e)}+10 \,{\mathrm e}^{i (f x+e)}+3\right)}{5 f \,a^3 \left({\mathrm e}^{i (f x+e)}+1\right)^5}+\frac{c^4 \ln \left({\mathrm e}^{i (f x+e)}+i\right)}{a^3 f}-\frac{c^4 \ln \left({\mathrm e}^{i (f x+e)}-i\right)}{a^3 f}$
norman	$\frac{\frac{c^4 x \left(\tan^6\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a}-\frac{c^4 x}{a}+\frac{4 c^4 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}-\frac{12 c^4 \left(\tan^3\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a f}+\frac{64 c^4 \left(\tan^5\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 a f}-\frac{32 c^4 \left(\tan^7\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 a f}+\frac{12 c^4 \left(\tan^9\left(\frac{f}{}\right.\right.}{5 a f}}{\left(\tan^2\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^3 a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*c^4/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-tan(1/2*f*x+1/2*e)-1/4*ln(tan(1/2*f*
x+1/2*e)-1)+1/2*arctan(tan(1/2*f*x+1/2*e))+1/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(152) = 304$.
time = 0.50, size = 430, normalized size = 2.91

$$-\frac{c^4 \left(\frac{\frac{105 \sin(fx+e)}{\cos(fx+e)+1}+\frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+c^4 \left(\frac{\frac{105 \sin(fx+e)}{\cos(fx+e)+1}-\frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{4 c^4 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1}+\frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{4 c^4 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1}-\frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{18 c^4 \left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) + c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arc
tan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 4*c^4*(15*sin(f*x + e)/(cos(f*x
 + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)/a^3 + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^
3 - 18*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3)/f
```

Fricas [A]
time = 3.19, size = 259, normalized size = 1.75

$$\frac{10 \, c^4 f x \cos\left(f x + e\right)^3 + 30 \, c^4 f x \cos\left(f x + e\right)^2 + 30 \, c^4 f x \cos\left(f x + e\right) + 10 \, c^4 f x + 5 \, \left(c^4 \cos\left(f x + e\right)^3 + 3 \, c^4 \cos\left(f x + e\right)^2 + 3 \, c^4 \cos\left(f x + e\right) + c^4\right) \log\left(\sin\left(f x + e\right) + 1\right) - 5 \, \left(c^4 \cos\left(f x + e\right)^3 + 3 \, c^4 \cos\left(f x + e\right)^2 + 3 \, c^4 \cos\left(f x + e\right) + c^4\right) \log\left(-\sin\left(f x + e\right) + 1\right) - 16 \, \left(3 \, c^4 \cos\left(f x + e\right)^2 + 4 \, c^4 \cos\left(f x + e\right) + 3 \, c^4\right) \sin\left(f x + e\right)}{10 \, \left(a^3 f \cos\left(f x + e\right)^3 + 3 \, a^3 f \cos\left(f x + e\right)^2 + 3 \, a^3 f \cos\left(f x + e\right) + a^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(10*c^4*f*x*cos(f*x + e)^3 + 30*c^4*f*x*cos(f*x + e)^2 + 30*c^4*f*x*co
s(f*x + e) + 10*c^4*f*x + 5*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*
c^4*cos(f*x + e) + c^4)*log(sin(f*x + e) + 1) - 5*(c^4*cos(f*x + e)^3 + 3*c
^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*log(-sin(f*x + e) + 1) - 16*(
```

```
3*c^4*cos(f*x + e)^2 + 4*c^4*cos(f*x + e) + 3*c^4)*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{4\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^4\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{1}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [**A**]
time = 0.56, size = 102, normalized size = 0.69

$$\frac{\frac{5\,(fx+e)c^4}{a^3}+\frac{5\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{5\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{4\left(a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/5*(5*(f*x + e)*c^4/a^3 + 5*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 5
*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 4*(a^12*c^4*tan(1/2*f*x + 1/2
*e)^5 + 5*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [**B**]
time = 1.42, size = 50, normalized size = 0.34

$$\frac{c^4\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}+f\,x\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^4*(2*atanh(tan(e/2 + (f*x)/2)) - 4*tan(e/2 + (f*x)/2) - (4*tan(e/2 + (f*
x)/2)^5)/5 + f*x))/(a^3*f)
```

3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=96

$$\frac{c^3x}{a^3}-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{26c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

```
[Out] c^3*x/a^3-8/5*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+4/15*c^3*tan(f*x+e)/a^3
/f/(1+sec(f*x+e))^2-26/15*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A]
time = 0.29, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882, 3884, 4085}

$$-\frac{26c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)}+\frac{4c^3\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2}-\frac{8c^3\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{c^3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^3*x)/a^3 - (8*c^3*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) + (4*c^3*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (26*c^3*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{c^3}{(1+\sec(e+fx))^3}-\frac{3c^3\sec(e+fx)}{(1+\sec(e+fx))^3}+\frac{3c^3\sec^2(e+fx)}{(1+\sec(e+fx))^3}-\frac{c^3\sec^3(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$
$$=\frac{c^3\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{c^3\int \frac{\sec^3(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{(3c^3)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}+\frac{(3c^3)\int$$
$$=-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{c^3\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}-\frac{c^3\int \frac{\sec(e+fx)(-3+5\sec(e+fx))}{(1+\sec(e+fx))^2}\,d}{5a^3}$$
$$=-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}+\frac{c^3\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}-$$
$$=\frac{c^3x}{a^3}-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{4c^3\tan(e+f}{15a^3f(1+\sec(e+}$$
$$=\frac{c^3x}{a^3}-\frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}+\frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{26c^3\tan(e+f}{15a^3f(1+\sec(e+}$$

Mathematica [A]
time = 0.08, size = 90, normalized size = 0.94

$$-\frac{c^3\left(-\frac{2\text{ArcTan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
[Out] -((c^3*((-2*ArcTan[Tan[e/2 + (f*x)/2]])/f + (2*Tan[e/2 + (f*x)/2])/f - (2*T
an[e/2 + (f*x)/2]^3)/(3*f) + (2*Tan[e/2 + (f*x)/2]^5)/(5*f)))/a^3)
```

Maple [A]
time = 0.14, size = 60, normalized size = 0.62

method	result
derivativedivides	$\frac{2c^3\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^3}$
default	$\frac{2c^3\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^3}$

risch	$\frac{c^3 x}{a^3} - \frac{4 i c^3 \left(45\, e^{4 i (f x+e)}+90\, e^{3 i (f x+e)}+140\, e^{2 i (f x+e)}+70\, e^{i (f x+e)}+23\right)}{15 f\, a^3 \left(e^{i (f x+e)}+1\right)^5}$
norman	$\frac{\frac{c^3 x}{a}+\frac{c^3 x \left(\tan^4\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a}-\frac{2 c^3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}+\frac{14 c^3 \left(\tan^3\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 a f}-\frac{56 c^3 \left(\tan^5\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 a f}+\frac{22 c^3 \left(\tan^7\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 a f}-\frac{2 c^3 \left(\tan^9 (\right.}{5 a}}{\left(\tan^2\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^2 a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 2/f*c^3/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5+1/3*tan(1/2*f*x+1/2*e)^3-tan(1/2*f*x +1/2*e)+arctan(tan(1/2*f*x+1/2*e)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(96) = 192$.
time = 0.53, size = 301, normalized size = 3.14

$$-\frac{c^3 \left(\frac{\frac{105 \sin(fx+e)}{\cos(fx+e)+1}-\frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^3 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1}+\frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{3 c^3 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1}-\frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{9 c^3 \left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f *x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si n(f*x + e)/(cos(f*x + e) + 1))/a^3) + c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e) ^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^ 3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5) /a^3)/f

Fricas [A]
time = 3.12, size = 147, normalized size = 1.53

$$\frac{15\, c^3 f x \cos(fx+e)^3 + 45\, c^3 f x \cos(fx+e)^2 + 45\, c^3 f x \cos(fx+e) + 15\, c^3 f x - 2\left(23\, c^3 \cos(fx+e)^2 + 24\, c^3 \cos(fx+e) + 13\, c^3\right) \sin(fx+e)}{15\left(a^3 f \cos(fx+e)^3 + 3\, a^3 f \cos(fx+e)^2 + 3\, a^3 f \cos(fx+e) + a^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(15*c^3*f*x*cos(f*x + e)^3 + 45*c^3*f*x*cos(f*x + e)^2 + 45*c^3*f*x*co s(f*x + e) + 15*c^3*f*x - 2*(23*c^3*cos(f*x + e)^2 + 24*c^3*cos(f*x + e) + 13*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^ 3*f*cos(f*x + e) + a^3*f)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3 \left(\int \frac{3 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\, dx + \int \left(-\frac{3\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right) dx + \int \frac{\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\, dx + \int \left(-\frac{1}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right) dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-1/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.54, size = 80, normalized size = 0.83

$$\frac{\frac{15\,(fx+e)c^3}{a^3}-\frac{2\left(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^3/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*
c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.40, size = 93, normalized size = 0.97

$$\frac{c^3\,x}{a^3}-\frac{\frac{46\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}-\frac{22\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15}+\frac{2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^3*x)/a^3 - ((2*c^3*sin(e/2 + (f*x)/2))/5 - (22*c^3*cos(e/2 + (f*x)/2)^2*
sin(e/2 + (f*x)/2))/15 + (46*c^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/1
5)/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

3.34 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=96

$$\frac{c^2x}{a^3}-\frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2}-\frac{23c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

```
[Out] c^2*x/a^3-4/5*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-8/15*c^2*tan(f*x+e)/a^3
/f/(1+sec(f*x+e))^2-23/15*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A]
time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.269$, Rules used = {3988, 3862, 4007, 4004, 3879, 3881, 3882}

$$-\frac{23c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)}-\frac{8c^2\tan(e+fx)}{15a^3f(\sec(e+fx)+1)^2}-\frac{4c^2\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{c^2x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*x)/a^3 - (4*c^2*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (8*c^2*
Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^2) - (23*c^2*Tan[e + f*x])/(15*a
^3*f*(1 + Sec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \left(\frac{c^2}{(1+\sec(e+fx))^3} - \frac{2c^2\sec(e+fx)}{(1+\sec(e+fx))^3} + \frac{c^2\sec^2(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}$$

$$= \frac{c^2\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3} + \frac{c^2\int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3} - \frac{(2c^2)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}$$

$$= -\frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{c^2\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} + \frac{(3c^2)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3} -$$

$$= -\frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} + \frac{c^2\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}$$

$$= \frac{c^2x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{c^2\tan(e+f}{15a^3f(1+\sec(e}$$

$$= \frac{c^2x}{a^3} - \frac{4c^2\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{8c^2\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{23c^2\tan(e+}{15a^3f(1+\sec(e}$$

Mathematica [A]
time = 0.52, size = 171, normalized size = 1.78

$$\frac{c^2\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(150fx\cos\left(\frac{fx}{2}\right)+150fx\cos\left(e+\frac{fx}{2}\right)+75fx\cos\left(e+\frac{3fx}{2}\right)+75fx\cos\left(2e+\frac{3fx}{2}\right)+15fx\cos\left(2e+\frac{5fx}{2}\right)+15fx\cos\left(3e+\frac{5fx}{2}\right)-500\sin\left(\frac{fx}{2}\right)+360\sin\left(e+\frac{fx}{2}\right)-280\sin\left(e+\frac{3fx}{2}\right)+150\sin\left(2e+\frac{3fx}{2}\right)-86\sin\left(2e+\frac{5fx}{2}\right)\right)}{480a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*Sec[e/2]*Sec[(e + f*x)/2]^5*(150*f*x*Cos[(f*x)/2] + 150*f*x*Cos[e + (f
*x)/2] + 75*f*x*Cos[e + (3*f*x)/2] + 75*f*x*Cos[2*e + (3*f*x)/2] + 15*f*x*C
os[2*e + (5*f*x)/2] + 15*f*x*Cos[3*e + (5*f*x)/2] - 500*Sin[(f*x)/2] + 360*
Sin[e + (f*x)/2] - 280*Sin[e + (3*f*x)/2] + 150*Sin[2*e + (3*f*x)/2] - 86*S
in[2*e + (5*f*x)/2]))/(480*a^3*f)
```

Maple [A]
time = 0.14, size = 61, normalized size = 0.64

method	result	size
derivativedivides	$\frac{c^2\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^3}$	61
default	$\frac{c^2\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^3}$	61
risch	$\frac{c^2x}{a^3}-\frac{2ic^2\left(75\,e^{4i(fx+e)}+180\,e^{3i(fx+e)}+250\,e^{2i(fx+e)}+140\,e^{i(fx+e)}+43\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	81

norman	$\frac{\frac{c^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{c^2x}{a}+\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{8c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{13c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15af}-\frac{c^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)a^2}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^2/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5+2/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f
*x+1/2*e)+2*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(96) = 192$.
time = 0.50, size = 229, normalized size = 2.39

$$-\frac{c^2\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{2c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c^2*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^2*(15*sin(f*x + e)/(cos(f*x + e)
+ 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/a^3 - 3*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 2.57, size = 147, normalized size = 1.53

$$\frac{15\,c^2fx\cos\left(fx+e\right)^3+45\,c^2fx\cos\left(fx+e\right)^2+45\,c^2fx\cos\left(fx+e\right)+15\,c^2fx-\left(43\,c^2\cos\left(fx+e\right)^2+54\,c^2\cos\left(fx+e\right)+23\,c^2\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*c^2*f*x*cos(f*x + e)^3 + 45*c^2*f*x*cos(f*x + e)^2 + 45*c^2*f*x*co
s(f*x + e) + 15*c^2*f*x - (43*c^2*cos(f*x + e)^2 + 54*c^2*cos(f*x + e) + 23
*c^2)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*
f*cos(f*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{1}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.52, size = 80, normalized size = 0.83

$$\frac{\frac{15\,(fx+e)c^2}{a^3} - \frac{3\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*c^2/a^3 - (3*a^12*c^2*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c
^2*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^2*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.40, size = 93, normalized size = 0.97

$$\frac{c^2\,x}{a^3} - \frac{\frac{43\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15} - \frac{16\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15} + \frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c^2*x)/a^3 - ((c^2*sin(e/2 + (f*x)/2))/5 - (16*c^2*cos(e/2 + (f*x)/2)^2*si
n(e/2 + (f*x)/2))/15 + (43*c^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/15)
/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

```
[Out] c*x/a^3-2/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-3/5*c*tan(f*x+e)/a^3/f/(1+s
ec(f*x+e))^2-8/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3988, 3862, 4007, 4004, 3879, 3881}

$$-\frac{8c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)}-\frac{3c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^2}-\frac{2c\tan(e+fx)}{5a^3f(\sec(e+fx)+1)^3}+\frac{cx}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*x)/a^3 - (2*c*Tan[e + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3) - (3*c*Tan[e
 + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^2) - (8*c*Tan[e + f*x])/(5*a^3*f*(1 + S
ec[e + f*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d/c)*csc[e +
 f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx &= \frac{\int \left(\frac{c}{(1+\sec(e+fx))^3}-\frac{c\sec(e+fx)}{(1+\sec(e+fx))^3}\right)dx}{a^3}\\
&= \frac{c\int \frac{1}{(1+\sec(e+fx))^3}\,dx}{a^3}-\frac{c\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3}\,dx}{a^3}\\
&= -\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{c\int \frac{-5+2\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}-\frac{(2c)\int \frac{\sec(e+fx)}{(1+\sec(e+fx))^2}\,dx}{5a^3}\\
&= -\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}+\frac{c\int \frac{15-7\sec(e+fx)}{1+\sec(e+fx)}\,dx}{15a^3}-\\
&= \frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{2c\tan(e+fx)}{15a^3f(1+\sec(e+}\\
&= \frac{cx}{a^3}-\frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3}-\frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+}
\end{aligned}$$

Mathematica [A]

time = 0.50, size = 169, normalized size = 1.92

$$\frac{c\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(50fx\cos\left(\frac{fx}{2}\right)+50fx\cos\left(e+\frac{fx}{2}\right)+25fx\cos\left(e+\frac{3fx}{2}\right)+25fx\cos\left(2e+\frac{3fx}{2}\right)+5fx\cos\left(2e+\frac{5fx}{2}\right)+5fx\cos\left(3e+\frac{5fx}{2}\right)-150\sin\left(\frac{fx}{2}\right)+110\sin\left(e+\frac{fx}{2}\right)-90\sin\left(e+\frac{3fx}{2}\right)+40\sin\left(2e+\frac{3fx}{2}\right)-26\sin\left(2e+\frac{5fx}{2}\right)\right)}{160a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^5*(50*f*x*Cos[(f*x)/2] + 50*f*x*Cos[e + (f*x)/
2] + 25*f*x*Cos[e + (3*f*x)/2] + 25*f*x*Cos[2*e + (3*f*x)/2] + 5*f*x*Cos[2*
e + (5*f*x)/2] + 5*f*x*Cos[3*e + (5*f*x)/2] - 150*Sin[(f*x)/2] + 110*Sin[e
+ (f*x)/2] - 90*Sin[e + (3*f*x)/2] + 40*Sin[2*e + (3*f*x)/2] - 26*Sin[2*e +
 (5*f*x)/2]))/(160*a^3*f)
```

Maple [A]
time = 0.13, size = 58, normalized size = 0.66

method	result	size
derivativedivides	$\frac{c\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,a^3}$	58
default	$\frac{c\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,a^3}$	58
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2af}-\frac{c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10af}}{a^2}$	70
risch	$\frac{cx}{a^3}-\frac{2ic\left(20\,e^{4i(fx+e)}+55\,e^{3i(fx+e)}+75\,e^{2i(fx+e)}+45\,e^{i(fx+e)}+13\right)}{5f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*c/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5+tan(1/2*f*x+1/2*e)^3-4*tan(1/2*f*x+1
/2*e)+4*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [A]
time = 0.50, size = 173, normalized size = 1.97

$$-\frac{c\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(c*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(sin(
```

```
f*x + e)/(cos(f*x + e) + 1))/a^3) + c*(15*sin(f*x + e)/(cos(f*x + e) + 1) -
 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3)/f
```

Fricas [A]
time = 3.65, size = 133, normalized size = 1.51

$$\frac{5\,cfx\cos\left(fx+e\right)^{3}+15\,cfx\cos\left(fx+e\right)^{2}+15\,cfx\cos\left(fx+e\right)+5\,cfx-\left(13\,c\cos\left(fx+e\right)^{2}+19\,c\cos\left(fx+e\right)+8\,c\right)\sin\left(fx+e\right)}{5\,\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
[Out] 1/5*(5*c*f*x*cos(f*x + e)^3 + 15*c*f*x*cos(f*x + e)^2 + 15*c*f*x*cos(f*x +
e) + 5*c*f*x - (13*c*cos(f*x + e)^2 + 19*c*cos(f*x + e) + 8*c)*sin(f*x + e)
)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a
^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\frac{\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{1}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x))/a**3
```

Giac [A]
time = 0.49, size = 71, normalized size = 0.81

$$\frac{\frac{10\,(fx+e)c}{a^{3}}-\frac{a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-5\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+20\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac")
[Out] 1/10*(10*(f*x + e)*c/a^3 - (a^12*c*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*c*tan(1/
2*f*x + 1/2*e)^3 + 20*a^12*c*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.38, size = 85, normalized size = 0.97

$$\frac{c\,x}{a^{3}}-\frac{\frac{13\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{4}}{5}-\frac{7\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{2}}{10}+\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{10}}{a^{3}\,f\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^3,x)
```

```
[Out] (c*x)/a^3 - ((c*sin(e/2 + (f*x)/2))/10 - (7*c*cos(e/2 + (f*x)/2)^2*sin(e/2
+ (f*x)/2))/10 + (13*c*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/5)/(a^3*f*c
os(e/2 + (f*x)/2)^5)
```

3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=126

$$\frac{x}{a^3c}+\frac{\cot(e+fx)}{a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}$$

```
[Out] x/a^3/c+cot(f*x+e)/a^3/c/f-1/3*cot(f*x+e)^3/a^3/c/f+2/5*cot(f*x+e)^5/a^3/c/
f-2*csc(f*x+e)/a^3/c/f+4/3*csc(f*x+e)^3/a^3/c/f-2/5*csc(f*x+e)^5/a^3/c/f
```

Rubi [A]
time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{\cot(e+fx)}{a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{x}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] x/(a^3*c) + Cot[e + f*x]/(a^3*c*f) - Cot[e + f*x]^3/(3*a^3*c*f) + (2*Cot[e
+ f*x]^5)/(5*a^3*c*f) - (2*Csc[e + f*x])/(a^3*c*f) + (4*Csc[e + f*x]^3)/(3*
a^3*c*f) - (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^2\,dx}{a^3c^3}$$
$$= -\frac{\int \left(c^2\cot^6(e+fx)-2c^2\cot^5(e+fx)\csc(e+fx)+c^2\cot^4\right.}{a^3c^3}$$
$$= -\frac{\int \cot^6(e+fx)\,dx}{a^3c} - \frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3c} + \frac{2\int}{}$$
$$= \frac{\cot^5(e+fx)}{5a^3cf} + \frac{\int \cot^4(e+fx)\,dx}{a^3c} - \frac{\mathrm{Subst}\left(\int x^4\,dx,x,-\cot\right.}{a^3cf}$$
$$= -\frac{\cot^3(e+fx)}{3a^3cf} + \frac{2\cot^5(e+fx)}{5a^3cf} - \frac{\int \cot^2(e+fx)\,dx}{a^3c} - \frac{2\mathrm{Su}}{}$$
$$= \frac{\cot(e+fx)}{a^3cf} - \frac{\cot^3(e+fx)}{3a^3cf} + \frac{2\cot^5(e+fx)}{5a^3cf} - \frac{2\csc(e+f}{a^3cf}$$
$$= \frac{x}{a^3c} + \frac{\cot(e+fx)}{a^3cf} - \frac{\cot^3(e+fx)}{3a^3cf} + \frac{2\cot^5(e+fx)}{5a^3cf} - \frac{2\csc}{}$$

Mathematica [A]

time = 1.03, size = 197, normalized size = 1.56

$$-\frac{\csc\left(\frac{e}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(-150fx\cos(fx)+150fx\cos(2e+fx)-120fx\cos(e+2fx)+120fx\cos(3e+2fx)-30fx\cos(2e+3fx)+30fx\cos(4e+3fx)+80\sin(e)+280\sin(fx)-445\sin(e+fx)-356\sin(2(e+fx))-89\sin(3(e+fx))+240\sin(2e+fx)+296\sin(e+2fx)+120\sin(3e+2fx)+104\sin(2e+3fx)\right)}{3840a^3cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/3840*(Csc[e/2]*Csc[(e + f*x)/2]*Sec[e/2]*Sec[(e + f*x)/2]^5*(-150*f*x*Co
s[f*x] + 150*f*x*Cos[2*e + f*x] - 120*f*x*Cos[e + 2*f*x] + 120*f*x*Cos[3*e
+ 2*f*x] - 30*f*x*Cos[2*e + 3*f*x] + 30*f*x*Cos[4*e + 3*f*x] + 80*Sin[e] +
280*Sin[f*x] - 445*Sin[e + f*x] - 356*Sin[2*(e + f*x)] - 89*Sin[3*(e + f*x)
] + 240*Sin[2*e + f*x] + 296*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 104*Si
n[2*e + 3*f*x]))/(a^3*c*f)
```

Maple [A]

time = 0.12, size = 73, normalized size = 0.58

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{5\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\,a^3c}$	73
default	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{5\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\,a^3c}$	73
risch	$\frac{x}{a^3c}-\frac{4i\left(15\,\mathrm{e}^{5i(fx+e)}+30\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{3i(fx+e)}-35\,\mathrm{e}^{2i(fx+e)}-37\,\mathrm{e}^{i(fx+e)}-13\right)}{15f\,a^3c\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)}$	105
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{8acf}-\frac{11\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}+\frac{5\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24acf}-\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{40acf}}{a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/a^3/c*(-1/5*tan(1/2*f*x+1/2*e)^5+5/3*tan(1/2*f*x+1/2*e)^3-11*tan(1/2*
f*x+1/2*e)+1/tan(1/2*f*x+1/2*e)+16*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [A]

time = 0.48, size = 132, normalized size = 1.05

$$-\frac{\frac{\frac{165\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{25\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}-\frac{240\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c}-\frac{15\,(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/120*((165*sin(f*x + e)/(cos(f*x + e) + 1) - 25*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) - 240*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/(a^3*c) - 15*(cos(f*x + e) + 1)/(a^3*c*sin(f
*x + e)))/f
```

Fricas [A]
time = 3.56, size = 118, normalized size = 0.94

$$\frac{26\,\cos\left(fx+e\right)^{3}+22\,\cos\left(fx+e\right)^{2}+15\left(fx\cos\left(fx+e\right)^{2}+2\,fx\cos\left(fx+e\right)+fx\right)\sin\left(fx+e\right)-17\,\cos\left(fx+e\right)-16}{15\left(a^{3}cf\cos\left(fx+e\right)^{2}+2\,a^{3}cf\cos\left(fx+e\right)+a^{3}cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/15*(26*cos(f*x + e)^3 + 22*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^2 + 2*f*
x*cos(f*x + e) + f*x)*sin(f*x + e) - 17*cos(f*x + e) - 16)/((a^3*c*f*cos(f*
x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{\sec^{4}\left(e+fx\right)+2\sec^{3}\left(e+fx\right)-2\sec\left(e+fx\right)-1}\,dx}{a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1), x)/
(a**3*c)
```

Giac [A]
time = 0.56, size = 102, normalized size = 0.81

$$\frac{\frac{120\,(fx+e)}{a^{3}c}+\frac{15}{a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-\frac{3\,a^{12}c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-25\,a^{12}c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+165\,a^{12}c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/120*(120*(f*x + e)/(a^3*c) + 15/(a^3*c*tan(1/2*f*x + 1/2*e)) - (3*a^12*c^
4*tan(1/2*f*x + 1/2*e)^5 - 25*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*a^12*c^
4*tan(1/2*f*x + 1/2*e))/(a^15*c^5))/f
```

Mupad [B]
time = 1.43, size = 82, normalized size = 0.65

$$\frac{x}{a^{3}\,c}+\frac{\frac{26\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}}{15}-\frac{28\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}}{15}+\frac{17\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}}{60}-\frac{1}{40}}{a^{3}\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

```
[Out] x/(a^3*c) + ((17*cos(e/2 + (f*x)/2)^2)/60 - (28*cos(e/2 + (f*x)/2)^4)/15 +
(26*cos(e/2 + (f*x)/2)^6)/15 - 1/40)/(a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2
+ (f*x)/2))
```

3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{x}{a^3c^2}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}-\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}$$

```
[Out] x/a^3/c^2+1/15*cot(f*x+e)*(15-8*sec(f*x+e))/a^3/c^2/f-1/15*cot(f*x+e)^3*(5-
4*sec(f*x+e))/a^3/c^2/f+1/5*cot(f*x+e)^5*(1-sec(f*x+e))/a^3/c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3967, 8}

$$\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}-\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f}+\frac{x}{a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] x/(a^3*c^2) + (Cot[e + f*x]*(15 - 8*Sec[e + f*x]))/(15*a^3*c^2*f) - (Cot[e
+ f*x]^3*(5 - 4*Sec[e + f*x]))/(15*a^3*c^2*f) + (Cot[e + f*x]^5*(1 - Sec[e
+ f*x]))/(5*a^3*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))\,dx}{a^3c^3}$$
$$= \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\int \cot^4(e+fx)(-5c+4c}{5a^3c^3}$$
$$= -\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot^5(e+fx)(1-\sec}{5a^3c^2f}$$
$$= \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec}{15a^3c^2f}$$
$$= \frac{x}{a^3c^2} + \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5}{15a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 257 vs. $2(100) = 200$.
time = 1.11, size = 257, normalized size = 2.57

$$\frac{\csc\left(\frac{e}{2}\right)\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(360fx\cos(fx)-360fx\cos(2e+fx)+120fx\cos(e+2fx)-120fx\cos(3e+2fx)-120fx\cos(2e+3fx)+120fx\cos(4e+3fx)-60fx\cos(3e+4fx)+60fx\cos(5e+4fx)-200\sin(e)-584\sin(fx)+534\sin(e+fx)+178\sin(2(e+fx))-178\sin(3(e+fx))-89\sin(4(e+fx))-520\sin(2e+fx)-248\sin(e+2fx)-120\sin(3e+2fx)+248\sin(2e+3fx)+120\sin(4e+3fx)+184\sin(3e+4fx))}{30720a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^3*Sec[e/2]*Sec[(e + f*x)/2]^5*(360*f*x*Cos[f*x]
- 360*f*x*Cos[2*e + f*x] + 120*f*x*Cos[e + 2*f*x] - 120*f*x*Cos[3*e + 2*f*x
] - 120*f*x*Cos[2*e + 3*f*x] + 120*f*x*Cos[4*e + 3*f*x] - 60*f*x*Cos[3*e +
4*f*x] + 60*f*x*Cos[5*e + 4*f*x] - 200*Sin[e] - 584*Sin[f*x] + 534*Sin[e +
f*x] + 178*Sin[2*(e + f*x)] - 178*Sin[3*(e + f*x)] - 89*Sin[4*(e + f*x)] -
520*Sin[2*e + f*x] - 248*Sin[e + 2*f*x] - 120*Sin[3*e + 2*f*x] + 248*Sin[2*
e + 3*f*x] + 120*Sin[4*e + 3*f*x] + 184*Sin[3*e + 4*f*x]))/(30720*a^3*c^2*f
)
```

Maple [A]
time = 0.12, size = 88, normalized size = 0.88

method	result	si
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$	88
default	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$	88
risch	$\frac{x}{a^3c^2}-\frac{2i\left(15\,e^{7i(fx+e)}-15\,e^{6i(fx+e)}-65\,e^{5i(fx+e)}-25\,e^{4i(fx+e)}+73\,e^{3i(fx+e)}+31\,e^{2i(fx+e)}-31\,e^{i(fx+e)}-23\right)}{15f\,c^2a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^3}$	12

norman	$\frac{\frac{x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}-\frac{1}{48acf}+\frac{3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}-\frac{\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}-\frac{\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)}{80acf}}{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/c^2/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5+2*tan(1/2*f*x+1/2*e)^3-16*tan(1/2
*f*x+1/2*e)+32*arctan(tan(1/2*f*x+1/2*e))-1/3/tan(1/2*f*x+1/2*e)^3+6/tan(1/
2*f*x+1/2*e))
```

Maxima [A]
time = 0.49, size = 158, normalized size = 1.58

$$-\frac{\frac{3\left(\frac{80\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^2}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^2}-\frac{5\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(3*(80*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) - 480*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^2) - 5*(18*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e)^3))/f
```

Fricas [A]
time = 4.16, size = 166, normalized size = 1.66

$$\frac{23\cos(fx+e)^4+8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3+fx\cos(fx+e)^2-fx\cos(fx+e)-fx\right)\sin(fx+e)-7\cos(fx+e)+8}{15\left(a^3c^2f\cos(fx+e)^3+a^3c^2f\cos(fx+e)^2-a^3c^2f\cos(fx+e)-a^3c^2f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^4 + 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*co
s(f*x + e)^3 + f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) - f*x)*sin(f*x + e) -
7*cos(f*x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 -
 a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^5(e+fx)+\sec^4(e+fx)-2\sec^3(e+fx)-2\sec^2(e+fx)+\sec(e+fx)+1}\,dx}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(e
 + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Giac [A]
time = 0.56, size = 116, normalized size = 1.16

$$\frac{\frac{240\,(fx+e)}{a^3c^2}+\frac{5\left(18\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}-\frac{3\left(a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+80\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}c^{10}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/240*(240*(f*x + e)/(a^3*c^2) + 5*(18*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2
*tan(1/2*f*x + 1/2*e)^3) - 3*(a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8
*tan(1/2*f*x + 1/2*e)^3 + 80*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

Mupad [B]
time = 1.50, size = 161, normalized size = 1.61

$$-\frac{5\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8+3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-30\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-90\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-240\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^3(e+f\,x)}{240\,a^3\,c^2\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

```
[Out] -(5*cos(e/2 + (f*x)/2)^8 + 3*sin(e/2 + (f*x)/2)^8 - 30*cos(e/2 + (f*x)/2)^2
*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 90*
cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 - 240*cos(e/2 + (f*x)/2)^5*sin(e/
2 + (f*x)/2)^3*(e + f*x))/(240*a^3*c^2*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*
x)/2)^3)
```

3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=67

$$\frac{x}{a^3c^3}+\frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}$$

```
[Out] x/a^3/c^3+cot(f*x+e)/a^3/c^3/f-1/3*cot(f*x+e)^3/a^3/c^3/f+1/5*cot(f*x+e)^5/
a^3/c^3/f
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3554, 8}

$$\frac{\cot^5(e+fx)}{5a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot(e+fx)}{a^3c^3f}+\frac{x}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] x/(a^3*c^3) + Cot[e + f*x]/(a^3*c^3*f) - Cot[e + f*x]^3/(3*a^3*c^3*f) + Cot
[e + f*x]^5/(5*a^3*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx &= -\frac{\int \cot^6(e+fx)\,dx}{a^3c^3} \\
&= \frac{\cot^5(e+fx)}{5a^3c^3f} + \frac{\int \cot^4(e+fx)\,dx}{a^3c^3} \\
&= -\frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f} - \frac{\int \cot^2(e+fx)\,dx}{a^3c^3} \\
&= \frac{\cot(e+fx)}{a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f} + \frac{\int 1\,dx}{a^3c^3} \\
&= \frac{x}{a^3c^3} + \frac{\cot(e+fx)}{a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 39, normalized size = 0.58

$$
\frac{\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5a^3c^3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*a^3*c
^3*f)
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.72

method	result
default	$-\frac{-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e}{c^3a^3f}$
risch	$\frac{x}{a^3c^3}+\frac{2i\left(45\,\mathrm{e}^{8i(fx+e)}-90\,\mathrm{e}^{6i(fx+e)}+140\,\mathrm{e}^{4i(fx+e)}-70\,\mathrm{e}^{2i(fx+e)}+23\right)}{15f\,c^3a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)^5}$
norman	$\frac{\frac{x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}+\frac{1}{160acf}-\frac{7\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96acf}+\frac{11\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}-\frac{11\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}+\frac{7\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96acf}-\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{160acf}}{a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$
derivativedivides	`error in RationalFunction: argument is not a rational function\`

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3/a^3/f*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)
```

Maxima [A]
time = 0.49, size = 60, normalized size = 0.90

$$\frac{\frac{15\,(fx+e)}{a^3c^3}+\frac{15\,\tan(fx+e)^4-5\,\tan(fx+e)^2+3}{a^3c^3\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(
a^3*c^3*tan(f*x + e)^5))/f
```

Fricas [A]
time = 4.07, size = 127, normalized size = 1.90

$$\frac{23\,\cos\left(fx+e\right)^5-35\,\cos\left(fx+e\right)^3+15\,\left(fx\cos\left(fx+e\right)^4-2\,fx\cos\left(fx+e\right)^2+fx\right)\sin\left(fx+e\right)+15\,\cos\left(fx+e\right)}{15\,\left(a^3c^3f\cos\left(fx+e\right)^4-2\,a^3c^3f\cos\left(fx+e\right)^2+a^3c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(f*x + e)^5 - 35*cos(f*x + e)^3 + 15*(f*x*cos(f*x + e)^4 - 2*f*
x*cos(f*x + e)^2 + f*x)*sin(f*x + e) + 15*cos(f*x + e))/((a^3*c^3*f*cos(f*x
 + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{\sec^6\left(e+fx\right)-3\sec^4\left(e+fx\right)+3\sec^2\left(e+fx\right)-1}\,dx}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),
x)/(a**3*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.
time = 0.55, size = 129, normalized size = 1.93

$$\frac{\frac{480\,(fx+e)}{a^3c^3}+\frac{330\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3}{a^3c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}-\frac{3\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+330\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^{15}}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/480*(480*(f*x + e)/(a^3*c^3) + (330*tan(1/2*f*x + 1/2*e)^4 - 35*tan(1/2*f
*x + 1/2*e)^2 + 3)/(a^3*c^3*tan(1/2*f*x + 1/2*e)^5) - (3*a^12*c^12*tan(1/2*
f*x + 1/2*e)^5 - 35*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 330*a^12*c^12*tan(1/
2*f*x + 1/2*e))/(a^15*c^15))/f
```

Mupad [B]
time = 1.57, size = 94, normalized size = 1.40

$$\frac{\frac{5\cos(e+f\,x)}{24}-\frac{5\cos(3\,e+3\,f\,x)}{48}+\frac{23\cos(5\,e+5\,f\,x)}{240}-\frac{5\sin(3\,e+3\,f\,x)\,(e+f\,x)}{16}+\frac{\sin(5\,e+5\,f\,x)\,(e+f\,x)}{16}+\frac{5\sin(e+f\,x)\,(e+f\,x)}{8}}{a^3\,c^3\,f\,\sin\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
```

```
[Out] ((5*cos(e + f*x))/24 - (5*cos(3*e + 3*f*x))/48 + (23*cos(5*e + 5*f*x))/240
- (5*sin(3*e + 3*f*x)*(e + f*x))/16 + (sin(5*e + 5*f*x)*(e + f*x))/16 + (5*
sin(e + f*x)*(e + f*x))/8)/(a^3*c^3*f*sin(e + f*x)^5)
```

3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=129

$$\frac{x}{a^3c^4}-\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f}+\frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f}+\frac{\cot(e+fx)(35+16\sec(e+fx))}{35a^3c^4f}-$$

```
[Out] x/a^3/c^4-1/7*cot(f*x+e)^7*(1+sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)^5*(7+6*
sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)*(35+16*sec(f*x+e))/a^3/c^4/f-1/105*co
t(f*x+e)^3*(35+24*sec(f*x+e))/a^3/c^4/f
```

Rubi [A]
time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3989, 3967, 8}

$$-\frac{\cot^7(e+fx)(\sec(e+fx)+1)}{7a^3c^4f}+\frac{\cot^5(e+fx)(6\sec(e+fx)+7)}{35a^3c^4f}-\frac{\cot^3(e+fx)(24\sec(e+fx)+35)}{105a^3c^4f}+\frac{\cot(e+fx)(16\sec(e+fx)+35)}{35a^3c^4f}+\frac{x}{a^3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] x/(a^3*c^4) - (Cot[e + f*x]^7*(1 + Sec[e + f*x]))/(7*a^3*c^4*f) + (Cot[e +
f*x]^5*(7 + 6*Sec[e + f*x]))/(35*a^3*c^4*f) + (Cot[e + f*x]*(35 + 16*Sec[e
+ f*x]))/(35*a^3*c^4*f) - (Cot[e + f*x]^3*(35 + 24*Sec[e + f*x]))/(105*a^3*
c^4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))\,dx}{a^4c^4}$$
$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\int \cot^6(e+fx)(-7a-}{7a^4c}$$
$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec}{35a^3c^4f}$$
$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec}{35a^3c^4f}$$
$$= -\frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec}{35a^3c^4f}$$
$$= \frac{x}{a^3c^4} - \frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+}{35a^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 362 vs. 2(129) = 258.
time = 1.48, size = 362, normalized size = 2.81

$$\frac{\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(16800fx\cos(fx)-16800fx\cos(2e+fx)-4200fx\cos(e+2fx)+4200fx\cos(3e+2fx)-8400fx\cos(2e+3fx)+8400fx\cos(4e+3fx)+3360fx\cos(3e+4fx)-3360fx\cos(5e+4fx)+1680fx\cos(4e+5fx)-1680fx\cos(6e+5fx)-840fx\cos(5e+6fx)+840fx\cos(7e+6fx)+3136\sin(e)-30112\sin(fx)-22860\sin(e+fx)+5715\sin(2(e+fx))+11430\sin(3(e+fx))-4572\sin(4(e+fx))-2286\sin(5(e+fx))+1143\sin(6(e+fx))-26208\sin(2e+fx)+14080\sin(e+2fx)+16400\sin(2e+3fx)+11760\sin(4e+3fx)-7904\sin(3e+4fx)-3360\sin(5e+4fx)-3952\sin(4e+5fx)-1680\sin(6e+5fx)+2816\sin(5e+6fx)\right)}{6881280a^3c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^7*Sec[e/2]*Sec[(e + f*x)/2]^5*(16800*f*x*Cos[f*x
] - 16800*f*x*Cos[2*e + f*x] - 4200*f*x*Cos[e + 2*f*x] + 4200*f*x*Cos[3*e +
 2*f*x] - 8400*f*x*Cos[2*e + 3*f*x] + 8400*f*x*Cos[4*e + 3*f*x] + 3360*f*x*
Cos[3*e + 4*f*x] - 3360*f*x*Cos[5*e + 4*f*x] + 1680*f*x*Cos[4*e + 5*f*x] -
1680*f*x*Cos[6*e + 5*f*x] - 840*f*x*Cos[5*e + 6*f*x] + 840*f*x*Cos[7*e + 6*
f*x] + 3136*Sin[e] - 30112*Sin[f*x] - 22860*Sin[e + f*x] + 5715*Sin[2*(e +
f*x)] + 11430*Sin[3*(e + f*x)] - 4572*Sin[4*(e + f*x)] - 2286*Sin[5*(e + f*
x)] + 1143*Sin[6*(e + f*x)] - 26208*Sin[2*e + f*x] + 14080*Sin[e + 2*f*x] +
 16400*Sin[2*e + 3*f*x] + 11760*Sin[4*e + 3*f*x] - 7904*Sin[3*e + 4*f*x] -
3360*Sin[5*e + 4*f*x] - 3952*Sin[4*e + 5*f*x] - 1680*Sin[6*e + 5*f*x] + 281
6*Sin[5*e + 6*f*x]))/(6881280*a^3*c^4*f)
```

Maple [A]
time = 0.14, size = 114, normalized size = 0.88

method	result

derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{8\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{}{3\tan}}{64fc^4a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{8\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+128\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{}{3\tan}}{64fc^4a^3}$
risch	$\frac{x}{a^3c^4}+\frac{2i\left(105\,e^{11i(fx+e)}+210\,e^{10i(fx+e)}-735\,e^{9i(fx+e)}+1638\,e^{7i(fx+e)}-196\,e^{6i(fx+e)}-1882\,e^{5i(fx+e)}+880\,e^{4i(fx+e)}\right.}{105fc^4a^3\left(e^{i(fx+e)}-1\right)^7\left(e^{i(fx+e)}+1\right)^5}$
norman	$\frac{\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}-\frac{1}{448acf}+\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{40acf}-\frac{29\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192acf}-\frac{29\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64acf}+\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{24acf}-\frac{\tan^{12}\left(\frac{f}{2}\right.}{320a}}{c^3a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/f/c^4/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5+8/3*tan(1/2*f*x+1/2*e)^3-29*tan(1
/2*f*x+1/2*e)+128*arctan(tan(1/2*f*x+1/2*e))-1/7/tan(1/2*f*x+1/2*e)^7+8/5/t
an(1/2*f*x+1/2*e)^5-29/3/tan(1/2*f*x+1/2*e)^3+64/tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.50, size = 203, normalized size = 1.57

$$-\frac{\frac{7\left(\frac{435\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}-\frac{13440\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^4}-\frac{\left(\frac{168\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1015\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{a^3c^4\sin(fx+e)^7}}{6720f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/6720*(7*(435*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) - 13440*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^4) - (168*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 1015*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6720*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin(f*x + e)
^7))/f
```

Fricas [A]
time = 4.05, size = 250, normalized size = 1.94

$$\frac{176\cos(fx+e)^6-71\cos(fx+e)^5-335\cos(fx+e)^4+125\cos(fx+e)^3+225\cos(fx+e)^2+105\left(fx\cos(fx+e)^5-fx\cos(fx+e)^4-2fx\cos(fx+e)^3+2fx\cos(fx+e)^2+fx\cos(fx+e)-fx\right)\sin(fx+e)-57\cos(fx+e)-48}{105\left(a^3c^4f\cos(fx+e)^5-a^3c^4f\cos(fx+e)^4-2a^3c^4f\cos(fx+e)^3+2a^3c^4f\cos(fx+e)^2+a^3c^4f\cos(fx+e)-a^3c^4f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(f*x + e)^6 - 71*cos(f*x + e)^5 - 335*cos(f*x + e)^4 + 125*co
s(f*x + e)^3 + 225*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^5 - f*x*cos(f*x +
```

```
 e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 + f*x*cos(f*x + e) - f*
x)*sin(f*x + e) - 57*cos(f*x + e) - 48)/((a^3*c^4*f*cos(f*x + e)^5 - a^3*c^
4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*cos(f*x + e)^
2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^7(e+fx)-\sec^6(e+fx)-3\sec^5(e+fx)+3\sec^4(e+fx)+3\sec^3(e+fx)-3\sec^2(e+fx)-\sec(e+fx)+1}\,dx}{a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(1/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**5 + 3*sec(e
 + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x) + 1), x)/
(a**3*c**4)
```

Giac [A]
time = 0.57, size = 142, normalized size = 1.10

$$\frac{\frac{6720\,(fx+e)}{a^3c^4}+\frac{6720\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1015\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+168\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15}{a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}-\frac{7\left(3\,a^{12}c^{16}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-40\,a^{12}c^{16}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+435\,a^{12}c^{16}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}c^{20}}}{6720\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/6720*(6720*(f*x + e)/(a^3*c^4) + (6720*tan(1/2*f*x + 1/2*e)^6 - 1015*tan(
1/2*f*x + 1/2*e)^4 + 168*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^3*c^4*tan(1/2*f*x
+ 1/2*e)^7) - 7*(3*a^12*c^16*tan(1/2*f*x + 1/2*e)^5 - 40*a^12*c^16*tan(1/2*
f*x + 1/2*e)^3 + 435*a^12*c^16*tan(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

Mupad [B]
time = 1.85, size = 209, normalized size = 1.62

$$-\frac{15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+21\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-280\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+3045\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-6720\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+1015\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-168\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-6720\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7(e+fx)}{6720\,a^3c^4f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(15*cos(e/2 + (f*x)/2)^12 + 21*sin(e/2 + (f*x)/2)^12 - 280*cos(e/2 + (f*x)
/2)^2*sin(e/2 + (f*x)/2)^10 + 3045*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^
8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1015*cos(e/2 + (f*x)/2
)^8*sin(e/2 + (f*x)/2)^4 - 168*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^2 -
 6720*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7*(e + f*x))/(6720*a^3*c^4*f*
cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7)
```

3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=210

$$\frac{x}{a^3c^5}+\frac{\cot(e+fx)}{a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}$$

```
[Out] x/a^3/c^5+cot(f*x+e)/a^3/c^5/f-1/3*cot(f*x+e)^3/a^3/c^5/f+1/5*cot(f*x+e)^5/
a^3/c^5/f-1/7*cot(f*x+e)^7/a^3/c^5/f+2/9*cot(f*x+e)^9/a^3/c^5/f+2*csc(f*x+e
)/a^3/c^5/f-8/3*csc(f*x+e)^3/a^3/c^5/f+12/5*csc(f*x+e)^5/a^3/c^5/f-8/7*csc(
f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/f
```

Rubi [A]
time = 0.18, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}+\frac{x}{a^3c^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] x/(a^3*c^5) + Cot[e + f*x]/(a^3*c^5*f) - Cot[e + f*x]^3/(3*a^3*c^5*f) + Cot
[e + f*x]^5/(5*a^3*c^5*f) - Cot[e + f*x]^7/(7*a^3*c^5*f) + (2*Cot[e + f*x]^
9)/(9*a^3*c^5*f) + (2*Csc[e + f*x])/(a^3*c^5*f) - (8*Csc[e + f*x]^3)/(3*a^3
*c^5*f) + (12*Csc[e + f*x]^5)/(5*a^3*c^5*f) - (8*Csc[e + f*x]^7)/(7*a^3*c^5
*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx &= -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^2\,dx}{a^5c^5} \\
&= -\frac{\int \left(a^2\cot^{10}(e+fx)+2a^2\cot^9(e+fx)\csc(e+fx)+a^2 \mathrm{c}\right.}{a^5c^5} \\
&= -\frac{\int \cot^{10}(e+fx)\,dx}{a^3c^5} - \frac{\int \cot^8(e+fx)\csc^2(e+fx)\,dx}{a^3c^5} - \frac{2}{} \\
&= \frac{\cot^9(e+fx)}{9a^3c^5f} + \frac{\int \cot^8(e+fx)\,dx}{a^3c^5} - \frac{\mathrm{Subst}\left(\int x^8\,dx, x, -\mathrm{co}\right.}{a^3c^5f} \\
&= -\frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e+fx)}{9a^3c^5f} - \frac{\int \cot^6(e+fx)\,dx}{a^3c^5} + \frac{2\mathrm{S}}{} \\
&= \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{2\csc(e+}{a^3c^5f} \\
&= -\frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{2\cot^9(e-}{9a^3c^5} \\
&= \frac{\cot(e+fx)}{a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7(e+fx}{7a^3c^5f} \\
&= \frac{x}{a^3c^5} + \frac{\cot(e+fx)}{a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^7}{7}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 441 vs. 2(210) = 420.
time = 1.80, size = 441, normalized size = 2.10

$$
\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\sec^7(e+fx)(453600fx\cos(fx)-453600fx\cos(2e+fx)-201600fx\cos(e+2fx)+201600fx\cos(3e+2fx)-191520fx\cos(2e+3fx)+191520fx\cos(4e+3fx)+161280fx\cos(3e+4fx)-161280fx\cos(5e+4fx)+10080fx\cos(4e+5fx)-10080fx\cos(6e+5fx)-40320fx\cos(5e+6fx)+40320fx\cos(7e+6fx)+10080fx\cos(6e+7fx)-10080fx\cos(8e+7fx)+259584\sin(e)-897024\sin(fx)-1152405\sin(e+fx)+512180\sin(2(e+fx))+486571\sin(3(e+fx))-409744\sin(4(e+fx))-25609\sin(5(e+fx))+102436\sin(6(e+fx))-25609\sin(7(e+fx))-825216\sin(2e+fx)+622976\sin(e+2fx)+142464\sin(3e+2fx)+297088\sin(2e+3fx)+430080\sin(4e+3fx)-424192\sin(3e+4fx)-188160\sin(5e+4fx)+2048\sin(4e+5fx)-40320\sin(6e+5fx)+112768\sin(5e+6fx)+40320\sin(7e+6fx)-38272\sin(6e+7fx))\tan(e+fx)}{2580480a^3c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^7*(453600*f*x*Cos[f*x] - 453600*f*x*Cos[2*e
 + f*x] - 201600*f*x*Cos[e + 2*f*x] + 201600*f*x*Cos[3*e + 2*f*x] - 191520*
f*x*Cos[2*e + 3*f*x] + 191520*f*x*Cos[4*e + 3*f*x] + 161280*f*x*Cos[3*e + 4
*f*x] - 161280*f*x*Cos[5*e + 4*f*x] + 10080*f*x*Cos[4*e + 5*f*x] - 10080*f*
x*Cos[6*e + 5*f*x] - 40320*f*x*Cos[5*e + 6*f*x] + 40320*f*x*Cos[7*e + 6*f*x
] + 10080*f*x*Cos[6*e + 7*f*x] - 10080*f*x*Cos[8*e + 7*f*x] + 259584*Sin[e]
 - 897024*Sin[f*x] - 1152405*Sin[e + f*x] + 512180*Sin[2*(e + f*x)] + 48657
1*Sin[3*(e + f*x)] - 409744*Sin[4*(e + f*x)] - 25609*Sin[5*(e + f*x)] + 102
436*Sin[6*(e + f*x)] - 25609*Sin[7*(e + f*x)] - 825216*Sin[2*e + f*x] + 622
976*Sin[e + 2*f*x] + 142464*Sin[3*e + 2*f*x] + 297088*Sin[2*e + 3*f*x] + 43
0080*Sin[4*e + 3*f*x] - 424192*Sin[3*e + 4*f*x] - 188160*Sin[5*e + 4*f*x] +
```

```
2048*Sin[4*e + 5*f*x] - 40320*Sin[6*e + 5*f*x] + 112768*Sin[5*e + 6*f*x] +
 40320*Sin[7*e + 6*f*x] - 38272*Sin[6*e + 7*f*x])*Tan[e + f*x])/(2580480*a^
3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

Maple [A]
time = 0.16, size = 127, normalized size = 0.60

method	result
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\cdots}{128f\,c^5a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\cdots}{128f\,c^5a^3}$
risch	$\frac{x}{a^3c^5}+\frac{4i\left(315\,\mathrm{e}^{13i(fx+e)}-315\,\mathrm{e}^{12i(fx+e)}-1470\,\mathrm{e}^{11i(fx+e)}+3360\,\mathrm{e}^{10i(fx+e)}+1113\,\mathrm{e}^{9i(fx+e)}-6447\,\mathrm{e}^{8i(fx+e)}+2028\,\mathrm{e}^{7\cdots}\right.}{315f\,c^5a^3\left(\mathrm{e}^{i(fx+e)}-\cdots\right.}$
norman	$\frac{\frac{x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}+\frac{1}{1152acf}-\frac{9\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{896acf}+\frac{37\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{640acf}-\frac{31\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{128acf}+\frac{163\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{128acf}-\frac{37\left(\tan^{10}\left(\frac{fx}{2}\cdots\right.\right.}{128acf}}{c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/f/c^5/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5+3*tan(1/2*f*x+1/2*e)^3-37*tan(1/
2*f*x+1/2*e)+256*arctan(tan(1/2*f*x+1/2*e))+1/9/tan(1/2*f*x+1/2*e)^9-9/7/ta
n(1/2*f*x+1/2*e)^7+37/5/tan(1/2*f*x+1/2*e)^5-31/tan(1/2*f*x+1/2*e)^3+163/ta
n(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.50, size = 223, normalized size = 1.06

$$-\frac{\frac{63\left(\frac{185\sin(fx+e)}{\cos(fx+e)+1}-\frac{15\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{80640\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^5}+\frac{\left(\frac{405\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2331\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9765\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{51345\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{a^3c^5\sin(fx+e)^9}}{40320\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/40320*(63*(185*sin(f*x + e)/(cos(f*x + e) + 1) - 15*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - 80640*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^5) + (405*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2331*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9765*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 51345*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35
)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Fricas [A]
time = 3.02, size = 292, normalized size = 1.39

$$\frac{598\cos(fx+e)^7-566\cos(fx+e)^6-1212\cos(fx+e)^5+1310\cos(fx+e)^4+860\cos(fx+e)^3-1014\cos(fx+e)^2+315\left(fx\cos(fx+e)^6-2fx\cos(fx+e)^5-fx\cos(fx+e)^4+4fx\cos(fx+e)^3-fx\cos(fx+e)^2-2fx\cos(fx+e)+fx\right)\sin(fx+e)-197\cos(fx+e)+256}{315\left(a^3c^5f\cos(fx+e)^6-2a^3c^5f\cos(fx+e)^5-a^3c^5f\cos(fx+e)^4+4a^3c^5f\cos(fx+e)^3-a^3c^5f\cos(fx+e)^2-2a^3c^5f\cos(fx+e)+a^3c^5f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(598*cos(f*x + e)^7 - 566*cos(f*x + e)^6 - 1212*cos(f*x + e)^5 + 1310
*cos(f*x + e)^4 + 860*cos(f*x + e)^3 - 1014*cos(f*x + e)^2 + 315*(f*x*cos(f
*x + e)^6 - 2*f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 + 4*f*x*cos(f*x + e)^
3 - f*x*cos(f*x + e)^2 - 2*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 197*cos(f
*x + e) + 256)/((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^
3*c^5*f*cos(f*x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e
)^2 - 2*a^3*c^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(1/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x)**6 + 6*se
c(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e + f*x) - 1)
, x)/(a**3*c**5)
```

Giac [A]
time = 0.59, size = 154, normalized size = 0.73

$$\frac{\frac{40320\,(fx+e)}{a^3c^5}+\frac{51345\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-9765\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+2331\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-405\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+35}{a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}-\frac{63\left(a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-15a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+185a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}c^{25}}}{40320\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/40320*(40320*(f*x + e)/(a^3*c^5) + (51345*tan(1/2*f*x + 1/2*e)^8 - 9765*t
an(1/2*f*x + 1/2*e)^6 + 2331*tan(1/2*f*x + 1/2*e)^4 - 405*tan(1/2*f*x + 1/2
*e)^2 + 35)/(a^3*c^5*tan(1/2*f*x + 1/2*e)^9) - 63*(a^12*c^20*tan(1/2*f*x +
1/2*e)^5 - 15*a^12*c^20*tan(1/2*f*x + 1/2*e)^3 + 185*a^12*c^20*tan(1/2*f*x
+ 1/2*e))/(a^15*c^25))/f
```

Mupad [B]
time = 2.04, size = 233, normalized size = 1.11

$$\frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}-63\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+945\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-11655\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+51345\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-9765\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+2331\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-405\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+40320\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9(e+fx)}{40320\,a^3c^5f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (35*cos(e/2 + (f*x)/2)^14 - 63*sin(e/2 + (f*x)/2)^14 + 945*cos(e/2 + (f*x)/
2)^2*sin(e/2 + (f*x)/2)^12 - 11655*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^
10 + 51345*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^8 - 9765*cos(e/2 + (f*x)
/2)^8*sin(e/2 + (f*x)/2)^6 + 2331*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^
4 - 405*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^2 + 40320*cos(e/2 + (f*x)/
2)^5*sin(e/2 + (f*x)/2)^9*(e + f*x))/(40320*a^3*c^5*f*cos(e/2 + (f*x)/2)^5*
sin(e/2 + (f*x)/2)^9)
```

3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=252

$$\frac{x}{a^3c^6}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{3\csc(e+f}{a^3c^6f}$$

```
[Out] x/a^3/c^6+cot(f*x+e)/a^3/c^6/f-1/3*cot(f*x+e)^3/a^3/c^6/f+1/5*cot(f*x+e)^5/
a^3/c^6/f-1/7*cot(f*x+e)^7/a^3/c^6/f+1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*
x+e)^11/a^3/c^6/f+3*csc(f*x+e)/a^3/c^6/f-16/3*csc(f*x+e)^3/a^3/c^6/f+34/5*c
sc(f*x+e)^5/a^3/c^6/f-36/7*csc(f*x+e)^7/a^3/c^6/f+19/9*csc(f*x+e)^9/a^3/c^6
/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Rubi [A]
time = 0.23, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3989, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{19\csc^9(e+fx)}{9a^3c^6f}-\frac{36\csc^7(e+fx)}{7a^3c^6f}+\frac{34\csc^5(e+fx)}{5a^3c^6f}-\frac{16\csc^3(e+fx)}{3a^3c^6f}+\frac{3\csc(e+fx)}{a^3c^6f}+\frac{x}{a^3c^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] x/(a^3*c^6) + Cot[e + f*x]/(a^3*c^6*f) - Cot[e + f*x]^3/(3*a^3*c^6*f) + Cot
[e + f*x]^5/(5*a^3*c^6*f) - Cot[e + f*x]^7/(7*a^3*c^6*f) + Cot[e + f*x]^9/(
9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + (3*Csc[e + f*x])/(a^3*c
^6*f) - (16*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (34*Csc[e + f*x]^5)/(5*a^3*c^6*
f) - (36*Csc[e + f*x]^7)/(7*a^3*c^6*f) + (19*Csc[e + f*x]^9)/(9*a^3*c^6*f)
- (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \cot^{12}(e+fx)(a+a\sec(e+fx))^3\,dx}{a^6c^6}$$

$$= \frac{\int \left(a^3\cot^{12}(e+fx)+3a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3 \mathrm{c}\right.}{a}$$

$$= \frac{\int \cot^{12}(e+fx)\,dx}{a^3c^6} + \frac{\int \cot^9(e+fx)\csc^3(e+fx)\,dx}{a^3c^6} + \frac{3\int}{}$$

$$= -\frac{\cot^{11}(e+fx)}{11a^3c^6f} - \frac{\int \cot^{10}(e+fx)\,dx}{a^3c^6} - \frac{\mathrm{Subst}\Big(\int x^2(-1+}{}$$

$$= \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{\int \cot^8(e+fx)\,dx}{a^3c^6} - \frac{\mathrm{Sub}}{}$$

$$= -\frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{3\csc(e}{a^3c}$$

$$= \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+}{11a^3c^6f}$$

$$= -\frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^9(e+}{9a^3c^6f}$$

$$= \frac{\cot(e+fx)}{a^3c^6f} - \frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7(e+fx}{7a^3c^6f}$$

$$= \frac{x}{a^3c^6} + \frac{\cot(e+fx)}{a^3c^6f} - \frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^7}{7}$$

Mathematica [A]
time = 2.31, size = 499, normalized size = 1.98

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^8*(24393600*f*x*Cos[f*x] - 24393600*f*x*Cos
[2*e + f*x] - 14636160*f*x*Cos[e + 2*f*x] + 14636160*f*x*Cos[3*e + 2*f*x] -
 7539840*f*x*Cos[2*e + 3*f*x] + 7539840*f*x*Cos[4*e + 3*f*x] + 11088000*f*x
*Cos[3*e + 4*f*x] - 11088000*f*x*Cos[5*e + 4*f*x] - 2217600*f*x*Cos[4*e + 5
*f*x] + 2217600*f*x*Cos[6*e + 5*f*x] - 2217600*f*x*Cos[5*e + 6*f*x] + 22176
00*f*x*Cos[7*e + 6*f*x] + 1330560*f*x*Cos[6*e + 7*f*x] - 1330560*f*x*Cos[8*
e + 7*f*x] - 221760*f*x*Cos[7*e + 8*f*x] + 221760*f*x*Cos[9*e + 8*f*x] + 17
677440*Sin[e] - 49287040*Sin[f*x] - 86058610*Sin[e + f*x] + 51635166*Sin[2*
(e + f*x)] + 26599934*Sin[3*(e + f*x)] - 39117550*Sin[4*(e + f*x)] + 782351
0*Sin[5*(e + f*x)] + 7823510*Sin[6*(e + f*x)] - 4694106*Sin[7*(e + f*x)] +
```

782351*Sin[8*(e + f*x)] - 55651200*Sin[2*e + f*x] + 47971968*Sin[e + 2*f*x] + 14990976*Sin[3*e + 2*f*x] + 8100992*Sin[2*e + 3*f*x] + 24334464*Sin[4*e + 3*f*x] - 28627840*Sin[3*e + 4*f*x] - 19071360*Sin[5*e + 4*f*x] + 9687680*Sin[4*e + 5*f*x] - 147840*Sin[6*e + 5*f*x] + 5548160*Sin[5*e + 6*f*x] + 3991680*Sin[7*e + 6*f*x] - 4393344*Sin[6*e + 7*f*x] - 1330560*Sin[8*e + 7*f*x] + 953984*Sin[7*e + 8*f*x])*Tan[e + f*x])/(113541120*a^3*c^6*f*(-1 + Sec[e + f*x])^6*(1 + Sec[e + f*x])^3)

Maple [A]

time = 0.16, size = 140, normalized size = 0.56

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{10\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-46\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-}{256f\,a^{3}c^{6}}$
default	$\frac{-\frac{\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{10\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-46\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-}{256f\,a^{3}c^{6}}$
risch	$\frac{x}{a^{3}c^{6}}+\frac{2i\left(10395\,\mathrm{e}^{15i(fx+e)}-31185\,\mathrm{e}^{14i(fx+e)}+1155\,\mathrm{e}^{13i(fx+e)}+148995\,\mathrm{e}^{12i(fx+e)}-190113\,\mathrm{e}^{11i(fx+e)}-117117\,\mathrm{e}^{10i(}\right.}{}$
norman	$\frac{\frac{x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{ca}-\frac{1}{2816acf}+\frac{5\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1152acf}-\frac{23\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{896acf}+\frac{13\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{128acf}-\frac{\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}+\frac{191\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{128acf}}{c^{5}a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOSE)

[Out] 1/256/f/a^3/c^6*(-1/5*tan(1/2*f*x+1/2*e)^5+10/3*tan(1/2*f*x+1/2*e)^3-46*tan(1/2*f*x+1/2*e)+512*arctan(tan(1/2*f*x+1/2*e))-1/11/tan(1/2*f*x+1/2*e)^11+10/9/tan(1/2*f*x+1/2*e)^9-46/7/tan(1/2*f*x+1/2*e)^7+26/tan(1/2*f*x+1/2*e)^5-256/3/tan(1/2*f*x+1/2*e)^3+382/tan(1/2*f*x+1/2*e))

Maxima [A]

time = 0.51, size = 247, normalized size = 0.98

$$-\frac{\frac{231\left(\frac{690\sin(fx+e)}{\cos(fx+e)+1}-\frac{50\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}c^{6}}-\frac{1774080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{3}c^{6}}-\frac{5\left(\frac{770\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{4554\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{18018\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{59136\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{264726\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-63\right)(\cos(fx+e)+1)^{11}}{a^{3}c^{6}\sin(fx+e)^{11}}}{887040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="maxima")

[Out] -1/887040*(231*(690*sin(f*x + e)/(cos(f*x + e) + 1) - 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) - 1774080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^6) - 5*(770*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4554*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 18018*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 59136*sin(f*x + e)^8/(cos(f*x + e) +

```
1)^8 + 264726*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1
)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Fricas [A]
time = 3.29, size = 334, normalized size = 1.33

$$\frac{7453 \cos\left(fx+e\right)^{8} - 11964 \cos\left(fx+e\right)^{7} - 11866 \cos\left(fx+e\right)^{6} + 30542 \cos\left(fx+e\right)^{5} + 90 \cos\left(fx+e\right)^{4} - 26438 \cos\left(fx+e\right)^{3} + 8539 \cos\left(fx+e\right)^{2} + 3465 \left(fx \cos\left(fx+e\right)^{7} - 3 fx \cos\left(fx+e\right)^{6} + fx \cos\left(fx+e\right)^{5} + 5 fx \cos\left(fx+e\right)^{4} - 5 fx \cos\left(fx+e\right)^{3} - fx \cos\left(fx+e\right)^{2} + 3 fx \cos\left(fx+e\right) - fx\right) \sin\left(fx+e\right) + 7671 \cos\left(fx+e\right) - 3712}{3465 \left(a^{3}c^{6}f \cos\left(fx+e\right)^{7} - 3 a^{3}c^{6}f \cos\left(fx+e\right)^{6} + a^{3}c^{6}f \cos\left(fx+e\right)^{5} + 5 a^{3}c^{6}f \cos\left(fx+e\right)^{4} - 5 a^{3}c^{6}f \cos\left(fx+e\right)^{3} - a^{3}c^{6}f \cos\left(fx+e\right)^{2} + 3 a^{3}c^{6}f \cos\left(fx+e\right) - a^{3}c^{6}f\right) \sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*cos(f*x + e)^8 - 11964*cos(f*x + e)^7 - 11866*cos(f*x + e)^6 +
 30542*cos(f*x + e)^5 + 90*cos(f*x + e)^4 - 26438*cos(f*x + e)^3 + 8539*cos
(f*x + e)^2 + 3465*(f*x*cos(f*x + e)^7 - 3*f*x*cos(f*x + e)^6 + f*x*cos(f*x
 + e)^5 + 5*f*x*cos(f*x + e)^4 - 5*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2
+ 3*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 7671*cos(f*x + e) - 3712)/((a^3*
c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x + e)^6 + a^3*c^6*f*cos(f*x + e)^
5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6*f*cos(f*x + e)^3 - a^3*c^6*f*cos
(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a^3*c^6*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{9}{\left(e+fx\right)} - 3\sec^{8}{\left(e+fx\right)} + 8\sec^{6}{\left(e+fx\right)} - 6\sec^{5}{\left(e+fx\right)} - 6\sec^{4}{\left(e+fx\right)} + 8\sec^{3}{\left(e+fx\right)} - 3\sec{\left(e+fx\right)} + 1}\, dx}{a^{3}c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Integral(1/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)**6 - 6*sec
(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e + f*x) + 1),
 x)/(a**3*c**6)
```

Giac [A]
time = 0.61, size = 169, normalized size = 0.67

$$\frac{\frac{887040\left(fx+e\right)}{a^{3}c^{6}} + \frac{5\left(264726 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{10} - 59136 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{8} + 18018 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{6} - 4554 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4} + 770 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2} - 63\right)}{a^{3}c^{6} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{11}} - \frac{231\left(3 a^{12}c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{5} - 50 a^{12}c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3} + 690 a^{12}c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)\right)}{a^{15}c^{30}}}{887040 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="giac")
```

```
[Out] 1/887040*(887040*(f*x + e)/(a^3*c^6) + 5*(264726*tan(1/2*f*x + 1/2*e)^10 -
59136*tan(1/2*f*x + 1/2*e)^8 + 18018*tan(1/2*f*x + 1/2*e)^6 - 4554*tan(1/2*
f*x + 1/2*e)^4 + 770*tan(1/2*f*x + 1/2*e)^2 - 63)/(a^3*c^6*tan(1/2*f*x + 1/
2*e)^11) - 231*(3*a^12*c^24*tan(1/2*f*x + 1/2*e)^5 - 50*a^12*c^24*tan(1/2*f
*x + 1/2*e)^3 + 690*a^12*c^24*tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

Mupad [B]
time = 2.33, size = 257, normalized size = 1.02

$$-\frac{315\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}+693\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}-11550\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}+159390\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-1323630\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+295680\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-90090\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+22770\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-3850\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-887040\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}(e+f\,x)}{887040\,a^3\,c^6\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -(315*cos(e/2 + (f*x)/2)^16 + 693*sin(e/2 + (f*x)/2)^16 - 11550*cos(e/2 + (
f*x)/2)^2*sin(e/2 + (f*x)/2)^14 + 159390*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*
x)/2)^12 - 1323630*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^10 + 295680*cos(
e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^8 - 90090*cos(e/2 + (f*x)/2)^10*sin(e/2
 + (f*x)/2)^6 + 22770*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^4 - 3850*cos
(e/2 + (f*x)/2)^14*sin(e/2 + (f*x)/2)^2 - 887040*cos(e/2 + (f*x)/2)^5*sin(e
/2 + (f*x)/2)^11*(e + f*x))/(887040*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2
+ (f*x)/2)^11)
```

3.42 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=175

$$\frac{2\sqrt{a}\,c^4\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^3c^4\tan^5(e}{5f(a+a\sec(e}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^4*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^4*tan(f*x+e)^3/f/(a+a*sec(f*x+
e))^(3/2)-2/5*a^3*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^4*c^4*tan
(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 308, 209}

$$\frac{2a^4c^4\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^3c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}\,c^4\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^4\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^4*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^4*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2)) + (2*a^4*c^4*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e +
 f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^4\,dx = (a^4c^4)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{(2a^5c^4)\,\text{Subst}\left(\int \frac{x^8}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{(2a^5c^4)\,\text{Subst}\left(\int \left(-\frac{1}{a^4}+\frac{x^2}{a^3}-\frac{x^4}{a^2}+\frac{x^6}{a}+\frac{1}{a^4(1+ax^2)}\right)\,dx, x\right.}{f}$$

$$= -\frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{}{5f}$$

$$= \frac{2\sqrt{a}\,c^4\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^4\tan(e+}{f\sqrt{a+a\sec(e}}$$

Mathematica [A]
time = 1.13, size = 121, normalized size = 0.69

$$\frac{2c^4\left(105\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^3(e+fx)+(76-198\cos(e+fx)+61\cos(2(e+fx))-44\cos(3(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{105f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*c^4*(105*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (76 - 198*Cos[
e + f*x] + 61*Cos[2*(e + f*x)] - 44*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x
```

```
]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*Sqrt
[-1 + Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(155) = 310$.
time = 1.24, size = 391, normalized size = 2.23

method	result
default	$\frac{c^4 \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}} \left(105 \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}} (\cos^3(fx+e)) \sin(fx+e)\sqrt{2} + 315}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/840*c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(105*arctanh(1/2*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(7/2)*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)+315*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(7/2)*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+315*arctanh(1/2*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(7/2)*cos(f*x+e)*sin(f*x+e)*2^(1/2)+105*2^(1/2)*arctanh
(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*sin(f*x+e)+2816*cos(f*x+e)^4-4768*cos(f*
x+e)^3+3008*cos(f*x+e)^2-1296*cos(f*x+e)+240)/cos(f*x+e)^3/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/210*(105*((c^4*cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(2
*f*x + 2*e) + c^4)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^4*
cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)
*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^4*f*cos(2*f*x + 2*
```

```
e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos(2*f*x + 2*e) + c^4*f)*integra
te((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x +
2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*
f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos
(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*co
s(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*c
os(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*
cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((
cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) +
 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e)
- cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e)
 - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e
))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*
e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6
*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*
e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x +
 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*c
os(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e
))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2
 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f
*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*
f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 +
8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x +
8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*s
in(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6
*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*
x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
)*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2
*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)
^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*
f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*si
n(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12
*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x +
6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
```

```
2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)
*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)),
 x) - 40*(c^4*f*cos(2*f*x + 2*e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos
(2*f*x + 2*e) + c^4*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4
*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*
e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x
+ 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x
+ 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*s...
```

Fricas [A]
time = 3.92, size = 404, normalized size = 2.31

$$\left[\frac{105\,\left(c^{4}\cos\left(fx+e\right)^{4}+c^{4}\cos\left(fx+e\right)^{3}\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)-2\,\left(176\,c^{4}\cos\left(fx+e\right)^{3}-122\,c^{4}\cos\left(fx+e\right)^{2}+66\,c^{4}\cos\left(fx+e\right)-15\,c^{4}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{105\,\left(f\cos\left(fx+e\right)^{4}+f\cos\left(fx+e\right)^{3}\right)},\,-\frac{2\,\left(105\,\left(c^{4}\cos\left(fx+e\right)^{4}+c^{4}\cos\left(fx+e\right)^{3}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{\sqrt{a}\,\sin\left(fx+e\right)}\right)+\left(176\,c^{4}\cos\left(fx+e\right)^{3}-122\,c^{4}\cos\left(fx+e\right)^{2}+66\,c^{4}\cos\left(fx+e\right)-15\,c^{4}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{105\,\left(f\cos\left(fx+e\right)^{4}+f\cos\left(fx+e\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(176*c^4*cos(
f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*
x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(a)*ar
ctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
+ e))) + (176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x
+ e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{4}\left(\int\left(-4\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)\right)dx+\int 6\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{2}\left(e+fx\right)dx+\int\left(-4\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{3}\left(e+fx\right)\right)dx+\int\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{4}\left(e+fx\right)dx+\int\sqrt{a\sec\left(e+fx\right)+a}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(6*sq
rt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-4*sqrt(a*sec(e + f*x
) + a)*sec(e + f*x)**3, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**4, x) + Integral(sqrt(a*sec(e + f*x) + a), x))
```

Giac [A]
time = 1.55, size = 295, normalized size = 1.69

$$-\frac{\frac{105\,\sqrt{-a}\,a c^{4}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}\,|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}\,|a|-6a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{|a|}-\frac{2\left(105\sqrt{2}\,a^{4}c^{4}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(385\sqrt{2}\,a^{4}c^{4}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+\left(379\sqrt{2}\,a^{4}c^{4}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-539\sqrt{2}\,a^{4}c^{4}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*a*c^4*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*
abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(105*sqrt(2)*a^4*c^4*sgn(cos(f*
x + e)) - (385*sqrt(2)*a^4*c^4*sgn(cos(f*x + e)) + (379*sqrt(2)*a^4*c^4*sgn
(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 539*sqrt(2)*a^4*c^4*sgn(cos(f*x + e
)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((
a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4, x)
```

3.43 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=140

$$\frac{2\sqrt{a}\,c^3\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^3c^3\tan^5(e}{5f(a+a\sec(e}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^3*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^3*tan(f*x+e)^3/f/(a+a*sec(f*x+
e))^(3/2)-2/5*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 308, 209}

$$-\frac{2a^3c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}\,c^3\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^3*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^3*c^3*Tan[e + f*x]^5)/(5*f*(
a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^3\,dx = -\left((a^3c^3)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx\right)$$

$$= \frac{(2a^4c^3)\,\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{(2a^4c^3)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)\,dx, x, -\frac{}{\sqrt{a\cdots}}\right.}{f}$$

$$= -\frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{\cdots}{5f\cdots}$$

$$= \frac{2\sqrt{a}\,c^3\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^3\tan(e+\cdots}{f\sqrt{a+a\sec(e\cdots}}$$

Mathematica [A]
time = 1.24, size = 111, normalized size = 0.79

$$\frac{c^3\left(30\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^2(e+fx)+(-29+22\cos(e+fx)-23\cos(2(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{15f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (c^3*(30*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (-29 + 22*Cos[e +
 f*x] - 23*Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2*Sqrt[a
*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(15*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(124) = 248$.
time = 0.24, size = 302, normalized size = 2.16

method	result
default	$-\frac{c^3\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(15\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}(\cos^2(fx+e))\sqrt{2}+30}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/60*c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(15*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)^2*2^(1/2)+30*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)*2^(1/2)+15*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*sin(f*x+e)-184*cos(f*x+e)^3+272*cos(f*x+e)^2-112*cos(f*x+e)+24)/sin(f*x+e)/cos(f*x+e)^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*f*x + 2*e)^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x +

```
4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x +
2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
 + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2
*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 +
6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x +
6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6
*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6
*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 18*(c^3*f*cos(2*f*x + 2*e)^
2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate(
(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
+ 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*
e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*
e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4
*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2
*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
+ 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
 + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
```

```
e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6
*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*...
```

Fricas [A]
time = 2.93, size = 376, normalized size = 2.69

$$\left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(23\,c^3\cos(fx+e)^2-11\,c^3\cos(fx+e)+3\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)},\ -\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(23\,c^3\cos(fx+e)^2-11\,c^3\cos(fx+e)+3\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(23*c^3*cos(f*x
 + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(c^3*cos
(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (23*c^3*cos(f*x + e)^2
 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin
(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)\,dx+\int\left(-3\sqrt{a\sec(e+fx)+a}\,\sec^2(e+fx)\right)dx+\int\sqrt{a\sec(e+fx)+a}\,\sec^3(e+fx)\,dx+\int\left(-\sqrt{a\sec(e+fx)+a}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-3*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x)
+ a)*sec(e + f*x)**3, x) + Integral(-sqrt(a*sec(e + f*x) + a), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(124) = 248.
time = 1.35, size = 263, normalized size = 1.88

$$-\frac{\frac{15\sqrt{-a}\,ac^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(15\sqrt{2}\,a^3c^3\mathrm{sgn}(\cos(fx+e))+\left(37\sqrt{2}\,a^3c^3\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-40\sqrt{2}\,a^3c^3\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*a*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(15*sqrt(2)*a^3*c^3*sgn(cos(f*x +
 e)) + (37*sqrt(2)*a^3*c^3*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 40*sq
rt(2)*a^3*c^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*
e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/
f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3, x)
```

3.44 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{a}\,c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c^2*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*x+
e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 308, 209}

$$\frac{2a^2c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{2\sqrt{a}\,c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*Sqrt[a]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*c^2*Tan[e + f
*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\left(2a^3c^2\right)\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{\left(2a^3c^2\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{}{\sqrt{a+}}\right.}{f}$$

$$= -\frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{(2}{}$$

$$= \frac{2\sqrt{a}\,c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^2\tan(e+}{f\sqrt{a+a\sec(e}}$$

Mathematica [A]
time = 0.88, size = 97, normalized size = 0.92

$$-\frac{2c^2\left(-3\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos(e+fx)+(-1+4\cos(e+fx))\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{3f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*c^2*(-3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x] + (-1 + 4*Cos[e +
f*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[
(e + f*x)/2])/(3*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [A]
time = 0.22, size = 142, normalized size = 1.35

method	result
default	$-\frac{c^{2}\left(3\sin(fx+e)\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}-8(\cos^{2}(fx+e))+10\cos(f$ … $}{3f\sin(fx+e)\cos(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c^2/f*(3*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ar
ctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2
))*2^(1/2)-8*cos(f*x+e)^2+10*cos(f*x+e)-2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1
/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*c^2*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*
x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x
 + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x
 + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x
+ 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x
 + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x +
 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(2*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 +
4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*
e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f
*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
)^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f
*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x +
```

```
6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)), x) + 4*c^2*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2
*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x
+ 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x
+ 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x +
 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x
+ 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x +
 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6
*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
+ 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x
+ 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos
(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*
e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*
f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 6*c^2*f*integrate((((cos(6*f*x + 6*
e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*
e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e
))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e
) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e)
 - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e)
+ 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*
x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*c
os(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin...
```

Fricas [A]
time = 2.68, size = 340, normalized size = 3.24

$$\left[\frac{3\,\left(c^{2} \cos\left(fx+e\right)^{2} + c^{2} \cos\left(fx+e\right)\right)\sqrt{-a}\,\log\left(\frac{2\,a \cos\left(fx+e\right)^{2} - 2\,\sqrt{-a}\,\sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)\sin\left(fx+e\right) + a\cos\left(fx+e\right) - a}{\cos\left(fx+e\right)+1}\right) - 2\,\left(4\,c^{2}\cos\left(fx+e\right) - c^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{3\,\left(f\cos\left(fx+e\right)^{2} + f\cos\left(fx+e\right)\right)}, -\frac{2\,\left(3\,\left(c^{2}\cos\left(fx+e\right)^{2} + c^{2}\cos\left(fx+e\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{\sqrt{a}\,\sin\left(fx+e\right)}\right) + \left(4\,c^{2}\cos\left(fx+e\right) - c^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{3\,\left(f\cos\left(fx+e\right)^{2} + f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(4*c^2*cos(f*x + e)
 - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-2\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)\right)dx + \int\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{2}\left(e+fx\right)dx + \int\sqrt{a\sec\left(e+fx\right)+a}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(sqrt
(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) + a
), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(93) = 186.
time = 1.27, size = 232, normalized size = 2.21

$$-\frac{\frac{3\,\sqrt{-a}\,ac^{2}\log\left(\frac{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - \sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2} - 4\,\sqrt{2}\left|a\right| - 6\,a\right|}{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - \sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2} + 4\,\sqrt{2}\left|a\right| - 6\,a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{\left|a\right|} + \frac{2\,\left(5\,\sqrt{2}\,a^{2}c^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2} - 3\,\sqrt{2}\,a^{2}c^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
```

```
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(5*sqrt(2)*a^2*c^2*sgn(cos(f*x + e)
)*tan(1/2*f*x + 1/2*e)^2 - 3*sqrt(2)*a^2*c^2*sgn(cos(f*x + e)))*tan(1/2*f*x
 + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2, x)
```

3.45 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}\,c\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f-2*a*c*tan(f
*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {3989, 3972, 327, 209}

$$\frac{2\sqrt{a}\,c\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2ac\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))\,dx = -\left((ac)\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\right)$$
$$= \frac{(2a^2c)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= -\frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2ac)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\ \ }{\sqrt{\ }}\right.}{f}$$
$$= \frac{2\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac\tan(e+f}{f\sqrt{a+a\sec(e+}}$$

Mathematica [A]
time = 0.35, size = 70, normalized size = 1.06

$$-\frac{2c\left(-\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)+\sqrt{-1+\sec(e+fx)}\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

```
[Out] (-2*c*(-ArcTan[Sqrt[-1 + Sec[e + f*x]]] + Sqrt[-1 + Sec[e + f*x]])*Sqrt[a*(
1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [A]
time = 0.17, size = 115, normalized size = 1.74

---

method	result
default	$-\frac{c\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)-2\cos(fx+e)+2\right)}{f\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2^(1/2)*arctanh(1/2*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)-2*cos(f*x+e)+2)/sin(f*x+e)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(62) = 124$.
time = 0.54, size = 159, normalized size = 2.41

$$\frac{\sqrt{a}\,c\arctan\left(\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2fx+2e),\cos(2fx+2e)+1)\right)+\sin(fx+e),\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(2fx+2e),\cos(2fx+2e)+1)\right)+\cos(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(
f*x + e))/f
```

Fricas [A]
time = 2.31, size = 255, normalized size = 3.86

$$\left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{f\cos(fx+e)+f},-\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)) - 2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
```

```
 + c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
+ f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \sqrt{a\sec(e+fx)+a}\,\sec(e+fx)\,dx + \int\left(-\sqrt{a\sec(e+fx)+a}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-sqrt(a*s
ec(e + f*x) + a), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(58) = 116.
time = 1.19, size = 193, normalized size = 2.92

$$\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,ac\,\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a}-\frac{\sqrt{-a}\,ac\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a*c*sgn(cos(f*x + e))*tan(1/
2*f*x + 1/2*e)/(a*tan(1/2*f*x + 1/2*e)^2 - a) - sqrt(-a)*a*c*log(abs(2*(sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqr
t(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f
*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a))/
f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)), x)
```

3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f+2*cot(f*x+e
)*(a+a*sec(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 331, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{ac} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} \\
&= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}
\end{aligned}
$$

Mathematica [A]
time = 0.66, size = 90, normalized size = 1.30

$$
\frac{2a\left(-\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)(-1+\cos(e+fx))+\cos(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)\tan(e+fx)}{cf(-1+\sec(e+fx))^{3/2}\sqrt{a(1+\sec(e+fx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + Cos[e + f*x]
*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x
])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.20, size = 116, normalized size = 1.68

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)-2\cos(fx+e)\right)}{cf\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2^(1/2)*arctanh(1/2*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)-2*cos(f*x+e))/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c), x)
```

Fricas [A]
time = 2.85, size = 290, normalized size = 4.20

$$\left[\frac{\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{2\,cf\sin(fx+e)},\ \frac{\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2\,a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{cf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(
f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos
(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{a\sec{\left(e+fx \right)}+a}}{\sec{\left(e+fx \right)}-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x)/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(61) = 122.

time = 1.10, size = 196, normalized size = 2.84

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}\,|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}\,|a|-6a\right|}\right)}{c|a|}+\frac{4\sqrt{-a}\,a}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)c}\right)\mathrm{sgn}(\cos(fx+e))}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/(c*abs(a)) + 4*sqrt(-a)*a/(((sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c))*sgn(cos(f*x + e))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)), x)
```

3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f}-\frac{2\cot^3(e+fx)(a+a\sec(e+fx)}{3ac^2f}$$

```
[Out] -2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^2/f+2*arctan(a^(1/2)*tan(f*x+e
)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c^2/f+2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)
/c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 331, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^2f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx &= \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{a^2c^2} \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^2f} \\
&= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f} + \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\text{ta}}{\sqrt{a+a}}\right.}{c^2f} \\
&= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f} - \\
&= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} -
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.25, size = 78, normalized size = 0.75

$$
-\frac{2\sqrt{\cos(e+fx)}\ {}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sec(e+fx))}\ \tan\left(\frac{1}{2}(e+fx)\right)}{3c^2f(-1+\cos(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-3/2, -3/2, -1/2, 2*Sin[(e + f*x)/
2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(3*c^2*f*(-1 + Cos[e + f
*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(92) = 184$.
time = 0.22, size = 214, normalized size = 2.06

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(3\sin(fx+e)\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}-3\sqrt{2}}{3c^2 f\sin(fx+e)(-1+\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(3*sin(f*x+e)*cos(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-3*2^(1/2)*arctanh(1/2*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-8*cos(f*x+e)^2+6*cos(f*x+e))/sin(f*x+e)/
(-1+cos(f*x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^2, x)
```

Fricas [A]
time = 3.78, size = 369, normalized size = 3.55

$$\left[\frac{3\sqrt{-a}\left(\cos(fx+e)-1\right)\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\left(4\cos(fx+e)^2-3\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{6\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}, \frac{3\sqrt{a}\left(\cos(fx+e)-1\right)\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(4\cos(fx+e)^2-3\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{3\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x
 + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(4*
cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*sqrt(a)*(cos(f*x + e) -
```

```
1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(4*c
os(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x
)/c**2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(92) = 184.
time = 1.12, size = 296, normalized size = 2.85

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c^2|a|}+\frac{2\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^4\sqrt{-a}\,a-12\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{-a}\,a^2+7\sqrt{-a}\,a^3\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)^3c^2}\right)\mathrm{sgn}(\cos(fx+e))}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
 - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*
sqrt(2)*abs(a) - 6*a))/(c^2*abs(a)) + 2*(9*(sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a - 12*(sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^2 + 7*sqrt(
-a)*a^3)/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a))^2 - a)^3*c^2))*sgn(cos(f*x + e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2, x)
```

3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3 f}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3 f}-\frac{2\cot^3(e+fx)(a+a\sec(e+f}{3ac^3 f}$$

```
[Out] -2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^3/f+2/5*cot(f*x+e)^5*(a+a*sec(
f*x+e))^(5/2)/a^2/c^3/f+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))
*a^(1/2)/c^3/f+2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f
```

Rubi [A]

time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 331, 209}

$$\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^3 f}+\frac{2\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3 f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^3 f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3 f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^3*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{a^3c^3}$$
$$= \frac{2\mathrm{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^3f}$$
$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f} - \frac{2\mathrm{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+}{\sqrt{a+a\sec}}\right.}{ac^3f}$$
$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f}$$
$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2}{}$$
$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.28, size = 78, normalized size = 0.56

$$-\frac{2\sqrt{\cos(e+fx)}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{5c^3f(-1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]

[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-5/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(5*c^3*f*(-1 + Cos[e + f*x])^3)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(123) = 246$.

time = 0.27, size = 311, normalized size = 2.24

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos(fx+e)+1)\left(15\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/15/c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)*(15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)-30*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+15*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-46*cos(f*x+e)^3+70*cos(f*x+e)^2-30*cos(f*x+e))/sin(f*x+e)^3/(-1+cos(f*x+e))

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^3, x)

Fricas [**A**]

time = 3.81, size = 441, normalized size = 3.17

$$\left[\frac{15\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\left(23\cos(fx+e)^3-35\cos(fx+e)^2+15\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{30\left(c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}, \frac{15\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(23\cos(fx+e)^3-35\cos(fx+e)^2+15\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{15\left(c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

```
[Out] [1/30*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x
+ e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))
*sin(f*x + e) + 4*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*
cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(cos(f*x + e)^2 - 2*cos(f*x +
 e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
 + e) + 2*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
+ e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3
*sec(e + f*x) - 1), x)/c**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(123) = 246.
time = 1.18, size = 391, normalized size = 2.81

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c^3|a|}+\frac{105\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^8\sqrt{-a}\,a-300\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^6\sqrt{-a}\,a^2+430\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^4\sqrt{-a}\,a^3-260\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{-a}\,a^4+73\sqrt{-a}\,a^5}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)^5c^3}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*(15*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/(c^3*abs(a)) + (105*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
 - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a - 300*(sqrt(-a)*tan(1/
2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2 + 430*
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqr
t(-a)*a^3 - 260*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*
e)^2 + a))^2*sqrt(-a)*a^4 + 73*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*f*x + 1/2*
```

```
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^5*c^3))*sgn(cos(f*x + e))/
f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(e+fx)}}}{\left(c-\frac{c}{\cos(e+fx)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3, x)
```

3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f}-\frac{2\cot^3(e+fx)(a+a\sec(e+fx)}{3ac^4f}$$

```
[Out] -2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(
f*x+e))^(5/2)/a^2/c^4/f-2/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a^3/c^4/f+2
*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c^4/f+2*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

Rubi [A]
time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 331, 209}

$$-\frac{2\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^3c^4f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^2c^4f}+\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3ac^4f}+\frac{2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*(a
 + a*Sec[e + f*x])^(3/2))/(3*a*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a^2*c^4*f) - (2*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7
*a^3*c^4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^4c^4}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^4f}$$

$$= -\frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f} + \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\text{ta}}{\sqrt{a+a}}\right.}{a^2c^4f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/}}{5a^2c^4f}$$

$$= \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} +$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.26, size = 78, normalized size = 0.45

$$-\frac{2\sqrt{\cos(e+fx)}\,{}_2F_1\left(-\frac{7}{2},-\frac{7}{2};-\frac{5}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{7c^4f(-1+\cos(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-7/2, -7/2, -5/2, 2*Sin[(e + f*x)/
2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(7*c^4*f*(-1 + Cos[e + f
*x])^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(154) = 308$.
time = 0.30, size = 402, normalized size = 2.31

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos(fx+e)+1)^2\left(-105\left(\cos^3(fx+e)\right)\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e}{2\cos(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105/c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-105*cos(
f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+315*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)-31
5*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+3
52*cos(f*x+e)^4+105*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)-812*cos(f*x+e)^3+700*cos(f*x+e)^2-210*cos(f*x+e))/sin(f*x+e)^5/(-1+co
s(f*x+e))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

[Out] Timed out

Fricas [A]
time = 4.25, size = 517, normalized size = 2.97

$$\left[\frac{105\,\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)^{2}+3\,\cos\left(fx+e\right)-1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos\left(fx+e\right)^{3}-4\,\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-7\,a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+4\,\left(176\,\cos\left(fx+e\right)^{4}-406\,\cos\left(fx+e\right)^{3}+350\,\cos\left(fx+e\right)^{2}-105\,\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}}{210\,\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)},\ \frac{105\,\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)^{2}+3\,\cos\left(fx+e\right)-1\right)\sqrt{a}\arctan\left(\frac{2\,\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,a\cos\left(fx+e\right)^{2}+a\cos\left(fx+e\right)-a}\right)\sin\left(fx+e\right)+2\,\left(176\,\cos\left(fx+e\right)^{4}-406\,\cos\left(fx+e\right)^{3}+350\,\cos\left(fx+e\right)^{2}-105\,\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}}{105\,\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(-
a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a
)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(176*cos(f*x + e)^4 - 406*cos(f*x +
e)^3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f
*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)
^2 + 3*cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x +
e) - a))*sin(f*x + e) + 2*(176*cos(f*x + e)^4 - 406*cos(f*x + e)^3 + 350*co
s(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4
*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a \sec{\left(e + fx \right)} + a}}{\sec^{4}{\left(e + fx \right)} - 4 \sec^{3}{\left(e + fx \right)} + 6 \sec^{2}{\left(e + fx \right)} - 4 \sec{\left(e + fx \right)} + 1}\, dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*
sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)/c**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(154) = 308.
time = 1.24, size = 487, normalized size = 2.80

$$-\frac{\sqrt{2}\left(\frac{210\,\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\left|a\right|-6\,a\right|}{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\left|a\right|-6\,a\right|}\right)}{c^{4}\left|a\right|}+\frac{1575\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{12}\sqrt{-a}\,a-7140\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{10}\sqrt{-a}\,a^{2}+16415\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{8}\sqrt{-a}\,a^{3}-19880\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{6}\sqrt{-a}\,a^{4}+14637\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{4}\sqrt{-a}\,a^{5}-5684\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{6}+1037\,\sqrt{-a}\,a^{7}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}-a\right)^{7}c^{4}}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/420*sqrt(2)*(210*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/(c^4*abs(a)) + (1575*(sqrt(-a)*tan(1/2*f*x + 1/2
*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)*a - 7140*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^2
+ 16415*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a
))^8*sqrt(-a)*a^3 - 19880*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*
f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^4 + 14637*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
- sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^5 - 5684*(sqrt(-a)*tan(
1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^6 + 10
37*sqrt(-a)*a^7)/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^2 - a)^7*c^4))*sgn(cos(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4, x)
```

3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=177

$$\frac{2a^{3/2}c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^4c^3\tan^5(}{5f(a+a\sec(e}$$

```
[Out] 2*a^(3/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^3
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^4*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-2/7*a^5*c^3*t
an(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3989, 3972, 470, 308, 209}

$$\frac{2a^{3/2}c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^5c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^4c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^2c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^4*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (2*a^5*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\right)$$

$$= \frac{\left(2a^5c^3\right)\operatorname{Subst}\left(\int \frac{x^6\left(2+ax^2\right)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} + \frac{\left(2a^5c^3\right)\operatorname{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x,\right.}{f}$$

$$= -\frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} + \frac{\left(2a^5c^3\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \right.\right.}{}$$

$$= -\frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{}{5f}$$

$$= \frac{2a^{3/2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^3\tan(e+}{f\sqrt{a+a\sec(e+}}$$

Mathematica [A]
time = 1.02, size = 122, normalized size = 0.69

$$\frac{ac^3\left(210\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^3(e+fx)+(2-171\cos(e+fx)+32\cos(2(e+fx))-73\cos(3(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{105f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a*c^3*(210*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (2 - 171*Cos[e
 + f*x] + 32*Cos[2*(e + f*x)] - 73*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]
])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*Sqrt[
-1 + Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(157) = 314$.
time = 0.23, size = 392, normalized size = 2.21

method	result
default	$\frac{c^3\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(105\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}(\cos^3(fx+e))\sin(fx+e)\sqrt{2}+3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/840*c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(105*arctanh(1/2*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(7/2)*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)+315*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(7/2)*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+315*arctanh(1/2*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(7/2)*cos(f*x+e)*sin(f*x+e)*2^(1/2)+105*2^(1/2)*arctanh
(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*sin(f*x+e)+2336*cos(f*x+e)^4-2848*cos(f*
x+e)^3+128*cos(f*x+e)^2+624*cos(f*x+e)-240)/cos(f*x+e)^3/sin(f*x+e)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^3
*cos(2*f*x + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1)
 - (a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*x
 + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a*c^
3*f*cos(2*f*x + 2*e)^2 + a*c^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x +
 2*e) + a*c^3*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x
 + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
+ 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x
 + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x
 + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*
x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*
x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x
 + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*
x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x
 + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*
x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x
 + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2
*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)
^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f
*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)
 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x
 + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x
```

```
+ 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos
(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)
 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2
*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin
(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x +
 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*
e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*
x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e...
```

Fricas [A]
time = 3.88, size = 416, normalized size = 2.35

$$\left[\frac{105\,\left(ac^{3}\cos\left(fx+e\right)^{4}+ac^{3}\cos\left(fx+e\right)^{3}\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(fx+e)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(146\,ac^{3}\cos\left(fx+e\right)^{3}-32\,ac^{3}\cos\left(fx+e\right)^{2}-24\,ac^{3}\cos\left(fx+e\right)+15\,ac^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{105\,\left(f\cos\left(fx+e\right)^{4}+f\cos\left(fx+e\right)^{3}\right)},-\frac{2\left(105\,\left(ac^{3}\cos\left(fx+e\right)^{4}+ac^{3}\cos\left(fx+e\right)^{3}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)}{\sqrt{a}\,\sin(fx+e)}\right)+\left(146\,ac^{3}\cos\left(fx+e\right)^{3}-32\,ac^{3}\cos\left(fx+e\right)^{2}-24\,ac^{3}\cos\left(fx+e\right)+15\,ac^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{105\,\left(f\cos\left(fx+e\right)^{4}+f\cos\left(fx+e\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(146*a*c^
3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2 - 24*a*c^3*cos(f*x + e) + 15*a*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e
)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) + (146*a*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2
 - 24*a*c^3*cos(f*x + e) + 15*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int\left(-a\sqrt{a\sec\left(e+fx\right)+a}\right)dx+\int 2a\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)dx+\int\left(-2a\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{3}\left(e+fx\right)\right)dx+\int a\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(2*a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x), x) + Integral(-2*a*sqrt(a*sec(e + f*x) + a)*sec(
e + f*x)**3, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x))
```

Giac [A]
time = 1.61, size = 297, normalized size = 1.68

$$-\frac{\frac{105\sqrt{-a}\,a^{2}c^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}\,|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}\,|a|-6a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{|a|}-\frac{2\left(105\sqrt{2}\,a^{5}c^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(385\sqrt{2}\,a^{5}c^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+\left(139\sqrt{2}\,a^{5}c^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-539\sqrt{2}\,a^{5}c^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}}}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*a^2*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqr
t(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(105*sqrt(2)*a^5*c^3*sgn(cos(
f*x + e)) - (385*sqrt(2)*a^5*c^3*sgn(cos(f*x + e)) + (139*sqrt(2)*a^5*c^3*s
gn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 539*sqrt(2)*a^5*c^3*sgn(cos(f*x +
 e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/
((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3, x)
```

3.51 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=142

$$\frac{2a^{3/2}c^2\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}+\frac{2a^4c^2\tan^5(}{5f(a+a\sec(e}$$

```
[Out] 2*a^(3/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^2
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^2*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)+2/5*a^4*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3989, 3972, 470, 308, 209}

$$\frac{2a^{3/2}c^2\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^4c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^3c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^4*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= -\frac{\left(2a^4c^2\right)\text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2a^4c^2\right)\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\right.}{f}$$

$$= \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2a^4c^2\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\right.\right.}{}$$

$$= -\frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{}{5f(}$$

$$= \frac{2a^{3/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^2\tan(e+}{f\sqrt{a+a\sec(e+}}$$

Mathematica [A]

time = 0.87, size = 112, normalized size = 0.79

$$-\frac{ac^2\left(-30\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^2(e+fx)+(11+2\cos(e+fx)+17\cos(2(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{15f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/15*(a*c^2*(-30*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (11 + 2*
Cos[e + f*x] + 17*Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2
*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [A]
time = 0.21, size = 232, normalized size = 1.63

method	result
default	$\frac{c^2\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(15(\cos^2(fx+e))\sin(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}+15\right.}{30}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(15*cos(f*x+e)^2*sin(f*x+e)*
(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+15*cos(f*x+e)*sin(f*x+e)*
(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+68*cos(f*x+e)^3-64*cos(f*
x+e)^2-16*cos(f*x+e)+12)/cos(f*x+e)^2/sin(f*x+e)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*c
os(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) -
 (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2*f*x +
 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
```

```
*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a*c^2*
f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2
*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x +
2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*
e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x +
 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2
*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*
e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 +
 (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2
*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*
x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)
*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
```

```
e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*
x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(
4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2),
x) + 6*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f
*cos(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))*cos(3/2*arc...
```

Fricas [A]
time = 2.64, size = 384, normalized size = 2.70

$$\left[\frac{15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{-a}\,\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(17ac^2\cos(fx+e)^2+ac^2\cos(fx+e)-3ac^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)},\ -\frac{2\left(15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(17ac^2\cos(fx+e)^2+ac^2\cos(fx+e)-3ac^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(17*a*c^2*c
os(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(
a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (17*a*c^2*c
os(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int a\sqrt{a\sec(e+fx)+a}\,dx+\int\left(-a\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx+\int\left(-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx+\int a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(a*sqrt(a*sec(e + f*x) + a), x) + Integral(-a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**2, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(126) = 252$.
time = 1.46, size = 265, normalized size = 1.87

$$-\frac{\frac{15\sqrt{-a}\,a^2c^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(15\sqrt{2}a^4c^2\mathrm{sgn}(\cos(fx+e))+\left(13\sqrt{2}a^4c^2\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-40\sqrt{2}a^4c^2\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*a^2*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*
abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(15*sqrt(2)*a^4*c^2*sgn(cos(f*x
 + e)) + (13*sqrt(2)*a^4*c^2*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 40*
sqrt(2)*a^4*c^2*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/
2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))
)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2, x)
```

3.52 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=101

$$\frac{2a^{3/2}c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c*tan
(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a^3*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^
(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3989, 3972, 470, 327, 209}

$$\frac{2a^{3/2}c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^3c\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^2c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^2*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -\left((ac)\int \sqrt{a+a\sec(e+fx)}\,\tan^2(e+fx)\,dx\right)$$

$$= \frac{(2a^3c)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{(2a^3c)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\right.}{f}$$

$$= -\frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{(2a^2}{}$$

$$= \frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c\tan(e+fx}{f\sqrt{a+a\sec(e+f}}$$

Mathematica [A]
time = 0.70, size = 96, normalized size = 0.95

$$-\frac{2ac\left(-3\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos(e+fx)+(1+2\cos(e+fx))\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{3f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (-2*a*c*(-3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x] + (1 + 2*Cos[e + f
*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(
e + f*x)/2])/(3*f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(89) = 178$.
time = 0.18, size = 212, normalized size = 2.10

method	result
default	$\frac{c\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(3\cos(fx+e)\sin(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}+3\sqrt{2}}{6f\sin(fx+e)\cos(fx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(3*cos(f*x+e)*sin(f*x+e)*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+3*2^(1/2)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(3/2)*sin(f*x+e)+8*cos(f*x+e)^2-4*cos(f*x+e)-4)/sin(f*x+e)/c
os(f*x+e)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(95) = 190$.
time = 0.59, size = 1076, normalized size = 10.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
```

```
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)
```

Fricas [**A**]
time = 3.45, size = 330, normalized size = 3.27

$$\left[\frac{3\left(ac\cos\left(fx+e\right)^{2}+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(2\,ac\cos\left(fx+e\right)+ac\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^{2}+f\cos\left(fx+e\right)\right)},-\frac{2\left(3\left(ac\cos\left(fx+e\right)^{2}+ac\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(2\,ac\cos\left(fx+e\right)+ac\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)\right)}{3\left(f\cos\left(fx+e\right)^{2}+f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(2*a*c*cos(f*x + e)
 + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqr
t(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (2*a*c*cos(f*x + e) + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-a\sqrt{a\sec\left(e+fx\right)+a}\right)dx+\int a\sqrt{a\sec\left(e+fx\right)+a}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(89) = 178.

time = 1.21, size = 227, normalized size = 2.25

$$-\frac{\frac{3\sqrt{-a}\,a^2c\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\operatorname{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(\sqrt{2}a^3c\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3\sqrt{2}a^3c\operatorname{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a^2*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(sqrt(2)*a^3*c*sgn(cos(f*x + e))*ta
n(1/2*f*x + 1/2*e)^2 - 3*sqrt(2)*a^3*c*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2
*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)), x)
```

3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+4*a*cot(f*x
+e)*(a+a*sec(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 464, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}+\frac{4a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (4*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{ac}$$

$$= \frac{(2a)\text{Subst}\left(\int \frac{2+ax^2}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

$$= \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\text{ta}}{\sqrt{a+a}}\right.}{cf}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

Mathematica [A]
time = 0.60, size = 93, normalized size = 1.33

$$\frac{2a^2\left(-\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)(-1+\cos(e+fx))+2\cos(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)\tan(e+fx)}{cf(-1+\sec(e+fx))^{3/2}\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^2*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + 2*Cos[e +
f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e +
 f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(62) = 124.
time = 0.18, size = 194, normalized size = 2.77

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left((\cos^2(fx+e))\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)-\sqrt{2}\sqrt{-\frac{2}{c}}\right.}{cf(\cos^2(fx+e)-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)^2*2^(1/2)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)/cos(f*x+e)*2^(1/2))-2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^
(1/2))+4*cos(f*x+e)*sin(f*x+e))/(cos(f*x+e)^2-1)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^(3/2)/(c*sec(f*x + e) - c), x)
```

Fricas [A]
time = 4.12, size = 293, normalized size = 4.19

$$\left[\frac{\sqrt{-a}\,a\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+8\,a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{2\,cf\sin(fx+e)},\ \frac{a^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2\,a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+4\,a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{cf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*co
s(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (a^(3/2)*arctan(2*sqr
t(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a
*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 4*a*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a\sqrt{a\sec{(e+fx)}+a}}{\sec{(e+fx)}-1}\,dx+\int \frac{a\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}}{\sec{(e+fx)}-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(a*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x))/c
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(62) = 124.
time = 1.16, size = 198, normalized size = 2.83

$$-\frac{\sqrt{2}\sqrt{-a}\,a^{3}\left(\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{ac|a|}+\frac{8}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-a\right)ac}\right)\mathrm{sgn}(\cos{(fx+e)})}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(-a)*a^3*(sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
 - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*
sqrt(2)*abs(a) - 6*a))/(a*c*abs(a)) + 8/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) -
sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*a*c))*sgn(cos(f*x + e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c-\frac{c}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)), x)
```

3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}+\frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f}-\frac{4\cot^3(e+fx)(a+a\sec(e+f}{3c^2f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-4/3*cot(f
*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f+2*a*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/
c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3989, 3972, 464, 331, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{4\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}+\frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (4*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx &= \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{a^2c^2}\\
&= -\frac{2\text{Subst}\left(\int \frac{2+ax^2}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}\\
&= -\frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} + \frac{(2a)\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\quad}{\sqrt{a}\cdots}\right.}{c^2f}\\
&= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}\\
&= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f}
\end{aligned}$$

Mathematica [A]

time = 0.68, size = 113, normalized size = 1.11

$$-\frac{2a\sqrt{\cos(e+fx)}\,\sqrt{a(1+\sec(e+fx))}\left(\sqrt{\cos(e+fx)}\,(-3+5\cos(e+fx))-6\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^3\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{3c^2f(-1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[Cos[e + f*x]]*(-3
 + 5*Cos[e + f*x]) - 6*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*
x)/2]^3)*Tan[(e + f*x)/2])/(3*c^2*f*(-1 + Cos[e + f*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(90) = 180$.
time = 0.21, size = 215, normalized size = 2.11

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(3\sin(fx+e)\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}-3\sqrt{2}\right.}{3c^2 f\sin(fx+e)(-1+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(3*sin(f*x+e)*cos(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-3*2^(1/2)*arctanh(1/2*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-10*cos(f*x+e)^2+6*cos(f*x+e))/sin(f*x+e)
/(-1+cos(f*x+e))*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.97, size = 381, normalized size = 3.74

$$\left[\frac{3\,(a\cos(fx+e)-a)\sqrt{-a}\,\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\left(5\,a\cos(fx+e)^2-3\,a\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{6\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)},\ \frac{3\,(a\cos(fx+e)-a)\sqrt{a}\,\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\cos(fx+e)\sin(fx+e)}{2\,a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(5\,a\cos(fx+e)^2-3\,a\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{3\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

[Out] [1/6*(3*(a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*(a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)

[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(90) = 180.
time = 1.28, size = 318, normalized size = 3.12

$$-\frac{\frac{3\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{c^2|a|}+\frac{4\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^4\sqrt{-a}\,a^2\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{-a}\,a^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+2\sqrt{-a}\,a^4\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)^3c^2}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] -1/3*(3*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/(c^2*abs(a)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*sgn(cos(f*x + e)) - 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(f*x + e)) + 2*sqrt(-a)*a^4*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3*c^2))/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2, x)
```

3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=137

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}-\frac{2\cot^3(e+fx)(a+a\sec(e+$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f-2/3*cot(f
*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^3/f+4/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2
)/a/c^3/f+2*a*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3989, 3972, 464, 331, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{4\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^3f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^3f}+\frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^3*f) + (4*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^3c^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{2+ax^2}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^3f}$$

$$= \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f} - \frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e}{\sqrt{a+a\,\text{se}}}\right.}{c^3f}$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f}$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} +$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.76, size = 102, normalized size = 0.74

$$\frac{2a\left(6\cos^{\frac{5}{2}}(e+fx)+5(-1+\cos(e+fx))\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{15c^3f\cos^{\frac{5}{2}}(e+fx)(-1+\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a*(6*Cos[e + f*x]^(5/2) + 5*(-1 + Cos[e + f*x])*Hypergeometric2F1[-3/2,
-3/2, -1/2, 2*Sin[(e + f*x)/2]^2])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)
/2])/(15*c^3*f*Cos[e + f*x]^(5/2)*(-1 + Sec[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(121) = 242.
time = 0.24, size = 304, normalized size = 2.22

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-15\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-15*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/c
os(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+30*sin(f*x+e)*cos(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-15*2^(1/2)*arctanh(1/2*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+52*cos(f*x+e)^3-70*cos(f*x+e)^2+30*
cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^2*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 2.77, size = 453, normalized size = 3.31

$$\left[\frac{15\,{\left(a \cos\left(f x + e\right)^{2} - 2\,a \cos\left(f x + e\right) + a\right)} \sqrt{-a} \log\left(-\frac{8\,a \cos\left(f x + e\right)^{3} - 4\,{\left(2\,\cos\left(f x + e\right)^{2} - \cos\left(f x + e\right)\right)} \sqrt{-a} \sqrt{\frac{a \cos\left(f x + e\right) + a}{\cos\left(f x + e\right)}} \sin\left(f x + e\right) - 7\,a \cos\left(f x + e\right) + a}{\cos\left(f x + e\right) + 1}\right) \sin\left(f x + e\right) + 4\,{\left(26\,a \cos\left(f x + e\right)^{3} - 35\,a \cos\left(f x + e\right)^{2} + 15\,a \cos\left(f x + e\right)\right)} \sqrt{\frac{a \cos\left(f x + e\right) + a}{\cos\left(f x + e\right)}}}{30\,{\left(c^{3} f \cos\left(f x + e\right)^{2} - 2\,c^{3} f \cos\left(f x + e\right) + c^{3} f\right)} \sin\left(f x + e\right)}, \frac{15\,{\left(a \cos\left(f x + e\right)^{2} - 2\,a \cos\left(f x + e\right) + a\right)} \sqrt{a} \arctan\left(\frac{2\,\sqrt{a} \sqrt{\frac{a \cos\left(f x + e\right) + a}{\cos\left(f x + e\right)}} \cos\left(f x + e\right) \sin\left(f x + e\right)}{2\,a \cos\left(f x + e\right)^{2} + a \cos\left(f x + e\right) - a}\right) \sin\left(f x + e\right) + 2\,{\left(26\,a \cos\left(f x + e\right)^{3} - 35\,a \cos\left(f x + e\right)^{2} + 15\,a \cos\left(f x + e\right)\right)} \sqrt{\frac{a \cos\left(f x + e\right) + a}{\cos\left(f x + e\right)}}}{15\,{\left(c^{3} f \cos\left(f x + e\right)^{2} - 2\,c^{3} f \cos\left(f x + e\right) + c^{3} f\right)} \sin\left(f x + e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-a)*log(-(8*a*cos(
f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) +
 1))*sin(f*x + e) + 4*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a*cos
(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2
- 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(a*cos(f*x + e)^2 -
 2*a*cos(f*x + e) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e)
 - a))*sin(f*x + e) + 2*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a*c
os(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^
2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2
+ 3*sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(121) = 242.
time = 1.46, size = 428, normalized size = 3.12

$$-\frac{\frac{15\,\sqrt{-a}\,a^{2}\log\left(\frac{{\left|2\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{2}-4\,\sqrt{2}{\left|a\right|}-6\,a\right|}}{{\left|2\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{2}+4\,\sqrt{2}{\left|a\right|}-6\,a\right|}}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{c^{3}{\left|a\right|}}+\frac{2\,\sqrt{2}\left(30\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{8}\sqrt{-a}\,a^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-75\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{6}\sqrt{-a}\,a^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+115\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{4}\sqrt{-a}\,a^{4}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-65\,{\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{2}\sqrt{-a}\,a^{5}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+19\,\sqrt{-a}\,a^{6}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)}{{\left({\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)}^{2}-a\right)}^{5}c^{3}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) + 2*sqrt(2)*(30*(sqrt(-a)*tan(1/2
*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a^2*sgn(cos
(f*x + e)) - 75*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*
e)^2 + a))^6*sqrt(-a)*a^3*sgn(cos(f*x + e)) + 115*(sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^4*sgn(cos(f*x + e
)) - 65*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a
))^2*sqrt(-a)*a^5*sgn(cos(f*x + e)) + 19*sqrt(-a)*a^6*sgn(cos(f*x + e)))/((
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a
)^5*c^3))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3, x)
```

3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=172

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4 f}+\frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4 f}-\frac{2\cot^3(e+fx)(a+a\sec(e+f}{3c^4 f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f-2/3*cot(f
*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2
)/a/c^4/f-4/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a^2/c^4/f+2*a*cot(f*x+e)*
(a+a*sec(f*x+e))^(1/2)/c^4/f
```

Rubi [A]
time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3989, 3972, 464, 331, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4 f}-\frac{4\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7a^2c^4f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5ac^4f}-\frac{2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^4f}+\frac{2a\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*
(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*
x])^(5/2))/(5*a*c^4*f) - (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/(7*a
^2*c^4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{11/2}\,dx}{a^4c^4}$$

$$= -\frac{2\text{Subst}\left(\int \frac{2+ax^2}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^4f}$$

$$= -\frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} + \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f} - \ldots$$

$$= -\frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} \ldots$$

$$= \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \ldots$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.32, size = 102, normalized size = 0.59

$$-\frac{2a\sqrt{\cos(e+fx)}\left(10\cos^{\frac{7}{2}}(e+fx)+7(-1+\cos(e+fx))\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{35c^4f(-1+\cos(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (-2*a*Sqrt[Cos[e + f*x]]*(10*Cos[e + f*x]^(7/2) + 7*(-1 + Cos[e + f*x])*Hyp
ergeometric2F1[-5/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2])*Sqrt[a*(1 + Sec[e +
 f*x])]*Tan[(e + f*x)/2])/(35*c^4*f*(-1 + Cos[e + f*x])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(152) = 304.
time = 0.26, size = 401, normalized size = 2.33

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos(fx+e)+1)\left(105\left(\cos^3(fx+e)\right)\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)}{2\cos(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105/c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)*(105*cos(f*x
+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-315*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+315*s
in(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-105*
2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x
+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-382*cos(f*x+e)
^4+812*cos(f*x+e)^3-700*cos(f*x+e)^2+210*cos(f*x+e))/sin(f*x+e)^3/(-1+cos(f
*x+e))^2*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.09, size = 537, normalized size = 3.12

$$\left[\frac{105\,(a\cos(fx+e)^3-3\,a\cos(fx+e)^2+3\,a\cos(fx+e)-a)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\,(191\,a\cos(fx+e)^4-406\,a\cos(fx+e)^3+350\,a\cos(fx+e)^2-105\,a\cos(fx+e))\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{210\,(c^4f\cos(fx+e)^3-3\,c^4f\cos(fx+e)^2+3\,c^4f\cos(fx+e)-c^4f)\sin(fx+e)},\ \frac{105\,(a\cos(fx+e)^3-3\,a\cos(fx+e)^2+3\,a\cos(fx+e)-a)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2\,a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\,(191\,a\cos(fx+e)^4-406\,a\cos(fx+e)^3+350\,a\cos(fx+e)^2-105\,a\cos(fx+e))\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{105\,(c^4f\cos(fx+e)^3-3\,c^4f\cos(fx+e)^2+3\,c^4f\cos(fx+e)-c^4f)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*
sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqr
t(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x +
e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(191*a*cos(f*x + e)^4 - 406*a*
```

```
cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(a*cos(f*x + e)^3
- 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x +
 e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(191*a*cos(f*x + e)^4 - 406*a
*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
+ 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}}{\sec^{4}\left(e+fx\right)-4\sec^{3}\left(e+fx\right)+6\sec^{2}\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)}{\sec^{4}\left(e+fx\right)-4\sec^{3}\left(e+fx\right)+6\sec^{2}\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 +
 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)*
*2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(152) = 304.
time = 1.60, size = 555, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(f*x + e))/(c^4*abs(a)) + (420*sqrt(2)*(sqrt(-a)*tan(1/
2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)*a^2*sgn(c
os(f*x + e)) - 1785*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/
2*f*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^3*sgn(cos(f*x + e)) + 4235*sqrt(2)*(sq
rt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-
a)*a^4*sgn(cos(f*x + e)) - 4970*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^5*sgn(cos(f*x + e)) + 3738*
sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a
))^4*sqrt(-a)*a^6*sgn(cos(f*x + e)) - 1421*sqrt(2)*(sqrt(-a)*tan(1/2*f*x +
```

```
1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^7*sgn(cos(f*x +
e)) + 263*sqrt(2)*sqrt(-a)*a^8*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(1/2*f*x +
 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^7*c^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4, x)
```

3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=212

$$\frac{2a^{5/2}c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^5c^3\tan^5(e}{5f(a+a\sec(e}$$

```
[Out] 2*a^(5/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^3
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^3*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^5*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-6/7*a^6*c^3*t
an(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)-2/9*a^7*c^3*tan(f*x+e)^9/f/(a+a*sec(f*
x+e))^(9/2)
```

Rubi [A]

time = 0.13, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^7c^3\tan^9(e+fx)}{9f(a\sec(e+fx)+a)^{9/2}}-\frac{6a^6c^3\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}-\frac{2a^5c^3\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^3*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c^3*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) - (6*a^6*c^3*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2)) - (2*a^7*c^3*Tan[e + f*x]^9)/(9*f*(a + a*Sec[e + f*x])^(9/2
))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx\right)$$

$$= \frac{\left(2a^6c^3\right)\text{Subst}\left(\int \frac{x^6\left(2+ax^2\right)^2}{1+ax^2}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{f}$$

$$= \frac{\left(2a^6c^3\right)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3(1+ax^2)}\right)\right.}{f}$$

$$= -\frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{}{5}$$

$$= \frac{2a^{5/2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^3\tan(e+}{f\sqrt{a+a\sec(e}}$$

Mathematica [A]
time = 1.28, size = 134, normalized size = 0.63

$$-\frac{a^2c^3\left(-2520\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^4(e+fx)+(901+164\cos(e+fx)+1004\cos(2(e+fx))+68\cos(3(e+fx))+383\cos(4(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{1260f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

[Out] -1/1260*(a^2*c^3*(-2520*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^4 + (901 + 164*Cos[e + f*x] + 1004*Cos[2*(e + f*x)] + 68*Cos[3*(e + f*x)] + 383*Cos[4*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(188) = 376$.
time = 0.30, size = 483, normalized size = 2.28

method	result
default	$-\frac{c^3 \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}} \left(315 \sin(fx+e)(\cos^4(fx+e)) \sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}} + \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] -1/5040*c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(315*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)+1260*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)+1890*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)+1260*sin(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)+315*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*sin(f*x+e)-12256*cos(f*x+e)^5+11168*cos(f*x+e)^4+5312*cos(f*x+e)^3-4064*cos(f*x+e)^2-1280*cos(f*x+e)+1120)/cos(f*x+e)^4/sin(f*x+e)*a^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -1/630*(315*((a^2*c^3*cos(2*f*x + 2*e)^4 + a^2*c^3*sin(2*f*x + 2*e)^4 + 4*a^2*c^3*cos(2*f*x + 2*e)^3 + 6*a^2*c^3*cos(2*f*x + 2*e)^2 + 4*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3 + 2*(a^2*c^3*cos(2*f*x + 2*e)^2 + 2*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e

```
), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)) + 1) - (a^2*c^3*cos(2*f*x + 2*e)^4 + a^2*c^3*sin(2*f*x + 2*e)^4 + 4*
a^2*c^3*cos(2*f*x + 2*e)^3 + 6*a^2*c^3*cos(2*f*x + 2*e)^2 + 4*a^2*c^3*cos(2
*f*x + 2*e) + a^2*c^3 + 2*(a^2*c^3*cos(2*f*x + 2*e)^2 + 2*a^2*c^3*cos(2*f*x
 + 2*e) + a^2*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - 1) - 2*(a^2*c^3*f*cos(2*f*x + 2*e)^4 + a^2*c^3*f*sin(2*f*x + 2*e)
^4 + 4*a^2*c^3*f*cos(2*f*x + 2*e)^3 + 6*a^2*c^3*f*cos(2*f*x + 2*e)^2 + 4*a^
2*c^3*f*cos(2*f*x + 2*e) + a^2*c^3*f + 2*(a^2*c^3*f*cos(2*f*x + 2*e)^2 + 2*
a^2*c^3*f*cos(2*f*x + 2*e) + a^2*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((((co
s(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*c
os(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*si
n(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(11/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*
sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*s
in(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*
sin(2*f*x + 2*e))*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*co
s(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)
*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e
)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(11/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) +
3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x
 + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3
 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8
*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x +
 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 +
 cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2
*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
```

```
(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*
e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2
*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6
*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*
cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2
*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*c
os(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2...
```

Fricas [A]
time = 2.75, size = 474, normalized size = 2.24

$$\left[\frac{315\left(a^2c^3\cos\left(fx+e\right)^5+a^2c^3\cos\left(fx+e\right)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(383\,a^2c^3\cos\left(fx+e\right)^4+34\,a^2c^3\cos\left(fx+e\right)^3-132\,a^2c^3\cos\left(fx+e\right)^2-5\,a^2c^3\cos\left(fx+e\right)+35\,a^2c^3\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{315\left(f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^4\right)},\ -\frac{2\left(315\left(a^2c^3\cos\left(fx+e\right)^5+a^2c^3\cos\left(fx+e\right)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(383\,a^2c^3\cos\left(fx+e\right)^4+34\,a^2c^3\cos\left(fx+e\right)^3-132\,a^2c^3\cos\left(fx+e\right)^2-5\,a^2c^3\cos\left(fx+e\right)+35\,a^2c^3\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)\right)}{315\left(f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(383*
a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x +
e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(315*(
a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (383*a^
2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x + e)
^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\left(-a^2\sqrt{a\sec\left(e+fx\right)+a}\right)dx+\int a^2\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)dx+\int 2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx+\int\left(-2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^3\left(e+fx\right)\right)dx+\int\left(-a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^4\left(e+fx\right)\right)dx+\int a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**3,x)
```

[Out] -c**3*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(a**2*sqrt(a*s
ec(e + f*x) + a)*sec(e + f*x), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a
)*sec(e + f*x)**2, x) + Integral(-2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**3, x) + Integral(-a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x) +
Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**5, x))

Giac [A]
time = 1.77, size = 329, normalized size = 1.55

$$-\frac{\frac{315\sqrt{-a}\,a^3c^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{|a|}+\frac{2\left(315\sqrt{2}a^7c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(1470\sqrt{2}a^7c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(2772\sqrt{2}a^7c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+\left(257\sqrt{2}a^7c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1314\sqrt{2}a^7c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] -1/315*(315*sqrt(-a)*a^3*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqr
t(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(315*sqrt(2)*a^7*c^3*sgn(cos(
f*x + e)) - (1470*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)) - (2772*sqrt(2)*a^7*c^3
*sgn(cos(f*x + e)) + (257*sqrt(2)*a^7*c^3*sgn(cos(f*x + e))*tan(1/2*f*x + 1
/2*e)^2 - 1314*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*t
an(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan
(1/2*f*x + 1/2*e)^2 - a)^4*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3, x)

3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=177

$$\frac{2a^{5/2}c^2\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}+\frac{6a^5c^2\tan^5(e}{5f(a+a\sec(e}$$

```
[Out] 2*a^(5/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^2
*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^2*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)+6/5*a^5*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^6*c^2*t
an(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}c^2\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^6c^2\tan^7(e+fx)}{7f(a\sec(e+fx)+a)^{7/2}}+\frac{6a^5c^2\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2a^4c^2\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^3c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (2*a^3*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*c^2*Tan[e +
 f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) + (6*a^5*c^2*Tan[e + f*x]^5)/(5*f
*(a + a*Sec[e + f*x])^(5/2)) + (2*a^6*c^2*Tan[e + f*x]^7)/(7*f*(a + a*Sec[e
 + f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \sqrt{a+a\sec(e+fx)}\,\tan^4(e+fx)\,dx$$

$$= -\frac{\left(2a^5c^2\right)\text{Subst}\left(\int \frac{x^4\left(2+ax^2\right)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{f}$$

$$= -\frac{\left(2a^5c^2\right)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2(1+ax^2)}\right) d\right.}{f}$$

$$= -\frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{}{5}$$

$$= \frac{2a^{5/2}c^2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^2\tan(e+}{f\sqrt{a+a\sec(e}}$$

Mathematica [A]
time = 0.97, size = 124, normalized size = 0.70

$$-\frac{2a^2c^2\left(-105\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^3(e+fx)+(8+51\cos(e+fx)+23\cos(2(e+fx))+23\cos(3(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{105f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*c^2*(-105*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (8 + 51*
Cos[e + f*x] + 23*Cos[2*(e + f*x)] + 23*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e +
 f*x]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*
Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(157) = 314$.
time = 0.21, size = 323, normalized size = 1.82

method	result
default	$-\frac{c^2\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(105\sin(fx+e)\left(\cos^3(fx+e)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}\sqrt{2}+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/420*c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(105*sin(f*x+e)*cos(f*x+e)
^3*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2
^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*2^(1/2)+210*sin(f*x+e)*arctanh
(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)^2*2^(1/2)+105*sin(f*x+e)*arct
anh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)*2^(1/2)-736*cos(f*x+e)^4+3
68*cos(f*x+e)^3+512*cos(f*x+e)^2-24*cos(f*x+e)-120)/cos(f*x+e)^3/sin(f*x+e)
*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a
^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a^2
*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x + 2*e)^2 + 2
*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
```

n(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(
9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*c
os(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin
(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*
x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6
e) + 4(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^
2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*co
s(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*co
s(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*
x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e
) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f
*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)), x) + 8*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*

```
f*sin(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate(
(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*
e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*
e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e
)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*c
os(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*...
```

Fricas [A]
time = 2.86, size = 440, normalized size = 2.49

$$\left[\frac{105\,\left(a^2c^2\cos\left(fx+e\right)^4+a^2c^2\cos\left(fx+e\right)^3\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\,\left(92\,a^2c^2\cos\left(fx+e\right)^3+46\,a^2c^2\cos\left(fx+e\right)^2-18\,a^2c^2\cos\left(fx+e\right)-15\,a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{105\,\left(f\cos\left(fx+e\right)^4+f\cos\left(fx+e\right)^3\right)},\,-\frac{2\,\left(105\,\left(a^2c^2\cos\left(fx+e\right)^4+a^2c^2\cos\left(fx+e\right)^3\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)}{\sqrt{a}\,\sin(fx+e)}\right)+\left(92\,a^2c^2\cos\left(fx+e\right)^3+46\,a^2c^2\cos\left(fx+e\right)^2-18\,a^2c^2\cos\left(fx+e\right)-15\,a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{105\,\left(f\cos\left(fx+e\right)^4+f\cos\left(fx+e\right)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(92*a
^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e)
 - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*
c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (92*a^2*c^2*cos(f*x + e)^3 + 46*a^2*
c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int a^2\sqrt{a\sec\left(e+fx\right)+a}\,dx+\int\left(-2a^2\sqrt{a\sec\left(e+fx\right)+a}\,\sec^2\left(e+fx\right)\right)dx+\int a^2\sqrt{a\sec\left(e+fx\right)+a}\,\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-2*a**2*sqrt(a*
sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x)**4, x))
```

Giac [A]
time = 1.52, size = 297, normalized size = 1.68

$$-\frac{\frac{105\,\sqrt{-a}\,a^3c^2\log\left(\frac{\left|2\,\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\,\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}-\frac{2\,\left(105\,\sqrt{2}\,a^6c^2\mathrm{sgn}(\cos(fx+e))-\left(385\,\sqrt{2}\,a^6c^2\mathrm{sgn}(\cos(fx+e))+\left(43\,\sqrt{2}\,a^6c^2\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-203\,\sqrt{2}\,a^6c^2\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*a^3*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqr
t(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(105*sqrt(2)*a^6*c^2*sgn(cos(
f*x + e)) - (385*sqrt(2)*a^6*c^2*sgn(cos(f*x + e)) + (43*sqrt(2)*a^6*c^2*sg
n(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 203*sqrt(2)*a^6*c^2*sgn(cos(f*x +
e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/(
(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2, x)
```

3.59 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$\frac{2a^{5/2}c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}-\frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c*tan
(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a^4*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(3
/2)-2/5*a^5*c*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{2a^5c\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}-\frac{2a^4c\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}-\frac{2a^3c\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (
2*a^3*c*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*c*Tan[e + f*x]^
3)/(f*(a + a*Sec[e + f*x])^(3/2)) - (2*a^5*c*Tan[e + f*x]^5)/(5*f*(a + a*Se
c[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = -\left((ac)\int (a+a\sec(e+fx))^{3/2}\tan^2(e+fx)\,dx\right)$$
$$= \frac{(2a^4c)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{(2a^4c)\,\text{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{}{\sqrt{a-}}\right.}{f}$$
$$= -\frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}-\frac{2}{5f(}$$
$$= \frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c\tan(e+f}{f\sqrt{a+a\sec(e+}}$$

Mathematica [A]
time = 0.84, size = 110, normalized size = 0.83

$$-\frac{a^2c\left(-10\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^2(e+fx)+(3+6\cos(e+fx)+\cos(2(e+fx)))\sqrt{-1+\sec(e+fx)}\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{5f\sqrt{-1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
[Out] -1/5*(a^2*c*(-10*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (3 + 6*Co
s[e + f*x] + Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2*Sqrt
[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(118) = 236$.
time = 0.18, size = 303, normalized size = 2.30

method	result
default	$-\frac{c\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(5\sin(fx+e)\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}(\cos^2(fx+e))\sqrt{2}+10\,\mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(5*sin(f*x+e)*arctanh(1/2*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)^2*2^(1/2)+10*sin(f*x+e)*arctanh(1/2*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)*2^(1/2)+5*2^(1/2)*arctanh(1/2*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(5/2)*sin(f*x+e)-8*cos(f*x+e)^3-16*cos(f*x+e)^2+16*cos(f
*x+e)+8)/sin(f*x+e)/cos(f*x+e)^2*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(126) = 252$.
time = 0.61, size = 1501, normalized size = 11.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3*a^2*cos
(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a) + 3*((a
^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x + 2*e) + a
^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x +
```

```
2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(
2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e
)^2 + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 1) + (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*
f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*sqrt(a)
)*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 3.35, size = 382, normalized size = 2.89

$$\left[\frac{5\left(a^2c\cos\left(fx+e\right)^3+a^2c\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(a^2c\cos\left(fx+e\right)^2+3\,a^2c\cos\left(fx+e\right)+a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{5\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}, -\frac{2\left(5\left(a^2c\cos\left(fx+e\right)^3+a^2c\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(a^2c\cos\left(fx+e\right)^2+3\,a^2c\cos\left(fx+e\right)+a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)\right)}{5\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(a^2*c*cos(f*
x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/5*(5*(a^2*c*c
os(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c*cos(f*x + e
)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-a^2\sqrt{a\sec(e+fx)+a}\right)dx+\int\left(-a^2\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)\right)dx+\int a^2\sqrt{a\sec(e+fx)+a}\,\sec^2(e+fx)\,dx+\int a^2\sqrt{a\sec(e+fx)+a}\,\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-a**2*sqrt(a*sec
(e + f*x) + a)*sec(e + f*x), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3,
 x))
```

Giac [A]
time = 1.43, size = 227, normalized size = 1.72

$$-\frac{\frac{5\sqrt{-a}\,a^3c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(\sqrt{2}\,a^5c\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-5\sqrt{2}\,a^5c\operatorname{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/5*(5*sqrt(-a)*a^3*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(sqrt(2)*a^5*c*sgn(cos(f*x + e))*ta
n(1/2*f*x + 1/2*e)^4 - 5*sqrt(2)*a^5*c*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2
*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))
/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)), x)
```

3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=103

$$\frac{2a^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+8*a^2*cot(f
*x+e)*(a+a*sec(f*x+e))^(1/2)/c/f-2*a^3*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}}+\frac{8a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (8*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c*f) - (2*a^3*Tan[e + f*x])
/(c*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx &= -\frac{\int \cot^2(e+fx)(a+a\sec(e+fx))^{7/2}\,dx}{ac}\\
&= \frac{(2a^2)\,\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{(2a^2)\,\text{Subst}\left(\int \left(a+\frac{4}{x^2}-\frac{a}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}\\
&= \frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}-\frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}-\frac{(2a^3)\,\text{Subst}}{}\\
&= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}
\end{aligned}$$

Mathematica [A]
time = 0.74, size = 96, normalized size = 0.93

$$\frac{2a^3\left(-\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)(-1+\cos(e+fx))+(-1+5\cos(e+fx))\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)\tan(e+fx)}{cf(-1+\sec(e+fx))^{3/2}\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^3*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + (-1 + 5*Co
s[e + f*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 +
Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.18, size = 120, normalized size = 1.17

method	result
default	$\frac{\left(-\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2 \cos(fx+e)}{\cos(fx+e)+1}}\, \sin(fx+e) \sqrt{2}}{2 \cos(fx+e)}\right) \sqrt{-\frac{2 \cos(fx+e)}{\cos(fx+e)+1}}\, \sin(fx+e)+10 \cos(fx+e)-2\right) \sqrt{\frac{a(\cos(fx+e)+1}{\cos(fx+e)}}}{cf \sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/f*(-2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+10*cos
(f*x+e)-2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^(5/2)/(c*sec(f*x + e) - c), x)
```

Fricas [A]
time = 3.15, size = 315, normalized size = 3.06

$$\left[\frac{\sqrt{-a}\, a^2 \log\left(-\frac{8 a \cos(fx+e)^3-4\left(2 \cos(fx+e)^2-\cos(fx+e)\right) \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}\, \sin(fx+e)-7 a \cos(fx+e)+a}{\cos(fx+e)+1}\right) \sin(fx+e)+4\left(5 a^2 \cos(fx+e)-a^2\right) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{2 c f \sin(fx+e)}, \frac{a^{\frac{5}{2}} \arctan\left(\frac{2 \sqrt{a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}\, \cos(fx+e) \sin(fx+e)}{2 a \cos(fx+e)^2+a \cos(fx+e)-a}\right) \sin(fx+e)+2\left(5 a^2 \cos(fx+e)-a^2\right) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{c f \sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x
 + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*
cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a^2*cos(f*x + e)
- a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e)), (a^(5/2
)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(5*a^
2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x
 + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx+\int \frac{2a^2\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{a^2\sqrt{a\sec(e+fx)+a}\,\sec^2(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(
2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integ
ral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x) - 1), x))/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(93) = 186.
time = 1.25, size = 268, normalized size = 2.60

$$\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,a^3\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)c}-\frac{\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{c|a|}-\frac{8\sqrt{2}\sqrt{-a}\,a^3\operatorname{sgn}(\cos(fx+e))}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a^3*sgn(cos(f*x + e))*tan(1/
2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*c) - sqrt(-a)*a^3*log(abs(2*
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4
*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/(c*a
bs(a)) - 8*sqrt(2)*sqrt(-a)*a^3*sgn(cos(f*x + e))/(((sqrt(-a)*tan(1/2*f*x +
 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)), x)
```

3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}-\frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-8/3*a*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^2f}-\frac{8a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f)
 - (8*a*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

```
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{9/2}\,dx}{a^2c^2}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= -\frac{(2a)\text{Subst}\left(\int \left(\frac{4}{x^4}+\frac{a^2}{1+ax^2}\right)\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f}$$

$$= -\frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\quad}{\sqrt{a+}}\right.}{c^2f}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} - \frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

Mathematica [**A**]
time = 4.36, size = 102, normalized size = 1.38

$$-\frac{\cos^{\frac{5}{2}}(e+fx)\left(-3\text{ArcSin}\left(\sqrt{1-\cos(e+fx)}\right)(1-\cos(e+fx))^{3/2}+4\cos^{\frac{3}{2}}(e+fx)\right)\csc^3\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}}{24c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/24*(Cos[e + f*x]^(5/2)*(-3*ArcSin[Sqrt[1 - Cos[e + f*x]]]*(1 - Cos[e + f
*x])^(3/2) + 4*Cos[e + f*x]^(3/2))*Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]^5*(a
*(1 + Sec[e + f*x]))^(5/2))/(c^2*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(64) = 128$.
time = 0.20, size = 351, normalized size = 4.74

method	result
default	$-\frac{\left(3\left(\cos^3(fx+e)\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}-3\left(\cos^2(fx+e)\right)\sqrt{2}\ \sqrt{-\frac{2\cos}{\cos(f}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/f*(3*cos(f*x+e)^3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2
)-3*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))-3*cos(f*
x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+8*cos(f*x+e)^2*sin(f
*x+e)+3*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2)))*(a*(cos(f*x+e)+1
)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))^2/(cos(f*x+e)+1)*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(68) = 136$.
time = 3.54, size = 367, normalized size = 4.96

$$\left[\frac{16\,a^{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+3\left(a^{2}\cos\left(fx+e\right)-a^{2}\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^{3}-4\left(2\cos(fx+e)^{2}-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin\left(fx+e\right)}{6\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)},\ \frac{8\,a^{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+3\left(a^{2}\cos\left(fx+e\right)-a^{2}\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{2\,a\cos(fx+e)^{2}+a\cos(fx+e)-a}\right)\sin\left(fx+e\right)}{3\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(16*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^
2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)
^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
 + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e))/((c^2*f*cos
```

```
(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(8*a^2*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan(2*sqrt
(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*
cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e))/((c^2*f*cos(f*x + e) -
c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx+\int \frac{2a^2\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx+\int \frac{a^2\sqrt{a\sec(e+fx)+a}\,\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] (Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) +
 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*
x)**2 - 2*sec(e + f*x) + 1), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(64) = 128.
time = 1.45, size = 243, normalized size = 3.28

$$-\frac{\sqrt{2}\sqrt{-a}\,a^5\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a^2c^2|a|}+\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^4+a^2\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)^3a^2c^2}\right)\mathrm{sgn}(\cos(fx+e))}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*sqrt(-a)*a^5*(3*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/(a^2*c^2*abs(a)) + 8*(3*(sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4 + a^2)/(((sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3*a^2*c^2))*sgn(co
s(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2, x)
```

3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{2a^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}+\frac{8\cot^5(e+fx)(a+a\sec(e+f$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f+8/5*cot(f
*x+e)^5*(a+a*sec(f*x+e))^(5/2)/c^3/f+2*a^2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2
)/c^3/f
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^3f}+\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^3f}+\frac{8\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) + (8*Cot[e + f*x]^
5*(a + a*Sec[e + f*x])^(5/2))/(5*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx &= -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{11/2}\,dx}{a^3c^3}\\
&= \frac{2\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^6(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{4}{x^6}+\frac{a^2}{x^2}-\frac{a^3}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}\\
&= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}+\frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3f}\\
&= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.47, size = 196, normalized size = 1.88

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}\left(30\text{ArcSin}\left(\sqrt{1-\cos(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{1-\cos(e+fx)}(-1+7\cos(e+fx))+4(-29+20\cos(e+fx)-15\cos(2(e+fx)))\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)+5\sqrt{\cos(e+fx)}(11\cos(e+fx)+3\cos(2(e+fx)))\sin^2(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{60c^3f(-1+\cos(e+fx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*(30*ArcSin[Sqrt[1 - Cos[
e + f*x]]]*Cos[(e + f*x)/2]^2*Sqrt[1 - Cos[e + f*x]]*(-1 + 7*Cos[e + f*x])
+ 4*(-29 + 20*Cos[e + f*x] - 15*Cos[2*(e + f*x)])*Hypergeometric2F1[-5/2, -
```

```
1/2, 1/2, 2*Sin[(e + f*x)/2]^2] + 5*Sqrt[Cos[e + f*x]]*(11*Cos[e + f*x] + 3
*Cos[2*(e + f*x)])*Sin[e + f*x]^2)*Tan[(e + f*x)/2])/(60*c^3*f*(-1 + Cos[e
+ f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(92) = 184$.
time = 0.24, size = 306, normalized size = 2.94

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-5\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}+10 \ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-5*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos
(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+10*sin(f*x+e)*cos(f*x+e)*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-5*2^(1/2)*arctanh(1/2*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+18*cos(f*x+e)^3-20*cos(f*x+e)^2+10*cos
(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^2*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(98) = 196$.
time = 3.51, size = 477, normalized size = 4.59

$$\left[\frac{5\left(a^2\cos(fx+e)^2-2a^2\cos(fx+e)+a^2\right)\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\left(9a^2\cos(fx+e)^3-10a^2\cos(fx+e)^2+5a^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{10\left(c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}, \frac{5\left(a^2\cos(fx+e)^2-2a^2\cos(fx+e)+a^2\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(9a^2\cos(fx+e)^3-10a^2\cos(fx+e)^2+5a^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{5\left(c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a
*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x +
 e) + 1))*sin(f*x + e) + 4*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 +
5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/5*(5*(a^2*cos(f*x
 + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a
*cos(f*x + e) - a))*sin(f*x + e) + 2*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x
 + e)^2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

<u>**Sympy**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{2a^2\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{a^2\sqrt{a\sec(e+fx)+a}\,\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec
(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) +
Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x)**3 - 3
*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

<u>**Giac**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(92) = 184.
time = 1.41, size = 428, normalized size = 4.12

$$-\frac{\frac{5\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{c^3|a|}+\frac{4\sqrt{2}\left(5\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^8\sqrt{-a}\,a^3\mathrm{sgn}(\cos(fx+e))-10\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^6\sqrt{-a}\,a^4\mathrm{sgn}(\cos(fx+e))+20\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^4\sqrt{-a}\,a^5\mathrm{sgn}(\cos(fx+e))-10\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{-a}\,a^6\mathrm{sgn}(\cos(fx+e))+3\sqrt{-a}\,a^7\mathrm{sgn}(\cos(fx+e))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)^5c^3}}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/5*(5*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) + 4*sqrt(2)*(5*(sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(f*
x + e)) - 10*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a))^6*sqrt(-a)*a^4*sgn(cos(f*x + e)) + 20*(sqrt(-a)*tan(1/2*f*x + 1/2*e
) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(f*x + e)) -
```

```
10*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
*sqrt(-a)*a^6*sgn(cos(f*x + e)) + 3*sqrt(-a)*a^7*sgn(cos(f*x + e)))/(((sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^5*c
^3))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3, x)
```

3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=140

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f}-\frac{2a\cot^3(e+fx)(a+a\sec(e-}{3c^4f}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f-2/3*a*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^4/f-8/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7
/2)/a/c^4/f+2*a^2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f
```

Rubi [A]
time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^4f}+\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^4f}-\frac{8\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}}{7ac^4f}-\frac{2a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^4*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^4*f) - (8*Cot[e + f*x]^7*(a + a*Sec[e
+ f*x])^(7/2))/(7*a*c^4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
```

```
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \cot^8(e+fx)(a+a\sec(e+fx))^{13/2}\,dx}{a^4c^4}$$

$$= -\frac{2\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^8(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= -\frac{2\text{Subst}\left(\int \left(\frac{4}{x^8}+\frac{a^2}{x^4}-\frac{a^3}{x^2}+\frac{a^4}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{ac^4f}$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.04, size = 361, normalized size = 2.58

$$-\frac{\csc^7\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(a(1+\sec(e+fx)))^{5/2}\sin^8\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\left(336\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\left(3-8\sin^2\left(\frac{1}{2}(e+fx)\right)+5\sin^4\left(\frac{1}{2}(e+fx)\right)\right)+4\,{}_2F_1\left(-\frac{7}{2},-\frac{3}{2};-\frac{1}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\left(15-42\sin^2\left(\frac{1}{2}(e+fx)\right)+35\sin^4\left(\frac{1}{2}(e+fx)\right)\right)-105\cos^4\left(\frac{1}{2}(e+fx)\right)\left(3\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sqrt{\sin^2\left(\frac{1}{2}(e+fx)\right)}\right)\sin^2\left(\frac{1}{2}(e+fx)\right)^{3/2}+2\sin^4\left(\frac{1}{2}(e+fx)\right)\left(5-4\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\right)\right)}{210f(c-c\sec(e+fx))^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/210*(Csc[(e + f*x)/2]^7*Sec[(e + f*x)/2]^5*Sec[e + f*x]^(3/2)*(a*(1 + Se
c[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2]^8*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1
)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(336*Hypergeometric2F1[-5/2, -1/2, 1/2, 2
```

```
*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2*(3 - 8*Sin[(e + f*x)/2]^2 + 5*Sin[(
e + f*x)/2]^4) + 4*Hypergeometric2F1[-7/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2
]*(15 - 42*Sin[(e + f*x)/2]^2 + 35*Sin[(e + f*x)/2]^4) - 105*Cos[(e + f*x)/
2]^4*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]]*(Sin[(e + f*x)/2]^
2)^(3/2) + 2*Sin[(e + f*x)/2]^4*(5 - 4*Sin[(e + f*x)/2]^2)*Sqrt[1 - 2*Sin[(
e + f*x)/2]^2])))/(f*(c - c*Sec[e + f*x])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(124) = 248$.
time = 0.25, size = 395, normalized size = 2.82

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(21\left(\cos^3(fx+e)\right)\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(21*cos(f*x+e)^3*sin(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-63*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+63*sin(f*x+e)*cos(f*x+
e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-21*2^(1/2)*arctanh(1/2
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-80*cos(f*x+e)^4+154*cos(f*x+e)^3-
140*cos(f*x+e)^2+42*cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^3*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(132) = 264$.
time = 3.19, size = 569, normalized size = 4.06

$$\left[\frac{21\left(a^2\cos(fx+e)^3-3a^2\cos(fx+e)^2+3a^2\cos(fx+e)-a^2\right)\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\left(40a^2\cos(fx+e)^4-77a^2\cos(fx+e)^3+70a^2\cos(fx+e)^2-21a^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{42\left(c^4f\cos(fx+e)^3-3c^4f\cos(fx+e)^2+3c^4f\cos(fx+e)-c^4f\right)\sin(fx+e)}, \frac{21\left(a^2\cos(fx+e)^3-3a^2\cos(fx+e)^2+3a^2\cos(fx+e)-a^2\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(40a^2\cos(fx+e)^4-77a^2\cos(fx+e)^3+70a^2\cos(fx+e)^2-21a^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{21\left(c^4f\cos(fx+e)^3-3c^4f\cos(fx+e)^2+3c^4f\cos(fx+e)-c^4f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/42*(21*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) -
 a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(
f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(40*a^2*cos(f*x + e)^4 -
 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*
x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/21*(21*(a^2*cos(f
*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan
(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(40*a^2*cos(f
*x + e)^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x
+ e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c
^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(124) = 248.
time = 1.60, size = 556, normalized size = 3.97

$$-\frac{\frac{21\sqrt{-a}\,a^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{c^{4}|a|}+\frac{4\left(21\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{12}\sqrt{-a}\,a^{3}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-84\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{10}\sqrt{-a}\,a^{4}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+217\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{8}\sqrt{-a}\,a^{5}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-238\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{6}\sqrt{-a}\,a^{6}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+189\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}\sqrt{-a}\,a^{7}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-70\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{8}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+13\sqrt{2}\sqrt{-a}\,a^{9}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-a\right)^{7}c^{4}}}{21\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/21*(21*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/(c^4*abs(a)) + 4*(21*sqrt(2)*(sqrt(-a)*tan(1/2
*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)*a^3*sgn(co
s(f*x + e)) - 84*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f
*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^4*sgn(cos(f*x + e)) + 217*sqrt(2)*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a
^5*sgn(cos(f*x + e)) - 238*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
```

```
*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^6*sgn(cos(f*x + e)) + 189*sqrt(2
)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*s
qrt(-a)*a^7*sgn(cos(f*x + e)) - 70*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^8*sgn(cos(f*x + e)) + 13
*sqrt(2)*sqrt(-a)*a^9*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^7*c^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4, x)
```

3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=172

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f}+\frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f}-\frac{2a\cot^3(e+fx)(a+a\sec(e+}{3c^5f}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^5/f-2/3*a*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^5/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5
/2)/c^5/f+8/9*cot(f*x+e)^9*(a+a*sec(f*x+e))^(9/2)/a^2/c^5/f+2*a^2*cot(f*x+e
)*(a+a*sec(f*x+e))^(1/2)/c^5/f
```

Rubi [A]

time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3989, 3972, 472, 209}

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{c^5f}+\frac{8\cot^9(e+fx)(a\sec(e+fx)+a)^{9/2}}{9a^2c^5f}+\frac{2a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{c^5f}+\frac{2\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5c^5f}-\frac{2a\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^5*f)
 + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^5*f) - (2*a*Cot[e + f*x
]^3*(a + a*Sec[e + f*x])^(3/2))/(3*c^5*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e
+ f*x])^(5/2))/(5*c^5*f) + (8*Cot[e + f*x]^9*(a + a*Sec[e + f*x])^(9/2))/(9
*a^2*c^5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = -\frac{\int \cot^{10}(e+fx)(a+a\sec(e+fx))^{15/2}\,dx}{a^5c^5}$$

$$= \frac{2\text{Subst}\left(\int \frac{(2+ax^2)^2}{x^{10}(1+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^5f}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{4}{x^{10}}+\frac{a^2}{x^6}-\frac{a^3}{x^4}+\frac{a^4}{x^2}-\frac{a^5}{1+ax^2}\right)dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^5f}$$

$$= \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/}}{3c^5f}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.60, size = 205, normalized size = 1.19

$$\frac{a^2\sqrt{a(1+\sec(e+fx))}\left((109+108\cos(e+fx)+63\cos(2(e+fx)))\,{}_2F_1\left(-\frac{9}{2},-\frac{5}{2};-\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)-15(2+\cos(e+fx)-2\cos(2(e+fx))-\cos(3(e+fx)))\,{}_3F_2\left(-\frac{7}{2},-\frac{3}{2},2;-\frac{1}{2},1;2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)+240(1+2\cos(e+fx))\,{}_2F_1\left(-\frac{7}{2},-\frac{3}{2};-\frac{1}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sin^2(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{315c^5f\cos^{\frac{9}{2}}(e+fx)(-1+\sec(e+fx))^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sec[e + f*x])]*((109 + 108*Cos[e + f*x] + 63*Cos[2*(e + f*
x)])*Hypergeometric2F1[-9/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2] - 15*(2 + Co
```

```
s[e + f*x] - 2*Cos[2*(e + f*x)] - Cos[3*(e + f*x)])*HypergeometricPFQ[{-7/2
, -3/2, 2}, {-1/2, 1}, 2*Sin[(e + f*x)/2]^2] + 240*(1 + 2*Cos[e + f*x])*Hyp
ergeometric2F1[-7/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2]*Sin[e + f*x]^2)*Tan[
(e + f*x)/2])/(315*c^5*f*Cos[e + f*x]^(9/2)*(-1 + Sec[e + f*x])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(152) = 304$.
time = 0.30, size = 492, normalized size = 2.86

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos(fx+e)+1)\left(-45\left(\cos^4(fx+e)\right)\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e}{2\cos(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/45/c^5/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)*(-45*cos(f*x
+e)^4*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+180*cos(
f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-270*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+17
8*cos(f*x+e)^5+180*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2
^(1/2))*2^(1/2)-486*cos(f*x+e)^4-45*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+648*cos(f*x+e)^3-390*cos(f*x+e)^2+90*cos(f*x+e))/sin(
f*x+e)^3/(-1+cos(f*x+e))^3*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.35, size = 649, normalized size = 3.77

$$\left[\frac{45\left(a^2\cos(fx+e)^4-4a^2\cos(fx+e)^3+6a^2\cos(fx+e)^2-4a^2\cos(fx+e)+a^2\right)\sqrt{-a}\,\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\left(89a^2\cos(fx+e)^5-243a^2\cos(fx+e)^4+324a^2\cos(fx+e)^3-195a^2\cos(fx+e)^2+45a^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{90\left(c^5f\cos(fx+e)^4-4c^5f\cos(fx+e)^3+6c^5f\cos(fx+e)^2-4c^5f\cos(fx+e)+c^5f\right)\sin(fx+e)}, \frac{45\left(a^2\cos(fx+e)^4-4a^2\cos(fx+e)^3+6a^2\cos(fx+e)^2-4a^2\cos(fx+e)+a^2\right)\sqrt{a}\,\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(89a^2\cos(fx+e)^5-243a^2\cos(fx+e)^4+324a^2\cos(fx+e)^3-195a^2\cos(fx+e)^2+45a^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{45\left(c^5f\cos(fx+e)^4-4c^5f\cos(fx+e)^3+6c^5f\cos(fx+e)^2-4c^5f\cos(fx+e)+c^5f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/90*(45*(a^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2
 - 4*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f
*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(
89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x + e)^3 - 1
95*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*
x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e)), 1/45*(45*(a^2*cos(f
*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2 - 4*a^2*cos(f*x + e
) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
 + e) + 2*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x
 + e)^3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(152) = 304.
time = 1.84, size = 672, normalized size = 3.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -1/45*(45*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/(c^5*abs(a)) + 4*(45*sqrt(2)*(sqrt(-a)*tan(1/2
*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^16*sqrt(-a)*a^3*sgn(co
s(f*x + e)) - 270*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*
f*x + 1/2*e)^2 + a))^14*sqrt(-a)*a^4*sgn(cos(f*x + e)) + 900*sqrt(2)*(sqrt(
```

```
-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)
*a^5*sgn(cos(f*x + e)) - 1575*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^6*sgn(cos(f*x + e)) + 1953*s
qrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)
)^8*sqrt(-a)*a^7*sgn(cos(f*x + e)) - 1452*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^8*sgn(cos(f*x + e
)) + 738*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^4*sqrt(-a)*a^9*sgn(cos(f*x + e)) - 207*sqrt(2)*(sqrt(-a)*tan(1/
2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^10*sgn(c
os(f*x + e)) + 28*sqrt(2)*sqrt(-a)*a^11*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(
1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^9*c^5))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5, x)
```

3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=185

$$\frac{2c^4\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{16\sqrt{2}\,c^4\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-16*c^4*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/
2)+14*c^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a*c^4*tan(f*x+e)^3/f/(a+a*s
ec(f*x+e))^(3/2)+2/5*a^2*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A]

time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 490, 596, 536, 209}

$$\frac{2a^2c^4\tan^5(e+fx)}{5f(a\sec(e+fx)+a)^{5/2}}+\frac{2c^4\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{16\sqrt{2}\,c^4\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2ac^4\tan^3(e+fx)}{f(a\sec(e+fx)+a)^{3/2}}+\frac{14c^4\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (16*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(Sqrt[a]*f) + (14*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
 - (2*a*c^4*Tan[e + f*x]^3)/(f*(a + a*Sec[e + f*x])^(3/2)) + (2*a^2*c^4*Tan
[e + f*x]^5)/(5*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{\left(2a^4c^4\right)\mathrm{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{\left(2a^2c^4\right)\mathrm{Subst}\left(\int \frac{x^4(10+15ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e}{\sqrt{a+a\sec}}\right.}{5f}$$

$$= -\frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{\left(2c^4\right)\mathrm{Subst}\left(\int \frac{x^2(90a}{(1+ax}\right.}{}$$

$$= \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{16\sqrt{2}\,c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+f}}\right.}{\sqrt{a}\,f}$$

Mathematica [A]
time = 1.59, size = 153, normalized size = 0.83

$$\frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left(100-155\cos(e+fx)+96\cos(2(e+fx))-41\cos(3(e+fx))+20\mathrm{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^3(e+fx)\sqrt{-1+\sec(e+fx)}-160\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos^3(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec^3(e+fx)}{10f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*Cot[(e + f*x)/2]*(100 - 155*Cos[e + f*x] + 96*Cos[2*(e + f*x)] - 41*Co
s[3*(e + f*x)] + 20*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3*Sqrt[-1
+ Sec[e + f*x]] - 160*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e
 + f*x]^3*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3)/(10*f*Sqrt[a*(1 + Sec[e
+ f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(162) = 324$.
time = 0.26, size = 544, normalized size = 2.94

method	result
default	$-\frac{c^4\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(5\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}(\cos^2(fx+e))\sqrt{2}+10}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(5*sin(f*x+e)*arctanh(1/2*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)^2*2^(1/2)+10*sin(f*x+e)*arctanh(1/2
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)*2^(1/2)+80*ln((sin(f*x+e)*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(5/2)*cos(f*x+e)^2*sin(f*x+e)+5*2^(1/2)*arctanh(1/2*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(5/2)*sin(f*x+e)+160*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*c
os(f*x+e)*sin(f*x+e)+80*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*sin(f*x+e)+
328*cos(f*x+e)^3-384*cos(f*x+e)^2+64*cos(f*x+e)-8)/cos(f*x+e)^2/sin(f*x+e)/
a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 4.98, size = 597, normalized size = 3.23

$$\left[\frac{40\sqrt{2}\left(ac^4\cos(fx+e)^3+ac^4\cos(fx+e)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-5\left(c^4\cos(fx+e)^3+c^4\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(41c^4\cos(fx+e)^2-7c^4\cos(fx+e)+c^4\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{5\left(af\cos(fx+e)^3+af\cos(fx+e)^2\right)}, -\frac{2\left(5\left(c^4\cos(fx+e)^3+c^4\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(41c^4\cos(fx+e)^2-7c^4\cos(fx+e)+c^4\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-\frac{40\sqrt{2}\left(ac^4\cos(fx+e)^3+ac^4\cos(fx+e)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}\right)}{5\left(af\cos(fx+e)^3+af\cos(fx+e)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5*(40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e)^2)*sqrt(-1/a)*l
og((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x +
e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 +
2*cos(f*x + e) + 1)) - 5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(-a)
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*
(41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2), -2
/5*(5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (41*c^4*
cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) - 40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e
)^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx+\int\frac{6\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx+\int\left(-\frac{4\sec^3(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx+\int\frac{\sec^4(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx+\int\frac{1}{\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(6*se
c(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(-4*sec(e + f*x)**3/sq
rt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**4/sqrt(a*sec(e + f*x) +
 a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2), x)
```

3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=152

$$\frac{2c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{8\sqrt{2}\,c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\cdot$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-8*c^3*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2
)+6*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a*c^3*tan(f*x+e)^3/f/(a+a*s
ec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 490, 596, 536, 209}

$$\frac{2c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{8\sqrt{2}\,c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2ac^3\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}+\frac{6c^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (8*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (6*c^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) -
 (2*a*c^3*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx\right)$$

$$= \frac{\left(2a^3c^3\right)\operatorname{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{\left(2ac^3\right)\operatorname{Subst}\left(\int \frac{x^2(6+9ax^2)}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(}{\sqrt{a+a\,\mathrm{s}}}\right.}{3f}$$

$$= \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{\left(2c^3\right)\operatorname{Subst}\left(\int \frac{18a+21a}{(1+ax^2)(2-}\right.}{}$$

$$= \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{\left(2c^3\right)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx\right.}{}$$

$$= \frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{8\sqrt{2}\,c^3\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx}}\right.}{\sqrt{a}\,f}$$

Mathematica [**A**]
time = 1.37, size = 166, normalized size = 1.09

$$\frac{4c^3\cos\left(\frac{e}{2}\right)\cos(e)\cot\left(\frac{1}{2}(e+fx)\right)\left(-6+11\cos(e+fx)-5\cos(2(e+fx))+3\mathrm{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^2(e+fx)\sqrt{-1+\sec(e+fx)}-12\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos^2(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec^2(e+fx)}{3f\left(\cos\left(\frac{e}{2}\right)+\cos\left(\frac{3e}{2}\right)\right)\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (4*c^3*Cos[e/2]*Cos[e]*Cot[(e + f*x)/2]*(-6 + 11*Cos[e + f*x] - 5*Cos[2*(e
+ f*x)] + 3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e
+ f*x]] - 12*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2
*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(3*f*(Cos[e/2] + Cos[(3*e)/2])*Sq
rt[a*(1 + Sec[e + f*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(131) = 262$.
time = 0.23, size = 372, normalized size = 2.45

method	result

default	$\frac{c^3\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(3\cos(fx+e)\sin(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}+3\sqrt{2}\ a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(3*cos(f*x+e)*sin(f*x+e)*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+3*2^(1/2)*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(3/2)*sin(f*x+e)+24*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(3/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)
/sin(f*x+e))*cos(f*x+e)+24*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*sin(f*x+
e)-40*cos(f*x+e)^2+44*cos(f*x+e)-4)/sin(f*x+e)/cos(f*x+e)/a
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

<u>Fricas</u> [A]
time = 3.66, size = 561, normalized size = 3.69

$$\left[\frac{12\sqrt{2}\left(ac^3\cos(fx+e)^2+ac^3\cos(fx+e)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-3\left(c^3\cos(fx+e)^2+c^3\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(10c^3\cos(fx+e)-c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{3\left(af\cos(fx+e)^2+af\cos(fx+e)\right)},-\frac{2\left(3\left(c^3\cos(fx+e)^2+c^3\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(10c^3\cos(fx+e)-c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-\frac{12\sqrt{2}\left(ac^3\cos(fx+e)^2+ac^3\cos(fx+e)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}\right)}{3\left(af\cos(fx+e)^2+af\cos(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(12*sqrt(2)*(a*c^3*cos(f*x + e)^2 + a*c^3*cos(f*x + e))*sqrt(-1/a)*log
((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)
*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - 3*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
```

```
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(10*
c^3*cos(f*x + e) - c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -2/3*(3*(c^3*cos(f*x + e)^2 + c
^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)/(sqrt(a)*sin(f*x + e))) - (10*c^3*cos(f*x + e) - c^3)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 12*sqrt(2)*(a*c^3*cos(f*x + e)^
2 + a*c^3*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2 + a*f
*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int \frac{3\sec{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\,dx+\int\left(-\frac{3\sec^2{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\right)dx+\int\frac{\sec^3{(e+fx)}}{\sqrt{a\sec{(e+fx)}+a}}\,dx+\int\left(-\frac{1}{\sqrt{a\sec{(e+fx)}+a}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-3*s
ec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**3/sqrt
(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^3}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=119

$$\frac{2c^2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{4\sqrt{2}\,c^2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-4*c^2*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2
)+2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3989, 3972, 490, 536, 209}

$$\frac{2c^2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{4\sqrt{2}\,c^2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}+\frac{2c^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
 - (4*Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f) + (2*c^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx &= \left(a^2c^2\right)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx \\
&= -\frac{\left(2a^2c^2\right)\operatorname{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} \\
&= \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{\left(2c^2\right)\operatorname{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} \\
&= \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{\left(2c^2\right)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} \\
&= \frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\sqrt{2}c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}
$$

Mathematica [A]

time = 0.48, size = 124, normalized size = 1.04

$$\frac{2c^2\cot\left(\frac{1}{2}(e+fx)\right)\left(1-\cos(e+fx)+\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos(e+fx)\sqrt{-1+\sec(e+fx)}-2\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)}{f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c^2*Cot[(e + f*x)/2]*(1 - Cos[e + f*x] + ArcTan[Sqrt[-1 + Sec[e + f*x]]]
*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e +
f*x]]/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(102) = 204$.
time = 0.23, size = 329, normalized size = 2.76

method	result
default	$-\frac{c^2\left(\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}+\sqrt{2}\ \sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c^2/f*(cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+2^(1/2
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+4*cos(f*x+e)*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(
f*x+e)+1)/sin(f*x+e))+4*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))-2*sin(f*x+e
))*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(cos(f*x+e)+1)/a
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 3.78, size = 473, normalized size = 3.97

$$\left[\frac{2\,c^{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+2\,\sqrt{2}\left(ac^{2}\cos\left(fx+e\right)+ac^{2}\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\,\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{a}}\cos\left(fx+e\right)\sin\left(fx+e\right)+3\,\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1}\right)-\left(c^{2}\cos\left(fx+e\right)+c^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}+2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)}{af\cos\left(fx+e\right)+af},\ \frac{2\left(c^{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-\left(c^{2}\cos\left(fx+e\right)+c^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\sqrt{a}\sin\left(fx+e\right)}\right)+\frac{2\,\sqrt{2}\left(ac^{2}\cos\left(fx+e\right)+ac^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\sqrt{a}\sin\left(fx+e\right)}\right)}{\sqrt{a}}\right)}{af\cos\left(fx+e\right)+af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 2*sqrt(2)*(a
*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 +
 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - (c^2*cos(f*x
+ e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos
(f*x + e) + 1)))/(a*f*cos(f*x + e) + a*f), 2*(c^2*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2
*sqrt(2)*(a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*
x + e) + a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-\frac{2\sec\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\right)dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int\frac{1}{\sqrt{a\sec\left(e+fx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(
e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x) +
 a), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2c\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{2\sqrt{2}\,c\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-2*c*arctan(
1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3989, 3972, 492, 209}

$$\frac{2c\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2\sqrt{2}\,c\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (2*Sqrt[2]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x
]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx\right)$$

$$= \frac{(2ac)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{(2c)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{(4c)\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx,\right.}{}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [A]
time = 0.28, size = 82, normalized size = 0.94

$$\frac{2c\left(\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}}{f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*(ArcTan[Sqrt[-1 + Sec[e + f*x]]] - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*
x]]/Sqrt[2]])*Cot[(e + f*x)/2]*Sqrt[-1 + Sec[e + f*x]])/(f*Sqrt[a*(1 + Sec[
e + f*x])])
```

Maple [A]
time = 0.18, size = 144, normalized size = 1.66

method	result
default	$-\frac{c\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)+2\ln\left(\frac{\sin(fx+e)\sqrt{-\frac{2c}{\cos}}}{\mathrm{si}}\right.\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos
(f*x+e)*2^(1/2))+2*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(
f*x+e)+1)/sin(f*x+e)))/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 4.27, size = 321, normalized size = 3.69

$$\left[\frac{\sqrt{2}\,ac\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-\sqrt{-a}\,c\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{af},\ \frac{2\left(\sqrt{2}\sqrt{a}\,c\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{a}\,c\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a)*c*log((2*a*cos(f*
x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), 2*(sqrt(2)*s
qrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))) - sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx+\int\left(-\frac{1}{\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a
*sec(e + f*x) + a), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c-\frac{c}{\cos(e+f\,x)}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=121

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,cf}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,cf}+\frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-1/2*arctan(
1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)+
cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3989, 3972, 491, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,cf}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}\,cf}+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]
*Sqrt[a]*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{\int \cot^2(e+fx)\sqrt{a+a\sec(e+fx)}\,dx}{ac}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{acf}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,d\ldots\right.}{\ldots}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\ldots\right.}{c\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,cf} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+\ldots}{\sqrt{2}\,\sqrt{a+a\sec\ldots}}\right.}{\sqrt{2}\,\sqrt{a}\,cf}$$

Mathematica [A]

time = 0.58, size = 101, normalized size = 0.83

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(2+4\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\sqrt{-1+\sec(e+fx)}-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{2cf\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] (Cot[(e + f*x)/2]*(2 + 4*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e +
f*x]] - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f
*x]]))/(2*c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.22, size = 195, normalized size = 1.61

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)-\sin(fx+e)\sqrt{-}}{2cf\sin(fx+e)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-2*2^(1/2)*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/si
n(f*x+e))+2*cos(f*x+e))/sin(f*x+e)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)), x)
```

Fricas [A]
time = 3.77, size = 474, normalized size = 3.92

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)-2\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3+4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{4acf\sin(fx+e)}, \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\sin(fx+e)+2\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{2acf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) - 2*sqrt(-a)*
log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(
cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*
sin(f*x + e) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))
*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*
c*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) +
a)), x)/c
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))), x)
```

3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{2\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,c^2f}-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}\sqrt{a}\,c^2f}+\frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^2/c^2/f+2*arctan(a^(1/2)*tan(f*x
+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f/a^(1/2)-1/4*arctan(1/2*a^(1/2)*tan(f*x+e)
*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c^2/f*2^(1/2)/a^(1/2)+3/2*cot(f*x+e)*(a+a*
sec(f*x+e))^(1/2)/a/c^2/f
```

Rubi [A]
time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 491, 597, 536, 209}

$$-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{3a^2c^2f}+\frac{2\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,c^2f}-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}\sqrt{a}\,c^2f}+\frac{3\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{2ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^2*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(2*Sqr
t[2]*Sqrt[a]*c^2*f) + (3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(2*a*c^2*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2c^2f}$$

$$= -\frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f} - \frac{\text{Subst}\left(\int \frac{-9a-3a}{x^2(1+ax^2)(}\right.}{}$$

$$= \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\,\text{se}}{3a^2c^2f}$$

$$= \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\,\text{se}}{3a^2c^2f}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,c^2f} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e}{\sqrt{2}\sqrt{a+a\,\text{se}}}\right.}{2\sqrt{2}\sqrt{a}\,c^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.25, size = 5576, normalized size = 34.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(136) = 272.
time = 0.25, size = 377, normalized size = 2.34

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(12\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}+3\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(12*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/co
s(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+3*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e
)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-12*2^(1/2)*arctanh(1/2*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)-3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(
f*x+e))-22*cos(f*x+e)^3-4*cos(f*x+e)^2+18*cos(f*x+e))/sin(f*x+e)^3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^2), x)
```

Fricas [A]
time = 4.86, size = 566, normalized size = 3.52

$$\left[-\frac{3\sqrt{2}\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+12\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{8a\cos(fx+e)^3+4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)-4\left(11\cos(fx+e)^2-9\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{24\left(ac^2f\cos(fx+e)-ac^2f\right)\sin(fx+e)},\ \frac{3\sqrt{2}\sqrt{a}(\cos(fx+e)-1)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\sin(fx+e)+12\sqrt{a}(\cos(fx+e)-1)\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(11\cos(fx+e)^2-9\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{12\left(ac^2f\cos(fx+e)-ac^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-a)*(cos(f*x + e) - 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(
f*x + e) + 12*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*c
os(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) -
 4*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*sqrt
(a)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 12*sqrt(a)*(cos(f*x
```

```
+ e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) +
 2*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{a\sec{(e+fx)}+a}\sec^2{(e+fx)}-2\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}+\sqrt{a\sec{(e+fx)}+a}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - 2*sqrt(a*sec(e + f*x
) + a)*sec(e + f*x) + sqrt(a*sec(e + f*x) + a)), x)/c**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2), x)
```

3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=196

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,c^3f}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\,\sqrt{a}\,c^3f}+\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx}}{4ac^3f}$$

```
[Out] -1/2*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^2/c^3/f+1/5*cot(f*x+e)^5*(a+a*se
c(f*x+e))^(5/2)/a^3/c^3/f+2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2
))/c^3/f/a^(1/2)-1/8*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))
^(1/2))/c^3/f*2^(1/2)/a^(1/2)+7/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c^3/f
```

Rubi [A]
time = 0.19, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 491, 597, 536, 209}

$$\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{5a^3c^3f}-\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{2a^2c^3f}+\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,c^3f}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}\,\sqrt{a}\,c^3f}+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4ac^3f}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c^3*f)
 - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(4*Sqr
t[2]*Sqrt[a]*c^3*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a*c^3*f)
 - (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(2*a^2*c^3*f) + (Cot[e + f*x
]^5*(a + a*Sec[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^3}\,dx=-\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$=\frac{2\mathrm{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3c^3f}$$

$$=\frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f}+\frac{\mathrm{Subst}\left(\int \frac{-15a-5a}{x^4(1+ax^2)(2}\right.}{}$$

$$=-\frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f}+\frac{\cot^5(e+fx)(a+}{5a^3}$$

$$=\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f}-\frac{\cot^3(e+fx)(a+a}{2a^2c^3}$$

$$=\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f}-\frac{\cot^3(e+fx)(a+a}{2a^2c^3}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,c^3f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a+a\,\mathrm{s}}}\right.}{4\sqrt{2}\sqrt{a}\,c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.13, size = 5592, normalized size = 28.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(167) = 334.
time = 0.34, size = 545, normalized size = 2.78

method	result

default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos(fx+e)+1)^2\left(40\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40/c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(40*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+5*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-80*sin(f*x+e)*co
s(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-10*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)+40*2^(1/2)*arctanh(1/2*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+5*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)
+1)/sin(f*x+e))-98*cos(f*x+e)^3+160*cos(f*x+e)^2-70*cos(f*x+e))/sin(f*x+e)^
5/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^3), x)
```

Fricas [A]
time = 2.30, size = 662, normalized size = 3.38

$$\left[-\frac{5\sqrt{2}\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+40\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3+4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)-4\left(49\cos(fx+e)^3-80\cos(fx+e)^2+35\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{80\left(ac^3f\cos(fx+e)^2-2ac^3f\cos(fx+e)+ac^3f\right)\sin(fx+e)},\ \frac{5\sqrt{2}\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\sin(fx+e)+40\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(49\cos(fx+e)^3-80\cos(fx+e)^2+35\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{40\left(ac^3f\cos(fx+e)^2-2ac^3f\cos(fx+e)+ac^3f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/80*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(
f*x + e) + 1))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt
(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) +
 a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(49*cos(f*x + e)^3 - 80*cos(f*x +
e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a*c^3*f*
cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/40*(5*s
qrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f
*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos
(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(49*cos(f*x + e)^3 - 80
*cos(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))
/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-3\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+3\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - 3*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*
sec(e + f*x) + a)), x)/c**3
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3), x)
```

3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{2c^4\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}+\frac{12\sqrt{2}\,c^4\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+12*c^4*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)
/f-14*c^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+8/3*c^4*tan(f*x+e)^3/f/(a+a
*sec(f*x+e))^(3/2)-a*c^4*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)*tan(f*x+e)^4/f/(a+
a*sec(f*x+e))^(5/2)
```

Rubi [A]

time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 481, 596, 536, 209}

$$\frac{2c^4\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{12\sqrt{2}\,c^4\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{8c^4\tan^3(e+fx)}{3f(a\sec(e+fx)+a)^{3/2}}-\frac{14c^4\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}-\frac{ac^4\sin(e+fx)\tan^4(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (12*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(3/2)*f) - (14*c^4*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]
]) + (8*c^4*Tan[e + f*x]^3)/(3*f*(a + a*Sec[e + f*x])^(3/2)) - (a*c^4*Sec[(
e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{11/2}}\,dx$$

$$= -\frac{\left(2a^3c^4\right)\text{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} - \frac{\left(ac^4\right)\text{Subst}\left(\int \frac{x^4(10+8ax^2}{(1+ax^2)(2+a}\right.}{}$$

$$= \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{f(a+a\sec(e+fx))^{5/2}} +$$

$$= -\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right.}{f(a+a}$$

$$= -\frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{ac^4\sec^2\left(\frac{1}{2}(e+fx)\right.}{f(a+a}$$

$$= \frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+}\right.}{a^{3/2}f}$$

Mathematica [A]
time = 1.56, size = 196, normalized size = 0.97

$$\frac{c^4\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\left(-22+20\cos(e+fx)-26\cos(2(e+fx))+28\cos(3(e+fx))+6\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)\right)^2\sqrt{-1+\sec(e+fx)}+36\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)\right)^2\sqrt{-1+\sec(e+fx)}\right)\sec^2(e+fx)}{12af\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^4*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-22 + 20*Cos[e + f*x] - 26*Cos[2*(e
 + f*x)] + 28*Cos[3*(e + f*x)] + 6*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(Cos[(e
+ f*x)/2] + Cos[(3*(e + f*x))/2])^2*Sqrt[-1 + Sec[e + f*x]] + 36*Sqrt[2]*Ar
cTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*(Cos[(e + f*x)/2] + Cos[(3*(e + f*x))
/2])^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(12*a*f*Sqrt[a*(1 + Sec[e +
 f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(179) = 358$.
time = 0.24, size = 552, normalized size = 2.72

method	result
default	$-\frac{c^4\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))\left(3(\cos^2(fx+e))\sin(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)}{2\cos(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(3*cos(f*x+e
)^2*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+6*cos(f*x+
e)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-36*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(3/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+3*2^(1/2)*arctanh(1
/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*sin(f*x+e)-72*sin(f*x+e)*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(3/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos
(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)-36*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2
)*sin(f*x+e)+112*cos(f*x+e)^3-52*cos(f*x+e)^2-64*cos(f*x+e)+4)/cos(f*x+e)/s
in(f*x+e)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^4/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A]
time = 3.66, size = 685, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(18*sqrt(2)*(a*c^4*cos(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos
(f*x + e))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 3*(c^4*cos(f*x + e)^3 + 2*c^
4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
 a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(28*c^4*cos(f*x + e)^2 + 15*c^
4*cos(f*x + e) - c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x + e)), -2/3
*(3*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e))) + (28*c^4*cos(f*x + e)^2 + 15*c^4*cos(f*x + e) - c^4)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 18*sqrt(2)*(a*c^4*cos(f*x + e)^3
 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(
a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{4}\left(\int\left(-\frac{4\sec\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\right)dx+\int\frac{6\sec^{2}\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int\left(-\frac{4\sec^{3}\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\right)dx+\int\frac{\sec^{4}\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int\frac{1}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-4*s
ec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e +
f*x) + a)), x) + Integral(sec(e + f*x)**4/(a*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2), x)
```

3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{2c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}+\frac{2\sqrt{2}\,c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+2*c^3*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/
f-4*c^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+c^3*sec(1/2*f*x+1/2*e)^2*sin(
f*x+e)*tan(f*x+e)^2/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 481, 596, 536, 209}

$$\frac{2c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}+\frac{2\sqrt{2}\,c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{4c^3\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}+\frac{c^3\sin(e+fx)\tan^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 + (2*Sqrt[2]*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(a^(3/2)*f) - (4*c^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]])
 + (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(a + a*Sec[e + f
*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx &= -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx\right)\\
&= \frac{\left(2a^2c^3\right)\mathrm{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{c^3\mathrm{Subst}\left(\int \frac{x^2(6+4ax^2)}{(1+ax^2)(2+ax^2)}\,dx,\right.}{f}\\
&= -\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}} -\\
&= -\frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{f(a+a\sec(e+fx))^{3/2}} -\\
&= \frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+f}\right.}{a^{3/2}f}
\end{aligned}$$

Mathematica [A]
time = 1.88, size = 132, normalized size = 0.78

$$\frac{2c^3\left(-3+\mathrm{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cot^2\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}+\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cot^2\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}-\sec(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^3*(-3 + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cot[(e + f*x)/2]^2*Sqrt[-1 + S
ec[e + f*x]] + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cot[(e + f*x
)/2]^2*Sqrt[-1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[(e + f*x)/2])/(a*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(149) = 298.
time = 0.23, size = 377, normalized size = 2.23

method	result

default	$-\frac{c^{3} \sqrt{\frac{a (\cos(fx+e)+1)}{\cos(fx+e)}}\left(-\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\, \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\, \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^{2}(fx+e)) \sin(fx+e)\sqrt{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+
e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+2*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/s
in(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)+6*cos(f*x+e)^3-2*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1
)/sin(f*x+e))-10*cos(f*x+e)^2+2*cos(f*x+e)+2)/sin(f*x+e)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A]
time = 4.45, size = 593, normalized size = 3.51

$$\left[\frac{\sqrt{2}\left(ac^{3}\cos(fx+e)^{2}+2\,ac^{3}\cos(fx+e)+ac^{3}\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^{2}-2\cos(fx+e)+1}{\cos(fx+e)^{2}+2\cos(fx+e)+1}\right)-\left(c^{3}\cos(fx+e)^{2}+2\,c^{3}\cos(fx+e)+c^{3}\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\left(3\,c^{3}\cos(fx+e)+c^{3}\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{a^{2}f\cos(fx+e)^{2}+2\,a^{2}f\cos(fx+e)+a^{2}f},\; -\frac{2\left(\left(c^{3}\cos(fx+e)^{2}+2\,c^{3}\cos(fx+e)+c^{3}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(3\,c^{3}\cos(fx+e)+c^{3}\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)+\frac{\sqrt{2}\left(ac^{3}\cos(fx+e)^{2}+2\,ac^{3}\cos(fx+e)+ac^{3}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}\right)}{a^{2}f\cos(fx+e)^{2}+2\,a^{2}f\cos(fx+e)+a^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*sqrt(-1/a)*
log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x
+ e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2
+ 2*cos(f*x + e) + 1)) - (c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sq
rt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
```

```
) - 2*(3*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -2*((c^3
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*c^3*cos(f
*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(
2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/s
qrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int \frac{3\sec (e+fx)}{a\sqrt{a\sec (e+fx)+a}\sec (e+fx)+a\sqrt{a\sec (e+fx)+a}}\,dx+\int \left(-\frac{3\sec^{2} (e+fx)}{a\sqrt{a\sec (e+fx)+a}\sec (e+fx)+a\sqrt{a\sec (e+fx)+a}}\right)dx+\int \frac{\sec^{3} (e+fx)}{a\sqrt{a\sec (e+fx)+a}\sec (e+fx)+a\sqrt{a\sec (e+fx)+a}}\,dx+\int \left(-\frac{1}{a\sqrt{a\sec (e+fx)+a}\sec (e+fx)+a\sqrt{a\sec (e+fx)+a}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec
(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*
x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqr
t(a*sec(e + f*x) + a)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^3}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{2c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{2}\,c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-c^2*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f-
2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 481, 12, 400, 209}

$$\frac{2c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\sqrt{2}\,c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f)
 - (Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f
*x]])])/(a^(3/2)*f) - (c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(a*f*Sqrt[a + a
*Sec[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = (a^2c^2)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{(2ac^2)\,\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{c^2\text{Subst}\left(\int \frac{2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan}{\sqrt{a+a}}\right.}{af}$$

$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{}{\sqrt{a}}\right.}{af}$$

$$= -\frac{c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+f}{\sqrt{a+a\sec(}}\right.}{af}$$

$$= \frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{2}\,c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right.}{a^{3/2}f}$$

Mathematica [A]
time = 1.12, size = 128, normalized size = 1.08

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-1+\cos(e+fx)+\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)(1+\cos(e+fx))\sqrt{-1+\sec(e+fx)}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{af\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(Sec[(e + f*x)/2]^2*(-1 + Cos[e + f*x] + ArcTan[Sqrt[
-1 + Sec[e + f*x]]]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]) - Sqrt[2]*A
rcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))/(a*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(102) = 204$.
time = 0.18, size = 369, normalized size = 3.10

method	result

default	$-\frac{c^2\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sin(fx+e)\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}+\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(sin(f*x+e)*cos(f*x+e)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*
x+e))*cos(f*x+e)*sin(f*x+e)+2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))-2*cos(f*x+
e)^2+2*cos(f*x+e))/(cos(f*x+e)+1)/sin(f*x+e)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(108) = 216$.
time = 2.81, size = 585, normalized size = 4.92

$$\left[-\frac{4c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\left(ac^2\cos(fx+e)^2+2ac^2\cos(fx+e)+ac^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\left(c^2\cos(fx+e)^2+2c^2\cos(fx+e)+c^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{2\left(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f\right)},-\frac{2c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+2\left(c^2\cos(fx+e)^2+2c^2\cos(fx+e)+c^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\frac{\sqrt{2}\left(ac^2\cos(fx+e)^2+2ac^2\cos(fx+e)+ac^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}}{a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^
2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
```

```
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(c^2*cos(
f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c^2*co
s(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a^2
*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int \left(-\frac{2\sec\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\right)dx+\int \frac{\sec^{2}\left(e+fx\right)}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int \frac{1}{a\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)+a\sqrt{a\sec\left(e+fx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**2/(a*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^2}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{3c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\,a^{3/2}f}-\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-3/2*c*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)-
c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3989, 3972, 482, 536, 209}

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{3c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\,a^{3/2}f}-\frac{c\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 (3*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(S
qrt[2]*a^(3/2)*f) - (c*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(2*a*f*Sqrt[a + a*S
ec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx &= -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx\right)\\
&= \frac{(2c)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\
&= -\frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}} - \frac{c\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e}{\sqrt{a+a\,\text{se}}}\right.}{af}\\
&= -\frac{c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{2af\sqrt{a+a\sec(e+fx)}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e}}\right.}{af}\\
&= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{3c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{3/2}f} -\end{aligned}$$

Mathematica [A]
time = 1.06, size = 130, normalized size = 1.15

$$\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-1+\cos(e+fx)+2\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)(1+\cos(e+fx))\sqrt{-1+\sec(e+fx)}\right)-3\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{2af\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c*Cot[(e + f*x)/2]*(Sec[(e + f*x)/2]^2*(-1 + Cos[e + f*x] + 2*ArcTan[Sqrt[
-1 + Sec[e + f*x]]]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]) - 3*Sqrt[2]
*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))/(2*a*f*S
qrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(96) = 192$.
time = 0.16, size = 371, normalized size = 3.28

method	result
default	$-\frac{c\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(2\sin(fx+e)\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}+3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2*sin(f*x+e)*cos(f*x+e)*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+3*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/
sin(f*x+e))*cos(f*x+e)*sin(f*x+e)+2*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(
(sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))-
2*cos(f*x+e)^2+2*cos(f*x+e))/(cos(f*x+e)+1)/sin(f*x+e)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(102) = 204$.
time = 4.32, size = 548, normalized size = 4.85

$$\left[-\frac{4c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3\sqrt{2}\left(c\cos(fx+e)^2+2c\cos(fx+e)+c\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+4\left(c\cos(fx+e)^2+2c\cos(fx+e)+c\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{4\left(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f\right)},\ -\frac{2c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3\sqrt{2}\left(c\cos(fx+e)^2+2c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+4\left(c\cos(fx+e)^2+2c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{2\left(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(
f*x + e) + 1)) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log((
2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a^2*f*co
s(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/2*(2*c*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*sqrt(2)*(c*cos(f*x + e)^
2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 4*(c*cos(f*x + e)^2 +
2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos
(f*x + e) + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx+\int\left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt
(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e +
 f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c-\frac{c}{\cos(e+f\,x)}}{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=177

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf}-\frac{7\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{3/2}cf}+\frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f-7/8*arctan(
1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f*2^(1/2)+
1/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c/f+1/4*cos(f*x+e)*cot(f*x+e)*sec
(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(1/2)/a^2/c/f
```

Rubi [A]
time = 0.16, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 483, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}cf}-\frac{7\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}a^{3/2}cf}+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4a^2cf}+\frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a\sec(e+fx)+a}}{4a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c*f) -
 (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(4*S
qrt[2]*a^(3/2)*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*a^2*c*f) +
 (Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f*x]])/(4
*a^2*c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{ac}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2cf}$$

$$= \frac{\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+f}{4a^2cf}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx}{}$$

$$= \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cos(e+fx)\cot(e+fx}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a+a\,\mathrm{s}}}\right.}{4\sqrt{2}\,a^{3/2}c}$$

Mathematica [A]
time = 1.25, size = 154, normalized size = 0.87

$$\frac{\left(1+3\cos(e+fx)-7\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}+8\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)(1+\cos(e+fx))\sqrt{-1+\sec(e+fx)}\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{2acf(-1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] ((1 + 3*Cos[e + f*x] - 7*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Co
s[(e + f*x)/2]^2*Sqrt[-1 + Sec[e + f*x]] + 8*ArcTan[Sqrt[-1 + Sec[e + f*x]]
]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]])*Sin[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/(2*a*c*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(151) = 302$.
time = 0.21, size = 377, normalized size = 2.13

method	result

default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(8\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}+7\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(8*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*
x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+7*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)
/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-8*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)-7*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e
))-6*cos(f*x+e)^3+4*cos(f*x+e)^2+2*cos(f*x+e))/sin(f*x+e)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)), x)
```

Fricas [A]
time = 3.48, size = 560, normalized size = 3.16

$$\left[-\frac{7\sqrt{2}\sqrt{-a}(\cos(fx+e)+1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+8\sqrt{-a}(\cos(fx+e)+1)\log\left(-\frac{8a\cos(fx+e)^3+4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)-4\left(3\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{16(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)},\ \frac{7\sqrt{2}\sqrt{a}(\cos(fx+e)+1)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\sin(fx+e)+8\sqrt{a}(\cos(fx+e)+1)\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(3\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{8(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/16*(7*sqrt(2)*sqrt(-a)*(cos(f*x + e) + 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(
f*x + e) + 8*sqrt(-a)*(cos(f*x + e) + 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*co
s(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) -
4*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
```

```
)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/8*(7*sqrt(2)*sqrt(a)*(
cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 8*sqrt(a)*(cos(f*x + e) +
 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(3*
cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a
^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e +
f*x) + a)), x)/c
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))), x)
```

3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=214

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f}-\frac{9\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{3/2}c^2f}+\frac{7\cot(e+fx)\sqrt{a+a\sec(e+f}}{8a^2c^2f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f+1/12*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-1/4*cos(f*x+e)*cot(f*x+e)^3*sec(1
/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-9/16*arctan(1/2*a^(1/2)*ta
n(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f*2^(1/2)+7/8*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/a^2/c^2/f
```

Rubi [A]
time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 483, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}c^2f}-\frac{9\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{8\sqrt{2}a^{3/2}c^2f}+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{12a^3c^2f}-\frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{3/2}}{4a^3c^2f}+\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^2*f)
 - (9*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(8
*Sqrt[2]*a^(3/2)*c^2*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*a^2*
c^2*f) + (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(12*a^3*c^2*f) - (Cos[
e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(3/2))/(4*a
^3*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^4(e+fx)\sqrt{a+a\sec(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{a^3c^2f}$$

$$= -\frac{\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e}{4a^3c^2f}$$

$$= \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cos(e+fx)\cot^3(e}{}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a}{12a^3c}$$

$$= \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f} + \frac{\cot^3(e+fx)(a+a}{12a^3c}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f} - \frac{9\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a+}}\right.}{8\sqrt{2}a^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.32, size = 5612, normalized size = 26.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(184) = 368.
time = 0.26, size = 387, normalized size = 1.81

method	result

default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(48\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(48*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/c
os(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+27*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x
+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-48*2^(1/2)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)-27*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/s
in(f*x+e))-62*cos(f*x+e)^3+4*cos(f*x+e)^2+42*cos(f*x+e))/sin(f*x+e)^5*(cos(
f*x+e)^2-1)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^2), x)
```

Fricas [A]
time = 2.99, size = 608, normalized size = 2.84

$$\left[-\frac{27\sqrt{2}\left(\cos(fx+e)^2-1\right)\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+48\left(\cos(fx+e)^2-1\right)\sqrt{-a}\,\log\left(-\frac{8a\cos(fx+e)^3+4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin(fx+e)-4\left(31\cos(fx+e)^3-2\cos(fx+e)^2-21\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{96\left(a^2c^2f\cos(fx+e)^2-a^2c^2f\right)\sin(fx+e)},\ \frac{27\sqrt{2}\left(\cos(fx+e)^2-1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\sin(fx+e)+48\left(\cos(fx+e)^2-1\right)\sqrt{a}\,\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{2a\cos(fx+e)^2+a\cos(fx+e)-a}\right)\sin(fx+e)+2\left(31\cos(fx+e)^3-2\cos(fx+e)^2-21\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{48\left(a^2c^2f\cos(fx+e)^2-a^2c^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/96*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(
f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4
*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
```

```
*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
 e) - 4*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(
f*x + e)), 1/48*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*s
in(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 +
 a*cos(f*x + e) - a))*sin(f*x + e) + 2*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^
2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*c
os(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - a*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt
(a*sec(e + f*x) + a)), x)/c**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^{3/2}\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2), x)
```

3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=249

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f}-\frac{11\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}\,a^{3/2}c^3f}+\frac{21\cot(e+fx)\sqrt{a+a\sec(e+}}{16a^2c^3f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^3/f-5/24*cot(
f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^3/f-3/20*cot(f*x+e)^5*(a+a*sec(f*x+e)
)^(5/2)/a^4/c^3/f+1/4*cos(f*x+e)*cot(f*x+e)^5*sec(1/2*f*x+1/2*e)^2*(a+a*sec
(f*x+e))^(5/2)/a^4/c^3/f-11/32*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*s
ec(f*x+e))^(1/2))/a^(3/2)/c^3/f*2^(1/2)+21/16*cot(f*x+e)*(a+a*sec(f*x+e))^(
1/2)/a^2/c^3/f
```

Rubi [A]
time = 0.23, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 483, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}c^3f}-\frac{11\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{16\sqrt{2}\,a^{3/2}c^3f}-\frac{3\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{20a^4c^3f}+\frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{5/2}}{4a^4c^3f}-\frac{5\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{24a^3c^3f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{16a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*c^3*f)
 - (11*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
16*Sqrt[2]*a^(3/2)*c^3*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(16*
a^2*c^3*f) - (5*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(24*a^3*c^3*f) -
 (3*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/(20*a^4*c^3*f) + (Cos[e + f*
x]*Cot[e + f*x]^5*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(5/2))/(4*a^4*c^3
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^6(e+fx)(a+a\sec(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^4c^3f}$$

$$= \frac{\cos(e+fx)\cot^5(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e+}{4a^4c^3f}$$

$$= -\frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f} + \frac{\cos(e+fx)\cot^5}{}$$

$$= -\frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f} - \frac{3\cot^5(e+fx)(a}{20}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} - \frac{5\cot^3(e+fx)(a+}{24a^3}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} - \frac{5\cot^3(e+fx)(a+}{24a^3}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f} - \frac{11\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a+}}\right.}{16\sqrt{2}\,a^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.23, size = 5629, normalized size = 22.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(215) = 430.
time = 0.45, size = 725, normalized size = 2.91

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))(\cos(fx+e)+1)^2\left(480(\cos^3(fx+e))\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(}{\cos(fx}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/480/c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)
+1)^2*(480*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arc
tanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2)
)*2^(1/2)-480*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)+165*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*sin(f*x+e)*cos(f*x
+e)^3-480*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan
h(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2
^(1/2)-165*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+
480*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos
(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-165*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)-898*cos(f*x+e)^4+16
5*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))+702*cos(f*x+e)^3+730*c
os(f*x+e)^2-630*cos(f*x+e))/sin(f*x+e)^7/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^3), x)
```

Fricas [A]
time = 3.14, size = 776, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/960*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f
*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f
*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*
(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x
 + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3
- a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x +
e)), 1/480*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f*x + e
)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e
) - a))*sin(f*x + e) + 2*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos
(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e
) + a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)+2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a*sqrt(a*sec(e
+ f*x) + a)*sec(e + f*x)**3 + 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - a
*sqrt(a*sec(e + f*x) + a)), x)/c**3
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3),x)
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3), x)
```

3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$\frac{2c^5\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{23\sqrt{2}\,c^5\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}+\frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^5*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23*c^5*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5/2)
/f+21*c^5*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-19/6*c^5*tan(f*x+e)^3/a/f
/(a+a*sec(f*x+e))^(3/2)+3/4*c^5*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)*tan(f*x+e)^
4/f/(a+a*sec(f*x+e))^(5/2)+1/4*a*c^5*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)^2*tan(
f*x+e)^5/f/(a+a*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.23, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3989, 3972, 481, 592, 596, 536, 209}

$$\frac{2c^5\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23\sqrt{2}\,c^5\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}+\frac{21c^5\tan(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}}-\frac{19c^5\tan^3(e+fx)}{6af(a\sec(e+fx)+a)^{3/2}}+\frac{ac^5\sin^2(e+fx)\tan^5(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4f(a\sec(e+fx)+a)^{7/2}}+\frac{3c^5\sin(e+fx)\tan^4(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (23*Sqrt[2]*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
+ f*x]])])/(a^(5/2)*f) + (21*c^5*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*
x]]) - (19*c^5*Tan[e + f*x]^3)/(6*a*f*(a + a*Sec[e + f*x])^(3/2)) + (3*c^5*
Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(4*f*(a + a*Sec[e + f*x])^(
5/2)) + (a*c^5*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^5)/(4*f*(a +
a*Sec[e + f*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = -\left(\left(a^5c^5\right)\int \frac{\tan^{10}(e+fx)}{(a+a\sec(e+fx))^{15/2}}\,dx\right)$$

$$= \frac{\left(2a^3c^5\right)\text{Subst}\left(\int \frac{x^{10}}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^5(e+fx)}{4f(a+a\sec(e+fx))^{7/2}} + \frac{\left(ac^5\right)\text{Subst}\left(\int \frac{x^6(14+10ax^2)}{(1+ax^2)(2+ax^2)}\right.}{}$$

$$= \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} + \frac{ac^5\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+}{4f(a+a\sec(e+}$$

$$= -\frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^4(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

$$= \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f(a+a\text{s}}$$

$$= \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} - \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{3c^5\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f(a+a\text{s}}$$

$$= \frac{2c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23\sqrt{2}c^5\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx}}\right.}{a^{5/2}f}$$

Mathematica [A]
time = 3.70, size = 180, normalized size = 0.69

$$\frac{c^5\cot\left(\frac{1}{2}(e+fx)\right)\left((81-30\cos(e+fx)+52\cos(2(e+fx))-66\cos(3(e+fx))-37\cos(4(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)+96\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^2(e+fx)\sqrt{-1+\sec(e+fx)}-1104\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos^2(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec^2(e+fx)}{48a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^5*Cot[(e + f*x)/2]*((81 - 30*Cos[e + f*x] + 52*Cos[2*(e + f*x)] - 66*Cos
[3*(e + f*x)] - 37*Cos[4*(e + f*x)])*Sec[(e + f*x)/2]^4 + 96*ArcTan[Sqrt[-1
```

```
 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 1104*Sqrt[2]*Arc
Tan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]]
)*Sec[e + f*x]^2)/(48*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(229) = 458$.
time = 0.27, size = 726, normalized size = 2.79

method	result
default	$-\frac{c^5\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))^2\left(-3(\cos^3(fx+e))\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c^5/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(-3*cos(f*
x+e)^3*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+
e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*sin(f*x+e)*2^(1/2)-69*cos(
f*x+e)^3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*sin(f*x+e)-9*cos(f*x+e)^2*
sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-207*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(3/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-9*cos(f*x+e)*sin(f*x+
e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-207*sin(f*x+e)*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(3/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)-3*2^(1/2)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(3/2)*sin(f*x+e)-69*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*
sin(f*x+e)+148*cos(f*x+e)^4+132*cos(f*x+e)^3-200*cos(f*x+e)^2-84*cos(f*x+e)
+4)/cos(f*x+e)/sin(f*x+e)^5/a^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 7.11, size = 801, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*c
os(f*x + e)^2 + a*c^5*cos(f*x + e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x
 + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 6*(c
^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f
*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*
x + e) + 1)) + 4*(37*c^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*co
s(f*x + e) - c^5)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^
3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*
f*cos(f*x + e)), -1/3*(6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5
*cos(f*x + e)^2 + c^5*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(37*c^5*cos(f*x +
e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*cos(f*x + e) - c^5)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) - 69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*
a*c^5*cos(f*x + e)^3 + 3*a*c^5*cos(f*x + e)^2 + a*c^5*cos(f*x + e))*arctan(
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e)))/sqrt(a))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 + a^3*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(-10*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a
*sec(e + f*x) + a)), x) + Integral(10*sec(e + f*x)**3/(a**2*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-5*sec(e + f*x)**4/(a**2*sqrt
(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(
```

```
e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**5/(a
**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^5}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2), x)
```

3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=229

$$\frac{2c^4\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{11c^4\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\,a^{5/2}f}+\frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}}-$$

```
[Out] 2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/2*c^4*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(
1/2)+7/2*c^4*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-1/4*c^4*sec(1/2*f*x+1/
2*e)^2*sin(f*x+e)*tan(f*x+e)^2/a/f/(a+a*sec(f*x+e))^(3/2)-1/4*c^4*sec(1/2*f
*x+1/2*e)^4*sin(f*x+e)^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3989, 3972, 481, 592, 596, 536, 209}

$$\frac{2c^4\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{11c^4\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\,a^{5/2}f}+\frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a\sec(e+fx)+a}}-\frac{c^4\sin^2(e+fx)\tan^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4f(a\sec(e+fx)+a)^{5/2}}-\frac{c^4\sin(e+fx)\tan^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4af(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[2]*a^(5/2)*f) + (7*c^4*Tan[e + f*x])/(2*a^2*f*Sqrt[a + a*Sec[e + f
*x]]) - (c^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^2)/(4*a*f*(a + a*
Sec[e + f*x])^(3/2)) - (c^4*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^
3)/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \left(a^4c^4\right)\int \frac{\tan^8(e+fx)}{(a+a\sec(e+fx))^{13/2}}\,dx$$

$$= -\frac{\left(2a^2c^4\right)\operatorname{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan^3(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{c^4\operatorname{Subst}\left(\int \frac{x^4(10+6ax^2)}{(1+ax^2)(2+ax^2)^2}\,d\right.}{}$$

$$= -\frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c^4\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+}{4f(a+a\sec(e+}$$

$$= \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c}{}$$

$$= \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\tan^2(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} - \frac{c}{}$$

$$= \frac{2c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^4\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\,a^{5/2}f}$$

Mathematica [**A**]
time = 2.72, size = 164, normalized size = 0.72

$$\frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left((-4+19\cos(e+fx)-12\cos(2(e+fx))-3\cos(3(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)+32\operatorname{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos(e+fx)\sqrt{-1+\sec(e+fx)}-88\sqrt{2}\operatorname{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos(e+fx)\sqrt{-1+\sec(e+fx)}\right)\sec(e+fx)}{16a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^4*Cot[(e + f*x)/2]*((-4 + 19*Cos[e + f*x] - 12*Cos[2*(e + f*x)] - 3*Cos[
3*(e + f*x)])*Sec[(e + f*x)/2]^4 + 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e
 + f*x]*Sqrt[-1 + Sec[e + f*x]] - 88*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]
/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(16*a^2*f*Sqr
t[a*(1 + Sec[e + f*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(200) = 400$.
time = 0.24, size = 550, normalized size = 2.40

method	result
default	$-\frac{c^4 \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(2(\cos^4(fx+e))\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c^4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2*cos(f*x+e)^4*sin(f*x+e)*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+11*sin(f*x+e)*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^4-4*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)
*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)-22*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/si
n(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+6*cos(f*x+e)^5+2*2^(1/2)*arctanh(1/2*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+11*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)
+1)/sin(f*x+e))-32*cos(f*x+e)^3+36*cos(f*x+e)^2-6*cos(f*x+e)-4)/sin(f*x+e)^
5/a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 4.73, size = 706, normalized size = 3.08

$$\left[-\frac{11\sqrt{2}\left(c^4\cos(fx+e)^3+3c^4\cos(fx+e)^2+3c^4\cos(fx+e)+c^4\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+4\left(c^4\cos(fx+e)^3+3c^4\cos(fx+e)^2+3c^4\cos(fx+e)+c^4\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-4\left(3c^4\cos(fx+e)^2+9c^4\cos(fx+e)+2c^4\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{4\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}, \frac{11\sqrt{2}\left(c^4\cos(fx+e)^3+3c^4\cos(fx+e)^2+3c^4\cos(fx+e)+c^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-4\left(c^4\cos(fx+e)^3+3c^4\cos(fx+e)^2+3c^4\cos(fx+e)+c^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+2\left(3c^4\cos(fx+e)^2+9c^4\cos(fx+e)+2c^4\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{2\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*
x + e) + c^4)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
```

```
os(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x +
 e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 4*(c^4*cos(f*x + e)^3 + 3
*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*c^4*cos(f*x + e)
^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*
x + e) + a^3*f), 1/2*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2
 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 4*(c^4*cos(f*x + e)
^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*
(3*c^4*cos(f*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)
^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right)dx+\int\frac{6\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}dx+\int\left(-\frac{4\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\right)dx+\int\frac{\sec^4(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}dx+\int\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(6*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec
(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*s
ec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*
*2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^4}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2), x)
```

3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{7c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2}f}-\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+f}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-7/4*c^3*a
rctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(1
/2)-1/4*c^3*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)+1/
4*c^3*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/
2)
```

Rubi [A]
time = 0.16, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 481, 592, 536, 209}

$$\frac{2c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7c^3\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{5/2}f}-\frac{c^3\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{4a^2f\sqrt{a\sec(e+fx)+a}}+\frac{c^3\sin^2(e+fx)\tan(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4af(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (7*c^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(2*Sqrt[2]*a^(5/2)*f) - (c^3*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sec[e + f*x]]) + (c^3*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*
x])/(4*a*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = -\left(\left(a^3c^3\right)\int \frac{\tan^6(e+fx)}{(a+a\sec(e+fx))^{11/2}}\,dx\right)$$

$$= \frac{\left(2ac^3\right)\text{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}} + \frac{c^3\text{Subst}\left(\int \frac{x^2(6+2ax^2)}{(1+ax^2)(2+ax^2)^2}\,dx, x\right.}{2af}$$

$$= -\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+}{4af(a+a\sec(e+fx))^{3/2}}$$

$$= -\frac{c^3\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} + \frac{c^3\sec^4\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tan(e+}{4af(a+a\sec(e+fx))^{3/2}}$$

$$= \frac{2c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{7c^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2}f}$$

Mathematica [A]
time = 1.61, size = 136, normalized size = 0.71

$$-\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\left(\left(-5+8\cos(e+fx)-3\cos(2(e+fx))\right)\sec^4\left(\frac{1}{2}(e+fx)\right)-32\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\sqrt{-1+\sec(e+fx)}+28\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{16a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/16*(c^3*Cot[(e + f*x)/2]*((-5 + 8*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sec
[(e + f*x)/2]^4 - 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]
] + 28*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*
x]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(164) = 328.
time = 0.23, size = 553, normalized size = 2.90

method	result

default	$-\frac{c^{3} \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\, (-1+\cos(fx+e)) \left(-4 \sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\, \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\, \sin(fx+e) \sqrt{2}}{2\cos(fx+e)}\right) (\cos^{2}(fx+ \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c^3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(-4*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)-7*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-8*sin(f*x+e)*cos(f
*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-14*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-co
s(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)-4*2^(1/2)*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+6*cos(f*x+e)^3-7*sin(f*x+e)*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
-cos(f*x+e)+1)/sin(f*x+e))-8*cos(f*x+e)^2+2*cos(f*x+e))/(cos(f*x+e)+1)/sin(
f*x+e)^3/a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 7.15, size = 696, normalized size = 3.64

$$\left[-\frac{7\sqrt{2}\left(c^{3}\cos(fx+e)^{3}+3c^{3}\cos(fx+e)^{2}+3c^{3}\cos(fx+e)+c^{3}\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^{2}-2a\cos(fx+e)+a}{\cos(fx+e)^{2}+2\cos(fx+e)+1}\right)+8\left(c^{3}\cos(fx+e)^{3}+3c^{3}\cos(fx+e)^{2}+3c^{3}\cos(fx+e)+c^{3}\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+4\left(3c^{3}\cos(fx+e)^{2}-c^{3}\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{8\left(a^{3}f\cos(fx+e)^{3}+3a^{3}f\cos(fx+e)^{2}+3a^{3}f\cos(fx+e)+a^{3}f\right)}, \frac{7\sqrt{2}\left(c^{3}\cos(fx+e)^{3}+3c^{3}\cos(fx+e)^{2}+3c^{3}\cos(fx+e)+c^{3}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-8\left(c^{3}\cos(fx+e)^{3}+3c^{3}\cos(fx+e)^{2}+3c^{3}\cos(fx+e)+c^{3}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(3c^{3}\cos(fx+e)^{2}-c^{3}\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{4\left(a^{3}f\cos(fx+e)^{3}+3a^{3}f\cos(fx+e)^{2}+3a^{3}f\cos(fx+e)+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x
 + e) + c^3)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x +
```

```
e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^3 + 3*
c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x +
e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(3*c^3*cos(f*x + e)^
2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^
3*f), 1/4*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos
(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8*(c^3*cos(f*x + e)^3 + 3*c^3*c
os(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(3*c^3*cos(f
*x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x +
 e) + a^3*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int \frac{3\sec(e+fx)}{a^{2}\sqrt{a\sec(e+fx)+a}\,\sec^{2}(e+fx)+2a^{2}\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)+a^{2}\sqrt{a\sec(e+fx)+a}}\,dx+\int\left(-\frac{3\sec^{2}(e+fx)}{a^{2}\sqrt{a\sec(e+fx)+a}\,\sec^{2}(e+fx)+2a^{2}\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)+a^{2}\sqrt{a\sec(e+fx)+a}}\right)dx+\int \frac{\sec^{3}(e+fx)}{a^{2}\sqrt{a\sec(e+fx)+a}\,\sec^{2}(e+fx)+2a^{2}\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)+a^{2}\sqrt{a\sec(e+fx)+a}}\,dx+\int\left(-\frac{1}{a^{2}\sqrt{a\sec(e+fx)+a}\,\sec^{2}(e+fx)+2a^{2}\sqrt{a\sec(e+fx)+a}\,\sec(e+fx)+a^{2}\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c**3*(Integral(3*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(-3*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*
sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*x)
 + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2
*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt
(a*sec(e + f*x) + a)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=189

$$\frac{2c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{11c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\,a^{5/2}f}-\frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+}{8a^2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/8*c^2*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(
1/2)-3/8*c^2*sec(1/2*f*x+1/2*e)^2*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-1
/4*c^2*cos(f*x+e)*sec(1/2*f*x+1/2*e)^4*sin(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3989, 3972, 481, 541, 536, 209}

$$\frac{2c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{11c^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{4\sqrt{2}\,a^{5/2}f}-\frac{3c^2\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2f\sqrt{a\sec(e+fx)+a}}-\frac{c^2\sin(e+fx)\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{4a^2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f)
 - (11*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(4*Sqrt[2]*a^(5/2)*f) - (3*c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(8*a^2*f
*Sqrt[a + a*Sec[e + f*x]]) - (c^2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f
*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = (a^2c^2)\int \frac{\tan^4(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{(2c^2)\,\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

$$= -\frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\text{Subst}\left(\int \frac{2-2ax^2}{(1+ax^2)(2+ax^2)^2}\,dx,\right.}{2a^2}$$

$$= -\frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{3c^2\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+}{4a^2f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f}$$

Mathematica [A]
time = 1.53, size = 136, normalized size = 0.72

$$-\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\left((-1+8\cos(e+fx)-7\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)-64\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\sqrt{-1+\sec(e+fx)}+44\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{32a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/32*(c^2*Cot[(e + f*x)/2]*((-1 + 8*Cos[e + f*x] - 7*Cos[2*(e + f*x)])*Sec
[(e + f*x)/2]^4 - 64*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]
] + 44*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*
x]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(162) = 324$.
time = 0.20, size = 545, normalized size = 2.88

method	result

default	$\frac{c^2 \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-8\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}\ -}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-8*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos
(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)-11*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e
)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-16*sin(f*x+e)*cos(f*x+e)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-22*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin
(f*x+e))*cos(f*x+e)*sin(f*x+e)-8*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+14*cos(f*x+e)^3-11*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1
)/sin(f*x+e))-8*cos(f*x+e)^2-6*cos(f*x+e))/(cos(f*x+e)+1)^2/sin(f*x+e)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

Fricas [A]
time = 5.08, size = 696, normalized size = 3.68

$$\left[-\frac{11\sqrt{2}\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+16\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+4\left(7c^2\cos(fx+e)^2+3c^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{16\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}, \frac{11\sqrt{2}\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-16\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(7c^2\cos(fx+e)^2+3c^2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{8\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f
*x + e) + c^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x
+ e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 16*(c^2*cos(f*x + e)^3 +
```

```
 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x
 + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(7*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e)
 + a^3*f), 1/8*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c
^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 16*(c^2*cos(f*x + e)^3 +
3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(7*c^
2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*
cos(f*x + e) + a^3*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-\frac{2\sec\left(e+fx\right)}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{2}\left(e+fx\right)+2a^{2}\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+a}}\right)dx+\int\frac{\sec^{2}\left(e+fx\right)}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{2}\left(e+fx\right)+2a^{2}\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+a}}\,dx+\int\frac{1}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\,\sec^{2}\left(e+fx\right)+2a^{2}\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x
) + a)), x) + Integral(sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec
(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^2}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{23c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}\,a^{5/2}f}-\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}}-\frac{}{8a}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23/16*c*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2
)-1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-7/8*c*tan(f*x+e)/a/f/(a+a*sec(f
*x+e))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3989, 3972, 482, 541, 536, 209}

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{23c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{8\sqrt{2}\,a^{5/2}f}-\frac{7c\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{16a^2f\sqrt{a\sec(e+fx)+a}}-\frac{c\sin(e+fx)\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{8a^2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 (23*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(
8*Sqrt[2]*a^(5/2)*f) - (7*c*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(16*a^2*f*Sqrt
[a + a*Sec[e + f*x]]) - (c*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(8
*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\left((ac)\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{7/2}}\,dx\right)$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{af}$$

$$= -\frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\text{Subst}\left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2}\,dx, x,\right.}{2a^2f}$$

$$= -\frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx}{8a^2f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{7c\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{16a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx}{8a^2f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{5/2}f} -$$

Mathematica [A]
time = 1.49, size = 134, normalized size = 0.91

$$-\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\left((3+8\cos(e+fx)-11\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)-128\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\sqrt{-1+\sec(e+fx)}+92\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{64a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/64*(c*Cot[(e + f*x)/2]*((3 + 8*Cos[e + f*x] - 11*Cos[2*(e + f*x)])*Sec[(
e + f*x)/2]^4 - 128*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]]
 + 92*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x
]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(123) = 246.
time = 0.17, size = 543, normalized size = 3.67

method	result

default	$-\frac{c\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(16\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(16*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos
(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+23*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e
)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+32*sin(f*x+e)*cos(f*x+e)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)+46*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin
(f*x+e))*cos(f*x+e)*sin(f*x+e)+16*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)+23*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((
sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))-2
2*cos(f*x+e)^3+8*cos(f*x+e)^2+14*cos(f*x+e))/(cos(f*x+e)+1)^2/sin(f*x+e)/a^
3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(131) = 262.
time = 4.86, size = 656, normalized size = 4.43

$$\left[-\frac{23\sqrt{2}\left(c\cos(fx+e)^3+3c\cos(fx+e)^2+3c\cos(fx+e)+c\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+32\left(c\cos(fx+e)^3+3c\cos(fx+e)^2+3c\cos(fx+e)+c\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+4\left(11c\cos(fx+e)^2+7c\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{32\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}, \frac{23\sqrt{2}\left(c\cos(fx+e)^3+3c\cos(fx+e)^2+3c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-32\left(c\cos(fx+e)^3+3c\cos(fx+e)^2+3c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(11c\cos(fx+e)^2+7c\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{16\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e
) + c)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
```

```
+ e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a
)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 32*(c*cos(f*x + e)^3 + 3*c*cos(f
*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt
(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*
cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(11*c*cos(f*x + e)^2 + 7*c*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/16*(23
*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - 32*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos
(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(11*c*cos(f*x + e)^2 + 7*c*cos(f*x + e)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{\sec{\left(e + fx \right)}}{a^{2}\sqrt{a\sec{\left(e + fx \right)} + a}\sec^{2}{\left(e + fx \right)} + 2a^{2}\sqrt{a\sec{\left(e + fx \right)} + a}\sec{\left(e + fx \right)} + a^{2}\sqrt{a\sec{\left(e + fx \right)} + a}}\, dx + \int \left(- \frac{1}{a^{2}\sqrt{a\sec{\left(e + fx \right)} + a}\sec^{2}{\left(e + fx \right)} + 2a^{2}\sqrt{a\sec{\left(e + fx \right)} + a}\sec{\left(e + fx \right)} + a^{2}\sqrt{a\sec{\left(e + fx \right)} + a}}\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c*(Integral(sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 +
2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a
)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**
2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x
))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c-\frac{c}{\cos\left(e+f\,x\right)}}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=230

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf}-\frac{71\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}\,a^{5/2}cf}-\frac{7\cot(e+fx)\sqrt{a+a\sec(e+f}}{32a^3cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f-71/64*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f*2^(1/2
)-7/32*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+13/32*cos(f*x+e)*cot(f*x+e
)*sec(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+1/16*cos(f*x+e)^2*cot
(f*x+e)*sec(1/2*f*x+1/2*e)^4*(a+a*sec(f*x+e))^(1/2)/a^3/c/f
```

Rubi [A]
time = 0.21, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3989, 3972, 483, 593, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}cf}-\frac{71\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}\,a^{5/2}cf}-\frac{7\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{32a^3cf}+\frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a\sec(e+fx)+a}}{16a^3cf}+\frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a\sec(e+fx)+a}}{32a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c*f) -
 (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32
*Sqrt[2]*a^(5/2)*c*f) - (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(32*a^3*c
*f) + (13*Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f
*x]])/(32*a^3*c*f) + (Cos[e + f*x]^2*Cot[e + f*x]*Sec[(e + f*x)/2]^4*Sqrt[a
 + a*Sec[e + f*x]])/(16*a^3*c*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\cot^2(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3cf}$$

$$= \frac{\cos^2(e+fx)\cot(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e+}}{16a^3cf}$$

$$= \frac{13\cos(e+fx)\cot(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{a+a\sec(e}}{32a^3cf}$$

$$= -\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{13\cos(e+fx)\cot(}{}$$

$$= -\frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{13\cos(e+fx)\cot(}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf} - \frac{71\tan^{-1}\left(\frac{\sqrt{a}\,\text{ta}}{\sqrt{2}\sqrt{a+a}}\right.}{32\sqrt{2}a^{5/2}}$$

Mathematica [A]
time = 1.51, size = 158, normalized size = 0.69

$$\frac{\left(13+24\cos(e+fx)+27\cos(2(e+fx))+512\text{ArcTan}\left(\sqrt{-1+\sec(e+fx)}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}-284\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+fx)}\right)\tan^3\left(\frac{1}{2}(e+fx)\right)}{64a^2cf(-1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

```
[Out] ((13 + 24*Cos[e + f*x] + 27*Cos[2*(e + f*x)] + 512*ArcTan[Sqrt[-1 + Sec[e +
 f*x]]]*Cos[(e + f*x)/2]^4*Sqrt[-1 + Sec[e + f*x]] - 284*Sqrt[2]*ArcTan[Sqr
t[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x)/2]^4*Sqrt[-1 + Sec[e + f*x]])*T
an[(e + f*x)/2]^3)/(64*a^2*c*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*
x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(199) = 398$.
time = 0.24, size = 545, normalized size = 2.37

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))^2\left(-64\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(-64*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)-71*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-128*sin(f*x+e)*c
os(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-142*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)-64*2^(1/2)*arctanh(1/2*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+54*cos(f*x+e)^3-71*sin(f*x+e)*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))+24*cos(f*x+e)^2-14*cos(f*x+e))/sin(f*x+e
)^5/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)), x)
```

Fricas [A]
time = 3.69, size = 662, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/128*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
```

```
*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sq
rt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e)
 + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(27*cos(f*x + e)^3 + 12*cos(f*x
+ e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c*f
*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/64*(71
*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin
(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*c
os(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(27*cos(f*x + e)^3 +
12*cos(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
)/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec^{3}\left(e+fx\right)+a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec^{2}\left(e+fx\right)-a^{2}\sqrt{a\sec\left(e+fx\right)+a}\sec\left(e+fx\right)-a^{2}\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 - a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
) - a**2*sqrt(a*sec(e + f*x) + a)), x)/c
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))), x)
```

3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=269

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f}-\frac{107\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{64\sqrt{2}\,a^{5/2}c^2f}+\frac{21\cot(e+fx)\sqrt{a+a\sec(e+}}{64a^3c^2f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f+43/96*cot
(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-15/32*cos(f*x+e)*cot(f*x+e)^3*se
c(1/2*f*x+1/2*e)^2*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-1/16*cos(f*x+e)^2*cot(f
*x+e)^3*sec(1/2*f*x+1/2*e)^4*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-107/128*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f*2^(1
/2)+21/64*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c^2/f
```

Rubi [A]

time = 0.23, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3989, 3972, 483, 593, 597, 536, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}c^2f}-\frac{107\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{64\sqrt{2}\,a^{5/2}c^2f}+\frac{43\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{96a^4c^2f}-\frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{3/2}}{16a^4c^2f}-\frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a\sec(e+fx)+a)^{3/2}}{32a^4c^2f}+\frac{21\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{64a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*c^2*f)
 - (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/
(64*Sqrt[2]*a^(5/2)*c^2*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(64
*a^3*c^2*f) + (43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(96*a^4*c^2*f)
 - (15*Cos[e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^
(3/2))/(32*a^4*c^2*f) - (Cos[e + f*x]^2*Cot[e + f*x]^3*Sec[(e + f*x)/2]^4*(
a + a*Sec[e + f*x])^(3/2))/(16*a^4*c^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{2\mathrm{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{a^4c^2f}$$

$$= -\frac{\cos^2(e+fx)\cot^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(a+a\sec(e}{16a^4c^2f}$$

$$= -\frac{15\cos(e+fx)\cot^3(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)(a+a\sec(}{32a^4c^2f}$$

$$= \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} - \frac{15\cos(e+fx)\,\mathrm{co}}{}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a}{96a^4}$$

$$= \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a}{96a^4}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f} - \frac{107\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a+}}\right.}{64\sqrt{2}a^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.26, size = 5650, normalized size = 21.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(234) = 468.
time = 0.30, size = 725, normalized size = 2.70

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos(fx+e)+1)(-1+\cos(fx+e))^2\left(384(\cos^3(fx+e))\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(}{\cos(fx}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/c^2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)*(-1+cos(f*x+
e))^2*(384*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arc
tanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2)
)*2^(1/2)+321*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)
^3+384*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e)*2^
(1/2)+321*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-3
84*sin(f*x+e)*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)-
321*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)*sin(f*x+e)-384*2^(1/
2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2
^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-410*cos(f*x+e)^4-32
1*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))-142*cos(f*x+e)^3+298*c
os(f*x+e)^2+126*cos(f*x+e))/sin(f*x+e)^7/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)^2), x)
```

Fricas [A]
time = 4.66, size = 768, normalized size = 2.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/768*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f
*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 384*(cos(f*x + e)^3 + cos(f
*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*
(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*x + e)^2 - 63*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/384*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*
sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 384*(cos(f*x + e)^3 + cos(f*x + e)^
2 - cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e)
- a))*sin(f*x + e) + 2*(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*
x + e)^2 - 63*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*
c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) -
a^3*c^2*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2\sqrt{a\sec{(e+fx)}+a}\,\sec^4{(e+fx)}-2a^2\sqrt{a\sec{(e+fx)}+a}\,\sec^2{(e+fx)}+a^2\sqrt{a\sec{(e+fx)}+a}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a**2*sqrt(a*s
ec(e + f*x) + a)*sec(e + f*x)**2 + a**2*sqrt(a*sec(e + f*x) + a)), x)/c**2
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2),x)
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2), x)
```

3.86 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=185

$$\frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{ac^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac^2(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a+a\sec(e+}}$$

```
[Out] -1/2*a*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-1/3*a
*c*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a*c^4*ln(cos(
f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a*c^3*(c
-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3991, 3990, 3556}

$$\frac{ac^4\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{ac^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{ac^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[a + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(
3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
```

```
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\,dx &= -\frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+c\int\sqrt{a+a\sec\\
&= -\frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e}{3f\sqrt{a}}\\
&= -\frac{ac^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac^2(c-c\sec(e}{2f\sqrt{a}}\\
&= -\frac{ac^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac^2(c-c\sec(e}{2f\sqrt{a}}\\
&= \frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{ac^3\sqrt{c-}}{f\sqrt{}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.43, size = 149, normalized size = 0.81

$$\frac{c^3\csc\left(\frac{1}{2}(e+fx)\right)\left(-22-18\cos(2(e+fx))+3ifx\cos(3(e+fx))+9\cos(e+fx)\left(2+ifx-\log\left(1+e^{2i(e+fx)}\right)\right)-3\cos(3(e+fx))\log\left(1+e^{2i(e+fx)}\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (c^3*Csc[(e + f*x)/2]*(-22 - 18*Cos[2*(e + f*x)] + (3*I)*f*x*Cos[3*(e + f*x
)] + 9*Cos[e + f*x]*(2 + I*f*x - Log[1 + E^((2*I)*(e + f*x))]) - 3*Cos[3*(e
 + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(24*f)
```

Maple [A]
time = 0.73, size = 194, normalized size = 1.05

method	result
default	$\frac{\left(6\left(\cos^3(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-6\left(\cos^3(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-6\left(\cos^3(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-29}{6f\sin(fx+e)(-1+\cos(fx+e)}$
risch	$\frac{c^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2c^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6/f*(6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^3*ln((-cos(f*x+e)+1 +sin(f*x+e))/sin(f*x+e))-6*cos(f*x+e)^3*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f *x+e))-29*cos(f*x+e)^3-18*cos(f*x+e)^2+9*cos(f*x+e)-2)*cos(f*x+e)*(c*(-1+co s(f*x+e))/cos(f*x+e))^(7/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/ (-1+cos(f*x+e))^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1393 vs. $2(178) = 356$.
time = 0.63, size = 1393, normalized size = 7.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim a")

[Out] -1/3*(3*(f*x + e)*c^3*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*c^3*cos(4*f*x + 4*e)^2 + 27*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*c^3*sin(6*f*x + 6*e)^2 + 27*(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*c^3*sin(2*f*x + 2* e)^2 + 18*(f*x + e)*c^3*cos(2*f*x + 2*e) + 3*(f*x + e)*c^3 + 18*c^3*sin(2*f *x + 2*e) - 3*(c^3*cos(6*f*x + 6*e)^2 + 9*c^3*cos(4*f*x + 4*e)^2 + 9*c^3*co s(2*f*x + 2*e)^2 + c^3*sin(6*f*x + 6*e)^2 + 9*c^3*sin(4*f*x + 4*e)^2 + 18*c ^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*c^3*sin(2*f*x + 2*e)^2 + 6*c^3*cos (2*f*x + 2*e) + c^3 + 2*(3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) + c^3)*cos(6*f*x + 6*e) + 6*(3*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e) + 6*(c^3*sin(4*f*x + 4*e) + c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*c^3*cos(4*f*x + 4* e) + 3*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 + 3*c^3*sin(4*f*x + 4 *e) + 3*c^3*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*cos(2* f*x + 2*e) + (f*x + e)*c^3)*cos(4*f*x + 4*e) + 18*(c^3*sin(6*f*x + 6*e) + 3 *c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 44*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4* e) + 3*c^3*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f* x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*((f*x + e)*c^3*sin(4*f*x + 4*e) + (f*x + e)*c^3*sin(2*f*x + 2*e) - c^3*cos(4*f*x + 4*e) - c^3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*sin(2* f*x + 2*e) + c^3)*sin(4*f*x + 4*e) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4 *f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 44*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) +

```
 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*
f*x + 2*e) + c^3)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqr
t(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9
*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e) + 1)*f)
```

Fricas [A]
time = 3.99, size = 497, normalized size = 2.69

$$\left[-\frac{\left(11\,c^{3}\cos\left(fx+e\right)^{2}-7\,c^{3}\cos\left(fx+e\right)+2\,c^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-3\left(c^{3}\cos\left(fx+e\right)^{3}+c^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{-ac}\log\left(\frac{ac\cos\left(fx+e\right)^{4}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+ac}{2\cos\left(fx+e\right)^{2}}\right)}{6\left(f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)^{2}\right)},\ -\frac{\left(11\,c^{3}\cos\left(fx+e\right)^{2}-7\,c^{3}\cos\left(fx+e\right)+2\,c^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-6\left(c^{3}\cos\left(fx+e\right)^{3}+c^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)}{ac\cos\left(fx+e\right)^{2}+ac}\right)}{6\left(f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) - 3*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c*co
s(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6*((11*c
^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 6*(c^3*
cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^3 + f*cos(
f*x + e)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2), x)
```

3.87 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=139

$$\frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a+a\sec(e+}}$$

```
[Out] -1/2*a*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a*c^3*l
n(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a*
c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3991, 3990, 3556}

$$\frac{ac^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]]) - (a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[a + a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx &= -\frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}+c\int \sqrt{a+a\sec(\\ &= -\frac{ac^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e+}{2f\sqrt{a+}\\ &= -\frac{ac^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e+}{2f\sqrt{a+}\\ &= \frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\sqrt{c-c}}{f\sqrt{}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.24, size = 162, normalized size = 1.17

$$-\frac{c^2e^{-3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^3\left(i+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(-1-ifx+4\cos(e+fx)+\log\left(1+e^{2i(e+fx)}\right)+\cos(2(e+fx))\left(-ifx+\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{16\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/16*(c^2*(1 + E^((2*I)*(e + f*x)))^3*(I + Cot[(e + f*x)/2])*(-1 - I*f*x +
 4*Cos[e + f*x] + Log[1 + E^((2*I)*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x
 + Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]
*Sqrt[c - c*Sec[e + f*x]])/(E^((3*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A]
time = 0.27, size = 184, normalized size = 1.32

method	result
default	$-\frac{\left(2\left(\cos^2(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+2\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-2\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+5}{2f\sin(fx+e)(-1+\cos(fx+e))^2}$
risch	$\frac{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*cos(f*x+e)^2*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+
e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^2*ln(2/(cos(f*x
+e)+1))+5*cos(f*x+e)^2+4*cos(f*x+e)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*
x+e))^(5/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))^
2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(135) = 270$.
time = 0.59, size = 767, normalized size = 5.52

$$-\frac{\left((fx+e)c^2\cos(4fx+4e)^2+4(fx+e)c^2\cos(2fx+2e)^2+(fx+e)c^2\sin(4fx+4e)^2+4(fx+e)c^2\sin(2fx+2e)^2+4(fx+e)c^2\cos(2fx+2e)+(fx+e)c^2+2c^2\sin(2fx+2e)-\left(c^2\cos(4fx+4e)^2+4c^2\cos(2fx+2e)^2+c^2\sin(4fx+4e)^2+4c^2\sin(4fx+4e)\sin(2fx+2e)+4c^2\sin(2fx+2e)^2+4c^2\cos(2fx+2e)+c^2+2\left(2c^2\cos(2fx+2e)+c^2\right)\cos(4fx+4e)\right)\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)+2\left(2(fx+e)c^2\cos(2fx+2e)+(fx+e)c^2+c^2\sin(2fx+2e)\right)\cos(4fx+4e)+4\left(c^2\sin(4fx+4e)+2c^2\sin(2fx+2e)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)\right)\right)+4\left(c^2\sin(4fx+4e)+2c^2\sin(2fx+2e)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)\right)\right)+2\left(2(fx+e)c^2\sin(2fx+2e)-c^2\cos(2fx+2e)\right)\sin(4fx+4e)-4\left(c^2\cos(4fx+4e)+2c^2\cos(2fx+2e)+c^2\right)\sin\left(\frac{3}{2}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)\right)\right)-4\left(c^2\cos(4fx+4e)+2c^2\cos(2fx+2e)+c^2\right)\sin\left(\frac{1}{2}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)\right)\right)\right)\sqrt{a}\sqrt{c}}{\left(2\left(2\cos(2fx+2e)+1\right)\cos(4fx+4e)+\cos(4fx+4e)^2+4\cos(2fx+2e)^2+\sin(4fx+4e)^2+4\sin(4fx+4e)\sin(2fx+2e)+4\sin(2fx+2e)^2+4\cos(2fx+2e)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e)^2 + (
f*x + e)*c^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + 2*c^2*sin(2*f*x + 2*e) - (c^
2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 +
4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*
cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*c^2*cos(2
*f*x + 2*e) + (f*x + e)*c^2 + c^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(c
^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*c^2*
sin(2*f*x + 2*e) - c^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 4*(c^2*cos(4*f*
x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2
*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f
*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*
sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 3.41, size = 461, normalized size = 3.32

$$\left[-\frac{\left(3c^2\cos(fx+e)-c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2\left(f\cos(fx+e)^2+f\cos(fx+e)\right)},\ -\frac{\left(3c^2\cos(fx+e)-c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)-2\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{2\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e)^2 +
 c^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3
 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*co
s(f*x + e)^2 + f*cos(f*x + e)), -1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) - 2*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x
+ e)^2 + f*cos(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.88 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=93

$$\frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] a*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-a*c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3991, 3990, 3556}

$$\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a +
 a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx = -\frac{ac\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + c\int \sqrt{a+a\sec(e
$$
$$= -\frac{ac\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(ac^2\tan(e+f}{\sqrt{a+a\sec(e+}$$
$$= \frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{c-cs}{f\sqrt{a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.33, size = 99, normalized size = 1.06

$$\frac{ic\left(i+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(i+\cos(e+fx)\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*c*(I + Cot[(e + f*x)/2])*(I + Cos[e + f*x]*(f*x + I*Log[1 + E^((2*I)*(e
+ f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/((1 + E^(I*
(e + f*x)))*f)
```

Maple [A]
time = 0.25, size = 164, normalized size = 1.76

method	result
default	$-\frac{\left(\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+1\right)}{f\sin(fx+e)(-1+\cos(fx+e))}$
risch	$\frac{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$ –

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(-(
cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*
x+e)+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*(a*(cos(f*x+e)+1)/c
os(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(92) = 184$.

time = 0.57, size = 264, normalized size = 2.84

$$-\frac{\left((fx+e)c\cos\left(2\,fx+2\,e\right)^2+(fx+e)c\sin\left(2\,fx+2\,e\right)^2+2\,(fx+e)c\cos\left(2\,fx+2\,e\right)+2\,c\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\sin\left(2\,fx+2\,e\right)+(fx+e)c-\left(c\cos\left(2\,fx+2\,e\right)^2+c\sin\left(2\,fx+2\,e\right)^2+2\,c\cos\left(2\,fx+2\,e\right)+c\right)\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-2\,\left(c\cos\left(2\,fx+2\,e\right)+c\right)\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\right)\sqrt{a}\,\sqrt{c}}{\left(\cos\left(2\,fx+2\,e\right)^2+\sin\left(2\,fx+2\,e\right)^2+2\,\cos\left(2\,fx+2\,e\right)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c*cos(2*f*x + 2*e)^2 + (f*x + e)*c*sin(2*f*x + 2*e)^2 + 2*(f*x
+ e)*c*cos(2*f*x + 2*e) + 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*c - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*
x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1) - 2*(c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]

time = 2.71, size = 380, normalized size = 4.09

$$\left[-\frac{2\,c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-\sqrt{-ac}\left(c\cos\left(fx+e\right)+c\right)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2\left(f\cos\left(fx+e\right)+f\right)},-\frac{c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-\sqrt{ac}\left(c\cos\left(fx+e\right)+c\right)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{f\cos\left(fx+e\right)+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*sin(f*x + e) - sqrt(-a*c)*(c*cos(f*x + e) + c)*log(1/2*(a*c
*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
 e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), -(c*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - s
qrt(a*c)*(c*cos(f*x + e) + c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c(\sec\left(e+fx\right)-1)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.89 $\int \sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=48

$$\frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3990, 3556}

$$\frac{ac\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}\,dx &= -\frac{(ac\tan(e+fx))\int \tan(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}\\ &= \frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.60, size = 102, normalized size = 2.12

$$\frac{ie^{\frac{1}{2}i(e+fx)}\cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (I*E^((I/2)*(e + f*x))*Cos[e + f*x]*Csc[(e + f*x)/2]*(f*x + I*Log[1 + E^((2
*I)*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/((1 +
 E^(I*(e + f*x)))*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(44) = 88$.
time = 0.25, size = 127, normalized size = 2.65

method	result
default	$-\frac{\left(\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)\right)\cos(fx+e)\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\sqrt{\frac{a(\cos(fx\cdots}{\cos(fx\cdots}}}{f\sin(fx+e)}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1))+ln(-(c
os(f*x+e)-1+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+
e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.88

$$-\frac{\left(fx-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)+e\right)\sqrt{a}\,\sqrt{c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(f*x - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + e)*sqrt(a)*sqrt(c
)/f
```

Fricas [A]
time = 2.75, size = 216, normalized size = 4.50

$$\left[\frac{\sqrt{-ac}\,\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\,\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2f},\ \frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e
))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/f, sqrt(a*c)*arctan(
sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*ln(1-cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3996, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{-c+cx}\,dx,x,\cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.93, size = 86, normalized size = 1.69

$$-\frac{\left(-1+e^{i(e+fx)}\right)\left(fx+2i\log\left(1-e^{i(e+fx)}\right)\right)\sqrt{a(1+\sec(e+fx))}}{\left(1+e^{i(e+fx)}\right)f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -(((-1 + E^(I*(e + f*x)))*(f*x + (2*I)*Log[1 - E^(I*(e + f*x))])*Sqrt[a*(1
+ Sec[e + f*x])])/((1 + E^(I*(e + f*x)))*f*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(47) = 94$.
time = 0.27, size = 100, normalized size = 1.96

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\cos(fx+e)}{f\sin(fx+e)c}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln(e^{i(f}}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2*ln(-(-1+cos(f*x+e))/sin(f*x+e))
-ln(2/(cos(f*x+e)+1)))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(
f*x+e)/c
```

Maxima [A]
time = 0.49, size = 69, normalized size = 1.35

$$\frac{\frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}-\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1))/sqrt(c) - sqrt(-a)*log(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e
) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Giac [A]
time = 1.45, size = 72, normalized size = 1.41

$$-\frac{\frac{\sqrt{-ac}\,a\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c|a|}-\frac{\sqrt{-ac}\,a\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right|\right)}{c|a|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] -(sqrt(-a*c)*a*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c*abs(a)) - sqrt(-a*c)*a
*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c*abs(a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*
x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3992, 3996, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)))
 + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
```

```
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$
$$= -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{-c-\cdots}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{\cdots}}$$
$$= -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.12, size = 107, normalized size = 1.11

$$\frac{\left(-1+ifx-2\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(-ifx+2\log\left(1-e^{i(e+fx)}\right)\right)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + I*f*x - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*((-I)*f*x + 2*Log[
1 - E^(I*(e + f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x
)/2])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Maple [A]
time = 0.25, size = 162, normalized size = 1.69

method	result
default	$\frac{(-1+\cos(fx+e))\left(2\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-4\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+4\ln\left(-\frac{-1+\cos\cdots}{\sin(\cdots}\right.\right.}{2f\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)x}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}} - \frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} + \frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}\cdots}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}\cdots\right.}{e^{2i(fx+e}\cdots}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))*(2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-4*cos(f*x+e)*ln(-(
-1+cos(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))+cos(f*x+e)+4*ln(-(-1+cos(
```

```
f*x+e))/sin(f*x+e))+1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(-
1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(95) = 190$.
time = 0.57, size = 440, normalized size = 4.58

$$-\frac{\left((fx+e)\cos\left(2\,fx+2\,e\right)^{2}+4\,(fx+e)\cos\left(fx+e\right)^{2}+(fx+e)\sin\left(2\,fx+2\,e\right)^{2}+4\,(fx+e)\sin\left(fx+e\right)^{2}+fx+2\,\left(2\,\left(2\,\cos\left(fx+e\right)-1\right)\cos\left(2\,fx+2\,e\right)-\cos\left(2\,fx+2\,e\right)^{2}-4\,\cos\left(fx+e\right)^{2}-\sin\left(2\,fx+2\,e\right)^{2}+4\,\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-4\,\sin\left(fx+e\right)^{2}+4\,\cos\left(fx+e\right)-1\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)+2\,\left(fx-2\,(fx+e)\cos\left(fx+e\right)+e+\sin\left(fx+e\right)\right)\cos\left(2\,fx+2\,e\right)-4\,(fx+e)\cos\left(fx+e\right)-2\,\left(2\,(fx+e)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\sin\left(2\,fx+2\,e\right)+e+2\,\sin\left(fx+e\right)\right)\sqrt{a}\,\sqrt{c}}{\left(c^{2}\cos\left(2\,fx+2\,e\right)^{2}+4\,c^{2}\cos\left(fx+e\right)^{2}+c^{2}\sin\left(2\,fx+2\,e\right)^{2}-4\,c^{2}\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+4\,c^{2}\sin\left(fx+e\right)^{2}-4\,c^{2}\cos\left(fx+e\right)+c^{2}-2\,\left(2\,c^{2}\cos\left(fx+e\right)-c^{2}\right)\cos\left(2\,fx+2\,e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin
(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(2*cos(f*x + e) -
 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x + e)^2 - sin(2*f*x +
2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e)^2 + 4*cos(f*x + e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 2*(f*x + e)*cos(f
*x + e) + e + sin(f*x + e))*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) - 2
*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e + 2*sin(f*x
 + e))*sqrt(a)*sqrt(c)/((c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(f*x + e)^2 + c^
2*sin(2*f*x + 2*e)^2 - 4*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(f*x + e) - c^2)*cos(2*f*x
+ 2*e))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x +
e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [A]
time = 1.57, size = 147, normalized size = 1.53

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-ac}\,a\log\left(2\left|a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right|\right)}{c^{2}|a|}-\frac{2\sqrt{2}\sqrt{-ac}\,a\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right|\right)}{c^{2}|a|}-\frac{\sqrt{2}\left(2\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right)\sqrt{-ac}\,a+\sqrt{-ac}\,a^{2}\right)}{ac^{2}|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a*c)*a*log(2*abs(a*tan(1/2*f*x + 1/2*e)^2))/(
c^2*abs(a)) - 2*sqrt(2)*sqrt(-a*c)*a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a)
)/(c^2*abs(a)) - sqrt(2)*(2*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a + s
qrt(-a*c)*a^2)/(a*c^2*abs(a)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}-\frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}+\frac{a\log(1}{c^2f\sqrt{a+a\,\mathrm{s}}}$$

```
[Out] -1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*x+e))*ta
n(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3992, 3996, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)) - (a*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3
/2)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+}{cf\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+}{cf\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+}{cf\sqrt{a+a\sec(e+fx)}\,(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.36, size = 152, normalized size = 1.07

$$\frac{\left(3-3ifx+\cos(e+fx)\left(-4+4ifx-8\log\left(1-e^{i(e+fx)}\right)\right)+6\log\left(1-e^{i(e+fx)}\right)+\cos(2(e+fx))\left(-ifx+2\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{2c^2f(-1+\cos(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((3 - (3*I)*f*x + Cos[e + f*x]*(-4 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x))]
) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E^(
I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(-1 +
 Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.27, size = 226, normalized size = 1.59

method	result
default	$\frac{(-1+\cos(fx+e))\left(8\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-16\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+7\left(\cos^2(fx+e)\right)-16\cos(fx+e)\ln\left(\frac{}{\cos(f}\right.\right.}{8f\sin(fx+e)\cos(fx+e)}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(2e^{3i(fx+e)}-3e^{2i}\right.}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c(e^{i}}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-16*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+7*cos(f*x+e)^2-16*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+32*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*cos(f*x+e)+8*ln(2/(cos(f*x+e)+1))-16*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(138) = 276$.
time = 0.75, size = 1288, normalized size = 9.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x + e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 4*(f*x + e)*cos(3*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) + e + 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*cos(4*f*x + 4*e) - 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) + e)*cos(3*f*x + 3*e) + 12*(f*x - 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) - 8*(f*x + e)*cos(f*x + e) - 2*(4*(f*x + e)*sin(3*f*x + 3*e) - 6*(f*x + e)*sin(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) - 3*cos(2*f*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) - 4*(12*(f*x + e)*sin(2*f*x + 2*e) - 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) - 6*(8*(f*x + e)*sin(f*x + e) + 1)*sin(2*f*x + 2*e) + e + 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*f*x + 2*e)^2 + 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(3*f*x + 3*e)^2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*c

```
^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*f*x + 3*e) -
6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x + 4*e) - 8*(6*
c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3*e) - 12*(4*c
^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x + 3*e) - 3*c^3
*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*(3*c^3*sin(2*
f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x +
 e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

Giac [A]
time = 1.85, size = 178, normalized size = 1.25

$$-\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-ac}\,a\log\left(2\left|a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right|\right)}{c^{3}|a|}-\frac{8\sqrt{2}\sqrt{-ac}\,a\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right|\right)}{c^{3}|a|}-\frac{\sqrt{2}\left(12\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right)^{2}\sqrt{-ac}\,a+18\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right)\sqrt{-ac}\,a^{2}+7\sqrt{-ac}\,a^{3}\right)}{a^{2}c^{3}|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}}\right)}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a*c)*a*log(2*abs(a*tan(1/2*f*x + 1/2*e)^2))/
(c^3*abs(a)) - 8*sqrt(2)*sqrt(-a*c)*a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a
))/(c^3*abs(a)) - sqrt(2)*(12*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a
 + 18*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^2 + 7*sqrt(-a*c)*a^3)/(a^
2*c^3*abs(a)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}-\frac{}{c^2f\sqrt{a+}}$$

```
[Out] -1/3*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a*tan
(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a*tan(f*x+e)/c^2/
f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a*ln(1-cos(f*x+e))*tan(f*x+
e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3992, 3996, 31}

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/3*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2
)) - (a*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^
(5/2)) - (a*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2)) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
```

```
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} + \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+f}{2cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+f}{2cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+f}{2cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

$$= -\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+f}{2cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.95, size = 198, normalized size = 1.05

$$\frac{\left(-40+30ifx-3ifx\cos(3(e+fx))+18i\cos(2(e+fx))\left(i+fx+2i\log\left(1-e^{i(e+fx)}\right)\right)-60\log\left(1-e^{i(e+fx)}\right)+6\cos(3(e+fx))\log\left(1-e^{i(e+fx)}\right)+9\cos(e+fx)\left(6-5ifx+10\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{12c^3f(-1+\cos(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((-40 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (18*I)*Cos[2*(e + f*x)]*(
I + f*x + (2*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))] + 6
*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + 9*Cos[e + f*x]*(6 - (5*I)*f*x
+ 10*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]
)/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.27, size = 288, normalized size = 1.53

method	result

default	$\frac{(-1+\cos(fx+e))\left(6(\cos^3(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-12(\cos^3(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+7(\cos^3(fx+e))-18(\cos^2(fx+e))\ln\left(\frac{}{\cos}\right.\right.}{}$
risch	$\frac{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)x}{c^3(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}}-\frac{2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)(fx+e)}{c^3(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2i\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(9\,e^{5i(fx+e)}-27\,e^{4}}{3c^3(e^{i(fx+e)}+1)(e^{i(fx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(-1+cos(f*x+e))*(6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-12*cos(f*x+e)^3*
ln(-(-1+cos(f*x+e))/sin(f*x+e))+7*cos(f*x+e)^3-18*cos(f*x+e)^2*ln(2/(cos(f*
x+e)+1))+36*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3*cos(f*x+e)^2+18*
cos(f*x+e)*ln(2/(cos(f*x+e)+1))-36*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e
))-6*cos(f*x+e)-6*ln(2/(cos(f*x+e)+1))+12*ln(-(-1+cos(f*x+e))/sin(f*x+e))+4
)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/
sin(f*x+e)/cos(f*x+e)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2681 vs. $2(181) = 362$.
time = 3.32, size = 2681, normalized size = 14.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*cos(6*f*x + 6*e)^2 + 108*(f*x + e)*cos(5*f*x + 5*e)^2 + 6
75*(f*x + e)*cos(4*f*x + 4*e)^2 + 1200*(f*x + e)*cos(3*f*x + 3*e)^2 + 675*(
f*x + e)*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*cos(f*x + e)^2 + 3*(f*x + e)*si
n(6*f*x + 6*e)^2 + 108*(f*x + e)*sin(5*f*x + 5*e)^2 + 675*(f*x + e)*sin(4*f
*x + 4*e)^2 + 1200*(f*x + e)*sin(3*f*x + 3*e)^2 + 675*(f*x + e)*sin(2*f*x +
 2*e)^2 + 108*(f*x + e)*sin(f*x + e)^2 + 3*f*x + 6*(2*(6*cos(5*f*x + 5*e) -
 15*cos(4*f*x + 4*e) + 20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6*cos(f*
x + e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 + 12*(15*cos(4*f*x + 4*e)
 - 20*cos(3*f*x + 3*e) + 15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(5*f*
x + 5*e) - 36*cos(5*f*x + 5*e)^2 + 30*(20*cos(3*f*x + 3*e) - 15*cos(2*f*x +
 2*e) + 6*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - 225*cos(4*f*x + 4*e)^2 + 40*
(15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 400*cos(3*f*x
 + 3*e)^2 + 30*(6*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 225*cos(2*f*x + 2*e)
^2 - 36*cos(f*x + e)^2 + 2*(6*sin(5*f*x + 5*e) - 15*sin(4*f*x + 4*e) + 20*s
in(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(6*f*x + 6*e) -
```

```
sin(6*f*x + 6*e)^2 + 12*(15*sin(4*f*x + 4*e) - 20*sin(3*f*x + 3*e) + 15*sin
(2*f*x + 2*e) - 6*sin(f*x + e))*sin(5*f*x + 5*e) - 36*sin(5*f*x + 5*e)^2 +
30*(20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(4*f*x +
 4*e) - 225*sin(4*f*x + 4*e)^2 + 120*(5*sin(2*f*x + 2*e) - 2*sin(f*x + e))*
sin(3*f*x + 3*e) - 400*sin(3*f*x + 3*e)^2 - 225*sin(2*f*x + 2*e)^2 + 180*si
n(2*f*x + 2*e)*sin(f*x + e) - 36*sin(f*x + e)^2 + 12*cos(f*x + e) - 1)*arct
an2(sin(f*x + e), cos(f*x + e) - 1) + 2*(3*f*x - 18*(f*x + e)*cos(5*f*x + 5
*e) + 45*(f*x + e)*cos(4*f*x + 4*e) - 60*(f*x + e)*cos(3*f*x + 3*e) + 45*(f
*x + e)*cos(2*f*x + 2*e) - 18*(f*x + e)*cos(f*x + e) + 3*e + 9*sin(5*f*x +
5*e) - 27*sin(4*f*x + 4*e) + 40*sin(3*f*x + 3*e) - 27*sin(2*f*x + 2*e) + 9*
sin(f*x + e))*cos(6*f*x + 6*e) - 6*(6*f*x + 90*(f*x + e)*cos(4*f*x + 4*e) -
 120*(f*x + e)*cos(3*f*x + 3*e) + 90*(f*x + e)*cos(2*f*x + 2*e) - 36*(f*x +
 e)*cos(f*x + e) + 6*e - 9*sin(4*f*x + 4*e) + 20*sin(3*f*x + 3*e) - 9*sin(2
*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(15*f*x - 300*(f*x + e)*cos(3*f*x + 3*e)
+ 225*(f*x + e)*cos(2*f*x + 2*e) - 90*(f*x + e)*cos(f*x + e) + 15*e + 20*si
n(3*f*x + 3*e) - 9*sin(f*x + e))*cos(4*f*x + 4*e) - 120*(f*x + 15*(f*x + e)
*cos(2*f*x + 2*e) - 6*(f*x + e)*cos(f*x + e) + e + sin(2*f*x + 2*e) - sin(f
*x + e))*cos(3*f*x + 3*e) + 18*(5*f*x - 30*(f*x + e)*cos(f*x + e) + 5*e - 3
*sin(f*x + e))*cos(2*f*x + 2*e) - 36*(f*x + e)*cos(f*x + e) - 2*(18*(f*x +
e)*sin(5*f*x + 5*e) - 45*(f*x + e)*sin(4*f*x + 4*e) + 60*(f*x + e)*sin(3*f*
x + 3*e) - 45*(f*x + e)*sin(2*f*x + 2*e) + 18*(f*x + e)*sin(f*x + e) + 9*co
s(5*f*x + 5*e) - 27*cos(4*f*x + 4*e) + 40*cos(3*f*x + 3*e) - 27*cos(2*f*x +
 2*e) + 9*cos(f*x + e))*sin(6*f*x + 6*e) - 6*(90*(f*x + e)*sin(4*f*x + 4*e)
 - 120*(f*x + e)*sin(3*f*x + 3*e) + 90*(f*x + e)*sin(2*f*x + 2*e) - 36*(f*x
 + e)*sin(f*x + e) + 9*cos(4*f*x + 4*e) - 20*cos(3*f*x + 3*e) + 9*cos(2*f*x
 + 2*e) - 3)*sin(5*f*x + 5*e) - 6*(300*(f*x + e)*sin(3*f*x + 3*e) - 225*(f*
x + e)*sin(2*f*x + 2*e) + 90*(f*x + e)*sin(f*x + e) + 20*cos(3*f*x + 3*e) -
 9*cos(f*x + e) + 9)*sin(4*f*x + 4*e) - 40*(45*(f*x + e)*sin(2*f*x + 2*e) -
 18*(f*x + e)*sin(f*x + e) - 3*cos(2*f*x + 2*e) + 3*cos(f*x + e) - 2)*sin(3
*f*x + 3*e) - 54*(10*(f*x + e)*sin(f*x + e) - cos(f*x + e) + 1)*sin(2*f*x +
 2*e) + 3*e + 18*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 3
6*c^4*cos(5*f*x + 5*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 400*c^4*cos(3*f*x +
 3*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x + e)^2 + c^4*sin(6*f*
x + 6*e)^2 + 36*c^4*sin(5*f*x + 5*e)^2 + 225*c^4*sin(4*f*x + 4*e)^2 + 400*c
^4*sin(3*f*x + 3*e)^2 + 225*c^4*sin(2*f*x + 2*e)^2 - 180*c^4*sin(2*f*x + 2*
e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*(6*
c^4*cos(5*f*x + 5*e) - 15*c^4*cos(4*f*x + 4*e) + 20*c^4*cos(3*f*x + 3*e) -
15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x + 6*e) - 12*(
15*c^4*cos(4*f*x + 4*e) - 20*c^4*cos(3*f*x + 3*e) + 15*c^4*cos(2*f*x + 2*e)
 - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 30*(20*c^4*cos(3*f*x + 3*e)
 - 15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*x + 4*e) - 4
0*(15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*x + 3*e) - 3
0*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 2*(6*c^4*sin(5*f*x + 5*e) -
 15*c^4*sin(4*f*x + 4*e) + 20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e
) + 6*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 12*(15*c^4*sin(4*f*x + 4*e) - 20
```

```
*c^4*sin(3*f*x + 3*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(f*x + e))*sin(5
*f*x + 5*e) - 30*(20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e) + 6*c^4
*sin(f*x + e))*sin(4*f*x + 4*e) - 120*(5*c^4*sin(2*f*x + 2*e) - 2*c^4*sin(f
*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x +
e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c
^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 1.93, size = 208, normalized size = 1.11

$$-\frac{\sqrt{2}\left(\frac{24\,\sqrt{2}\,\sqrt{-ac}\,a\log\left(2\left|a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right|\right)}{c^{4}|a|}-\frac{24\,\sqrt{2}\,\sqrt{-ac}\,a\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right|\right)}{c^{4}|a|}-\frac{\sqrt{2}\left(44\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{3}\sqrt{-ac}\,a+111\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{2}\sqrt{-ac}\,a^{2}+96\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)\sqrt{-ac}\,a^{3}+28\,\sqrt{-ac}\,a^{4}\right)}{a^{3}c^{4}|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/48*sqrt(2)*(24*sqrt(2)*sqrt(-a*c)*a*log(2*abs(a*tan(1/2*f*x + 1/2*e)^2))
/(c^4*abs(a)) - 24*sqrt(2)*sqrt(-a*c)*a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 -
 a))/(c^4*abs(a)) - sqrt(2)*(44*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*c)
*a + 111*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^2 + 96*(a*tan(1/2*f*
x + 1/2*e)^2 - a)*sqrt(-a*c)*a^3 + 28*sqrt(-a*c)*a^4)/(a^3*c^4*abs(a)*tan(1
/2*f*x + 1/2*e)^6))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=190

$$\frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{a^2c(c-c\sec(e+fx))^3}{2f\sqrt{a+a\sec(e}}$$

```
[Out] -1/2*a^2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/3*a
^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+a^2*c^3*ln(co
s(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^2*c^
2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3994, 3991, 3990, 3556}

$$\frac{a^2c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{a^2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[a + a*Sec[e + f*x]]) - (a^2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
```

```
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3994

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx &= \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+a\int\sqrt{a+a\sec(e\\
&= -\frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}+\frac{a^2(c-c\sec(e}{3f\sqrt{a+}}\\
&= -\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{a^2c(c-c\sec(e}{2f\sqrt{a-}}\\
&= -\frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{a^2c(c-c\sec(e}{2f\sqrt{a-}}\\
&= \frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\sqrt{c-}}{f\sqrt{}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.36, size = 157, normalized size = 0.83

$$\frac{iac^2\csc\left(\frac{1}{2}(e+fx)\right)\left(2i+6i\cos(2(e+fx))+3fx\cos(3(e+fx))+\cos(e+fx)\left(6i+9fx+9i\log\left(1+e^{2i(e+fx)}\right)\right)+3i\cos(3(e+fx))\log\left(1+e^{2i(e+fx)}\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/24)*a*c^2*Csc[(e + f*x)/2]*(2*I + (6*I)*Cos[2*(e + f*x)] + 3*f*x*Cos[3*
(e + f*x)] + Cos[e + f*x]*(6*I + 9*f*x + (9*I)*Log[1 + E^((2*I)*(e + f*x))]
) + (3*I)*Cos[3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*S
ec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/f
```

Maple [A]
time = 0.27, size = 189, normalized size = 0.99

method	result
default	$\frac{\left(6(\cos^3(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-6(\cos^3(fx+e))\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-6(\cos^3(fx+e))\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-(c}{6f\sin(fx+e)(-1+\cos(fx+e))^2}$
risch	$\frac{a\,c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2a\,c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(f}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^3*ln((-cos(f*x+e)+1
+sin(f*x+e))/sin(f*x+e))-6*cos(f*x+e)^3*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f
*x+e))-cos(f*x+e)^3-6*cos(f*x+e)^2-3*cos(f*x+e)+2)*(c*(-1+cos(f*x+e))/cos(f
*x+e))^(5/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))
^2*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(183) = 366$.
time = 0.64, size = 1460, normalized size = 7.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a*c^2*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*cos(4*f*x +
 4*e)^2 + 27*(f*x + e)*a*c^2*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a*c^2*sin(6*f
*x + 6*e)^2 + 27*(f*x + e)*a*c^2*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a*c^2*si
n(2*f*x + 2*e)^2 + 18*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a*c^2
- 6*a*c^2*sin(2*f*x + 2*e) - 3*(a*c^2*cos(6*f*x + 6*e)^2 + 9*a*c^2*cos(4*f*
x + 4*e)^2 + 9*a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(6*f*x + 6*e)^2 + 9*a*c^
2*sin(4*f*x + 4*e)^2 + 18*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a*c^2
*sin(2*f*x + 2*e)^2 + 6*a*c^2*cos(2*f*x + 2*e) + a*c^2 + 2*(3*a*c^2*cos(4*f
*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(6*f*x + 6*e) + 6*(3*a*c^2
*cos(2*f*x + 2*e) + a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(4*f*x + 4*e) + a
*c^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1) + 6*(3*(f*x + e)*a*c^2*cos(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*c
os(2*f*x + 2*e) + (f*x + e)*a*c^2 - a*c^2*sin(4*f*x + 4*e) - a*c^2*sin(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + (f*x
+ e)*a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*
x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 4*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a
```

```
*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x
 + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
 e)*a*c^2*sin(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*sin(2*f*x + 2*e) + a*c^2*cos
(4*f*x + 4*e) + a*c^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
*c^2*sin(2*f*x + 2*e) - a*c^2)*sin(4*f*x + 4*e) - 6*(a*c^2*cos(6*f*x + 6*e)
 + 3*a*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) + 3*a
*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(a*c^2*cos(6*f*x + 6*e) + 3*a*c^2*c
os(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 3.44, size = 505, normalized size = 2.66

$$\left[-\frac{\left(7\,ac^2\cos\left(fx+e\right)^2+ac^2\cos\left(fx+e\right)-2\,ac^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-3\left(ac^2\cos\left(fx+e\right)^3+ac^2\cos\left(fx+e\right)^2\right)\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)},-\frac{\left(7\,ac^2\cos\left(fx+e\right)^2+ac^2\cos\left(fx+e\right)-2\,ac^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-6\left(ac^2\cos\left(fx+e\right)^3+ac^2\cos\left(fx+e\right)^2\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) - 3*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*
(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6
*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 - 6*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*
c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x +
 e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=103

$$\frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^2*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+1/2*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3990, 3554, 3556}

$$\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^2*c^2*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{(a^2c^2\tan(e+fx))\int\tan^3(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(a^2c^2}{\sqrt{a+a}}$$
$$= \frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{2f\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.45, size = 159, normalized size = 1.54

$$\frac{iace^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2\left(i+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(i+fx+\cos(2(e+fx))\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)+i\log\left(1+e^{2i(e+fx)}\right)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{8\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((I/8)*a*c*(1 + E^((2*I)*(e + f*x)))^2*(I + Cot[(e + f*x)/2])*(I + f*x + Co
s[2*(e + f*x)]*(f*x + I*Log[1 + E^((2*I)*(e + f*x))]) + I*Log[1 + E^((2*I)*
(e + f*x))])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f
*x]])/(E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A]
time = 0.26, size = 171, normalized size = 1.66

method	result
default	$-\frac{\left(2\left(\cos^2(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+2\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-2\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-}{2f\sin(fx+e)(-1+\cos(fx+e))}$
risch	$\frac{ac\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2ac\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*cos(f*x+e)^2*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+
e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^2*ln(2/(cos(f*x
+e)+1))-cos(f*x+e)^2+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*(a*(cos(f*x+e)
+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(100) = 200.
time = 0.57, size = 518, normalized size = 5.03

$$-\frac{\left((fx+e)ac\cos(4fx+4e)^2+4(fx+e)ac\cos(2fx+2e)^2+(fx+e)ac\sin(4fx+4e)^2+4(fx+e)ac\sin(2fx+2e)^2+4(fx+e)ac\cos(2fx+2e)+(fx+e)ac-2ac\sin(2fx+2e)-\left(ac\cos(4fx+4e)^2+4ac\cos(2fx+2e)^2+ac\sin(4fx+4e)^2+4ac\sin(4fx+4e)\sin(2fx+2e)+4ac\sin(2fx+2e)^2+4ac\cos(2fx+2e)+ac+2(2ac\cos(2fx+2e)+ac)\cos(4fx+4e)\right)\arctan(\sin(2fx+2e),\cos(2fx+2e)+1)+2(2(fx+e)ac\cos(2fx+2e)+(fx+e)ac-ac\sin(2fx+2e))\cos(4fx+4e)+2(2(fx+e)ac\sin(2fx+2e)+ac\cos(2fx+2e))\sin(4fx+4e)\right)\sqrt{a}\sqrt{c}}{\left(2(2\cos(2fx+2e)+1)\cos(4fx+4e)+\cos(4fx+4e)^2+4\cos(2fx+2e)^2+\sin(4fx+4e)^2+4\sin(4fx+4e)\sin(2fx+2e)+4\sin(2fx+2e)^2+4\cos(2fx+2e)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*c*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*cos(2*f*x + 2*e)^2 + (
f*x + e)*a*c*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*a*c*cos(2*f*x + 2*e) + (f*x + e)*a*c - 2*a*c*sin(2*f*x + 2*e) - (a*
c*cos(4*f*x + 4*e)^2 + 4*a*c*cos(2*f*x + 2*e)^2 + a*c*sin(4*f*x + 4*e)^2 +
4*a*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*c*sin(2*f*x + 2*e)^2 + 4*a*c*
cos(2*f*x + 2*e) + a*c + 2*(2*a*c*cos(2*f*x + 2*e) + a*c)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a*c*cos(2
*f*x + 2*e) + (f*x + e)*a*c - a*c*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2
*(f*x + e)*a*c*sin(2*f*x + 2*e) + a*c*cos(2*f*x + 2*e))*sin(4*f*x + 4*e))*s
qrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x +
4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.72, size = 376, normalized size = 3.65

$$\left[\frac{\sqrt{-ac}\,ac\cos(fx+e)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)-ac\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)}{2f\cos(fx+e)}, \frac{2\sqrt{ac}\,ac\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)\cos(fx+e)-ac\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)}{2f\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(-a*c)*a*c*cos(f*x + e)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x +
e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2) -
a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)), 1/2*(2*sqrt(a*c)*a*c*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)
 - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^2*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-a*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3991, 3990, 3556}

$$\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + a\int \sqrt{a+a\sec$$
$$= -\frac{ac\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{(a^2c\tan(e+}{\sqrt{a+a\sec(e+}$$
$$= \frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac\sqrt{a+a}}{f\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.81, size = 128, normalized size = 1.38

$$\frac{ae^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\left(i+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(1+\cos(e+fx)\left(ifx-\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{2\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (a*(1 + E^((2*I)*(e + f*x)))*(I + Cot[(e + f*x)/2])*(1 + Cos[e + f*x]*(I*f*
x - Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*
Sqrt[c - c*Sec[e + f*x]])/(2*E^(I*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A]
time = 0.25, size = 149, normalized size = 1.60

method	result
default	$\frac{\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)+1\right)}{f\sin(fx+e)}$
risch	$\frac{a\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2a\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))-cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e
))/sin(f*x+e))-cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+cos(f*x
+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(
1/2)/sin(f*x+e)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(92) = 184$.
time = 0.58, size = 264, normalized size = 2.84

$$-\frac{\left((fx+e)a\cos(2fx+2e)^2+(fx+e)a\sin(2fx+2e)^2+2(fx+e)a\cos(2fx+2e)-2a\cos\left(\frac{1}{2}\arctan(\sin(2fx+2e),\cos(2fx+2e))\right)\sin(2fx+2e)+(fx+e)a-\left(a\cos(2fx+2e)^2+a\sin(2fx+2e)^2+2a\cos(2fx+2e)+a\right)\arctan(\sin(2fx+2e),\cos(2fx+2e)+1)+2\left(a\cos(2fx+2e)+a\right)\sin\left(\frac{1}{2}\arctan(\sin(2fx+2e),\cos(2fx+2e))\right)\right)\sqrt{a}\sqrt{c}}{\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*cos(2*f*x + 2*e)^2 + (f*x + e)*a*sin(2*f*x + 2*e)^2 + 2*(f*x
+ e)*a*cos(2*f*x + 2*e) - 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*a - (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*
x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1) + 2*(a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.96, size = 377, normalized size = 4.05

$$\left[\frac{2\,a\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+\sqrt{-ac}\left(a\cos\left(fx+e\right)+a\right)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2\left(f\cos\left(fx+e\right)+f\right)},\frac{a\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+\sqrt{ac}\left(a\cos\left(fx+e\right)+a\right)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{f\cos\left(fx+e\right)+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(2*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*sin(f*x + e) + sqrt(-a*c)*(a*cos(f*x + e) + a)*log(1/2*(a*c*
cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), (a*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + sqr
t(a*c)*(a*cos(f*x + e) + a)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=104

$$\frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 78}

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{a+ax}{x(c-cx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{2a}{c(-1+x)}+\frac{a}{cx}\right)dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\log(1-\sec(e+fx))\,\text{ta}}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.25, size = 105, normalized size = 1.01

$$-\frac{a\left(-1+e^{i(e+fx)}\right)\left(fx+4i\log\left(1-e^{i(e+fx)}\right)-i\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sec(e+fx))}}{\left(1+e^{i(e+fx)}\right)f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((a*(-1 + E^(I*(e + f*x)))*(f*x + (4*I)*Log[1 - E^(I*(e + f*x))] - I*Log[1
 + E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])])/((1 + E^(I*(e + f*x)))
*f*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A]
time = 0.23, size = 149, normalized size = 1.43

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-4\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)\right)\sqrt{}}{f\sin(fx+e)c}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}} - \frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} - \frac{4ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)}{e^{2i(fx+e)}+}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(ln((-cos(f*x+e)+1+sin(f*x+e))/sin(
f*x+e))-4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+ln(2/(cos(f*x+e)+1))+ln(-(cos(f*x
+e)-1+sin(f*x+e))/sin(f*x+e)))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x
+e)/sin(f*x+e)/c*a
```

Maxima [A]
time = 0.56, size = 65, normalized size = 0.62

$$-\frac{((fx+e)a+a\arctan(\sin(2fx+2e),\cos(2fx+2e)+1)-4a\arctan(\sin(fx+e),\cos(fx+e)-1))\sqrt{a}}{\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a + a*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*a*arc
tan2(sin(f*x + e), cos(f*x + e) - 1))*sqrt(a)/(sqrt(c)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x +
 e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}+\frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-
cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3993, 3996, 31}

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
```

```
qQ[m + n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{a\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c} \\
&= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{(a^2\tan(e+fx))\,\mathrm{Subst}\big(\int \cdots}{f\sqrt{a+a\sec(e+fx)}\,\cdots} \\
&= -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\cdots}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 115, normalized size = 1.15

$$
\frac{a\left(-2+ifx-2\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(-ifx+2\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{cf(-1+\cos(e+fx))\sqrt{c-c\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
[Out] (a*(-2 + I*f*x - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*((-I)*f*x + 2*Lo
g[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*
(-1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.23, size = 161, normalized size = 1.61

method	result
default	$\frac{(-1+\cos(fx+e))\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+2\ln\left(-\frac{-1+\cos(f\cdots}{\sin(fx+\cdots}\right.\right.}{f\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)}$
risch	$\frac{a\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)x}{c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}}-\frac{2a\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)(fx+e)}{c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{4ia\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+\cdots}}{c(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)\sqrt{\frac{c(e^{i(fx-\cdots}}{e^{2i(fx\cdots}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-1+cos(f*x+e))*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)*ln(-(-1+c
os(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1))+cos(f*x+e)+2*ln(-(-1+cos(f*x+e)
```

```
)/sin(f*x+e))+1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(-1+cos(
f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)*a
```

Maxima [A]
time = 0.53, size = 101, normalized size = 1.01

$$\frac{\frac{2\sqrt{-a}\,a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{-a}\,a\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}\,a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}}\sin(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(3/2) - sqrt(-a)*a*log
(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + sqrt(-a)*a*(cos(f*x + e
) + 1)^2/(c^(3/2)*sin(f*x + e)^2))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x
+ e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [A]
time = 1.68, size = 120, normalized size = 1.20

$$-\frac{\frac{\sqrt{-ac}\,a^2\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c^2|a|}-\frac{\sqrt{-ac}\,a^2\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right|\right)}{c^2|a|}-\frac{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)\sqrt{-ac}\,a}{c^2|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -(sqrt(-a*c)*a^2*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^2*abs(a)) - sqrt(-a*
c)*a^2*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^2*abs(a)) - (a*tan(1/2*f*
x + 1/2*e)^2 - a)*sqrt(-a*c)*a/(c^2*abs(a)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}+\frac{a^2\log(1 -}{c^2f\sqrt{a+a\,\mathrm{se}}}$$

```
[Out] -a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^2*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-cos(f*x+e))*
tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3993, 3992, 3996, 31}

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a^2\tan(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
)) - (a^2*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^
(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{a\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,d}{c}$$

$$= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+}{cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

$$= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+}{cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

$$= -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+}{cf\sqrt{a+a\sec(e+fx)}\,(c-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.24, size = 153, normalized size = 1.05

$$\frac{a\left(4-3ifx+\cos(e+fx)\left(-6+4ifx-8\log\left(1-e^{i(e+fx)}\right)\right)+6\log\left(1-e^{i(e+fx)}\right)+\cos(2(e+fx))\left(-ifx+2\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{2c^2f(-1+\cos(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(4 - (3*I)*f*x + Cos[e + f*x]*(-6 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x)
)]) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E
^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(-1
 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.25, size = 227, normalized size = 1.55

method	result
default	$\frac{(-1+\cos(fx+e))\left(4(\cos^2(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-8(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+5(\cos^2(fx+e))-8\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)\cdots}\right.\right.}{4f\sin(fx+e)\cos(fx+e)^2\left(\cdots\right.}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(3e^{3i(fx+e)}-4e^{\cdots}\right.}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c(\cdots}{e\cdots}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-8*cos(f*x+e)^2*l
n(-(-1+cos(f*x+e))/sin(f*x+e))+5*cos(f*x+e)^2-8*cos(f*x+e)*ln(2/(cos(f*x+e)
+1))+16*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*cos(f*x+e)+4*ln(2/(cos
(f*x+e)+1))-8*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3)*(a*(cos(f*x+e)+1)/cos(f*x+
e))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1937 vs. $2(144) = 288$.
time = 0.81, size = 1937, normalized size = 13.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 16*(
f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x
 + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*
a*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*a*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*a*cos(2*f
*x + 2*e) + (f*x + e)*a - 2*(a*cos(4*f*x + 4*e)^2 + 36*a*cos(2*f*x + 2*e)^2
 + 16*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*sin(4*f*x + 4*e)^2 +
12*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a*sin(2*f*x + 2*e)^2 + 16*a*sin
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*a*cos(2*f*x + 2*e) + a)*cos(4
*f*x + 4*e) + 12*a*cos(2*f*x + 2*e) - 8*(a*cos(4*f*x + 4*e) + 6*a*cos(2*f*x
 + 2*e) - 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*cos(4*f*x + 4*e) +
 6*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
```

```
e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - 4*a*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*arctan2(sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 1) + 2*(6*(f*x + e)*a*cos(2*f*x + 2*e) + (f*x + e)*a - 4*a*sin
(2*f*x + 2*e))*cos(4*f*x + 4*e) - 2*(4*(f*x + e)*a*cos(4*f*x + 4*e) + 24*(f
*x + e)*a*cos(2*f*x + 2*e) - 16*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*(f*x + e)*a + 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*
x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(4*(f*x
+ e)*a*cos(4*f*x + 4*e) + 24*(f*x + e)*a*cos(2*f*x + 2*e) + 4*(f*x + e)*a +
 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*a*sin(2*f*x + 2*e) + 2*a*cos(2*f*x
 + 2*e))*sin(4*f*x + 4*e) - 8*a*sin(2*f*x + 2*e) - 2*(4*(f*x + e)*a*sin(4*f
*x + 4*e) + 24*(f*x + e)*a*sin(2*f*x + 2*e) - 16*(f*x + e)*a*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*a*cos(4*f*x + 4*e) - 2*a*cos(2*f
*x + 2*e) - 3*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(
4*(f*x + e)*a*sin(4*f*x + 4*e) + 24*(f*x + e)*a*sin(2*f*x + 2*e) - 3*a*cos(
4*f*x + 4*e) - 2*a*cos(2*f*x + 2*e) - 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 36*c^3*cos
(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^3*
sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c^3*sin(
2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*c^
3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e
) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e)
+ c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*sin(4*
f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x
 + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}{(-c\,(\sec{(e+fx)}-1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

Giac [A]
time = 2.04, size = 170, normalized size = 1.16

$$-\frac{\frac{4\sqrt{-ac}\,a^2\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c^3|a|}-\frac{4\sqrt{-ac}\,a^2\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right|\right)}{c^3|a|}-\frac{6\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^2\sqrt{-ac}\,a^2+8\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)\sqrt{-ac}\,a^3+3\sqrt{-ac}\,a^4}{a^2c^3|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(4*sqrt(-a*c)*a^2*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^3*abs(a)) - 4*
sqrt(-a*c)*a^2*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^3*abs(a)) - (6*(a
*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^2 + 8*(a*tan(1/2*f*x + 1/2*e)^2
 - a)*sqrt(-a*c)*a^3 + 3*sqrt(-a*c)*a^4)/(a^2*c^3*abs(a)*tan(1/2*f*x + 1/2*
e)^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}-\frac{\quad}{c^2f\sqrt{a+}}$$

```
[Out] -2/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^2
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^2*tan(f*x+e
)/c^2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^2*ln(1-cos(f*x+e))*
tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3993, 3992, 3996, 31}

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a^2\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{a^2\tan(e+fx)}{2cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^2*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^2*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^2*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
```

```
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} + \frac{a\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^2\tan(e+\ldots}{2cf\sqrt{a+a\sec(e+fx)}\,(c\ldots}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^2\tan(e+\ldots}{2cf\sqrt{a+a\sec(e+fx)}\,(c\ldots}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^2\tan(e+\ldots}{2cf\sqrt{a+a\sec(e+fx)}\,(c\ldots}$$

$$= -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^2\tan(e+\ldots}{2cf\sqrt{a+a\sec(e+fx)}\,(c\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.21, size = 199, normalized size = 1.02

$$\frac{a\left(-50+30ifx-3ifx\cos(3(e+fx))+6i\cos(2(e+fx))\left(4i+3fx+6i\log\left(1-e^{i(e+fx)}\right)\right)-60\log\left(1-e^{i(e+fx)}\right)+6\cos(3(e+fx))\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(66-45ifx+90\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{12c^3f(-1+\cos(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*(-50 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (6*I)*Cos[2*(e + f*x)]*
(4*I + 3*f*x + (6*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))
] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(66 - (45*I)
```

```
*f*x + 90*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*
x)/2])/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.29, size = 289, normalized size = 1.47

method	result
default	$-\frac{(-1+\cos(fx+e))\left(48\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-24\left(\cos^3(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-35\left(\cos^3(fx+e)\right)-144\left(\cos^2(fx+e)\right)}{}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(12\,e^{5i(fx+e)}-\right.}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/f*(-1+cos(f*x+e))*(48*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-24
*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-35*cos(f*x+e)^3-144*cos(f*x+e)^2*ln(-(-1
+cos(f*x+e))/sin(f*x+e))+72*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+9*cos(f*x+e)^
2+144*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-72*cos(f*x+e)*ln(2/(cos(f*
x+e)+1))+27*cos(f*x+e)-48*ln(-(-1+cos(f*x+e))/sin(f*x+e))+24*ln(2/(cos(f*x+
e)+1))-17)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e
))^(7/2)/sin(f*x+e)/cos(f*x+e)^3*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3777 vs. $2(189) = 378$.
time = 3.33, size = 3777, normalized size = 19.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a*cos(4*f*x + 4*e)^2
 + 675*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a*cos(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a*sin(6*f*x + 6*e)^2 + 675*(
f*x + e)*a*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 108*(f
*x + e)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*
x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x
+ e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*(f*x + e
)*a*cos(2*f*x + 2*e) + 3*(f*x + e)*a - 6*(a*cos(6*f*x + 6*e)^2 + 225*a*cos(
```

```
4*f*x + 4*e)^2 + 225*a*cos(2*f*x + 2*e)^2 + 36*a*cos(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 400*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 36*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + a*sin(6*f*x + 6*e)^2 + 225*a*sin(4*f*x + 4*e)^2 + 450*a*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + 225*a*sin(2*f*x + 2*e)^2 + 36*a*sin(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*sin(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 36*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 2*(15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*cos(6*f*x
 + 6*e) + 30*(15*a*cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e) + 30*a*cos(2*f*x
+ 2*e) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x +
2*e) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*a*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*cos(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) - 6*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
2*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a*sin(4*f*x + 4*
e) + a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(a*sin(6*f*x + 6*e) + 15*a*s
in(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 20*a*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*sin(
6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 6*a*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
15*a*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + a)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 6*(15*(f*x + e)*a*cos(
4*f*x + 4*e) + 15*(f*x + e)*a*cos(2*f*x + 2*e) + (f*x + e)*a - 11*a*sin(4*f
*x + 4*e) - 11*a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 90*(15*(f*x + e)*a*co
s(2*f*x + 2*e) + (f*x + e)*a)*cos(4*f*x + 4*e) - 12*(3*(f*x + e)*a*cos(6*f*
x + 6*e) + 45*(f*x + e)*a*cos(4*f*x + 4*e) + 45*(f*x + e)*a*cos(2*f*x + 2*e
) - 60*(f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
8*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(f*x
 + e)*a + 2*a*sin(6*f*x + 6*e) - 3*a*sin(4*f*x + 4*e) - 3*a*sin(2*f*x + 2*e
) + 10*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*cos(6*f*x + 6*
e) + 90*(f*x + e)*a*cos(4*f*x + 4*e) + 90*(f*x + e)*a*cos(2*f*x + 2*e) - 36
*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(f*x
+ e)*a + 5*a*sin(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e)
 - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*cos(6*f*x + 6*e)
 + 45*(f*x + e)*a*cos(4*f*x + 4*e) + 45*(f*x + e)*a*cos(2*f*x + 2*e) + 3*(f
*x + e)*a + 2*a*sin(6*f*x + 6*e) - 3*a*sin(4*f*x + 4*e) - 3*a*sin(2*f*x + 2
*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(15*(f*x + e)
*a*sin(4*f*x + 4*e) + 15*(f*x + e)*a*sin(2*f*x + 2*e) + 11*a*cos(4*f*x + 4*
```

```
e) + 11*a*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(225*(f*x + e)*a*sin(2*f*x
 + 2*e) - 11*a)*sin(4*f*x + 4*e) - 66*a*sin(2*f*x + 2*e) - 12*(3*(f*x + e)*
a*sin(6*f*x + 6*e) + 45*(f*x + e)*a*sin(4*f*x + 4*e) + 45*(f*x + e)*a*sin(2
*f*x + 2*e) - 60*(f*x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 18*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 2*a*cos(6*f*x + 6*e) + 3*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e) - 10
*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*a)*sin(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*sin(6*f*x + 6*
e) + 90*(f*x + e)*a*sin(4*f*x + 4*e) + 90*(f*x + e)*a*sin(2*f*x + 2*e) - 36
*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a*cos
(6*f*x + 6*e) - 9*a*cos(4*f*x + 4*e) - 9*a*cos(2*f*x + 2*e) + 6*a*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x
+ e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) +
 c^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 1.72, size = 200, normalized size = 1.02

$$-\frac{\frac{24\,\sqrt{-ac}\,a^2\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c^4|a|}-\frac{24\,\sqrt{-ac}\,a^2\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right|\right)}{c^4|a|}-\frac{44\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^3\sqrt{-ac}\,a^2+108\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^2\sqrt{-ac}\,a^3+93\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)\sqrt{-ac}\,a^4+27\,\sqrt{-ac}\,a^5}{a^3c^4|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/24*(24*sqrt(-a*c)*a^2*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^4*abs(a)) -
24*sqrt(-a*c)*a^2*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^4*abs(a)) - (4
4*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*c)*a^2 + 108*(a*tan(1/2*f*x + 1/
2*e)^2 - a)^2*sqrt(-a*c)*a^3 + 93*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)
*a^4 + 27*sqrt(-a*c)*a^5)/(a^3*c^4*abs(a)*tan(1/2*f*x + 1/2*e)^6))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=153

$$\frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^3c}{4f\sqrt{a+a\sec(e}}$$

```
[Out] a^3*c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+1/2*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-1/4*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3990, 3554, 3556}

$$-\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^3*c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (a^3*c^3*Tan[e + f*x]^3)/(2*f*Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c - c*Sec[e + f*x]]) - (a^3*c^3*Tan[e + f*x]^5)/(4*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx &= -\frac{(a^3c^3\tan(e+fx))\int\tan^5(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{(a^3c^3}{\sqrt{a+a}} \\
&= \frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{}{4f\sqrt{a+}} \\
&= \frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{}{2f\sqrt{a+a}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.59, size = 164, normalized size = 1.07

$$
\frac{ia^2c^2\csc\left(\frac{1}{2}(e+fx)\right)\left(2i+3fx+\cos(4(e+fx))\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)+4\cos(2(e+fx))\left(i+fx+i\log\left(1+e^{2i(e+fx)}\right)\right)+3i\log\left(1+e^{2i(e+fx)}\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{16f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/16)*a^2*c^2*Csc[(e + f*x)/2]*(2*I + 3*f*x + Cos[4*(e + f*x)]*(f*x + I*L
og[1 + E^((2*I)*(e + f*x))]) + 4*Cos[2*(e + f*x)]*(I + f*x + I*Log[1 + E^((
2*I)*(e + f*x))]) + (3*I)*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Se
c[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/f
```

Maple [A]
time = 0.28, size = 191, normalized size = 1.25

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\left(4(\cos^4(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-4(\cos^4(fx+e))\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-4(\cos^4(\right.}{4f\sin(fx+e)(-1+\cos(fx+e))^2\cos(fx+e)}$
risch	$\frac{a^2c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2a^2c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(f}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5
/2)*(4*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-4*cos(f*x+e)^4*ln((-cos(f*x+e)+1+s
in(f*x+e))/sin(f*x+e))-4*cos(f*x+e)^4*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x
```

```
+e))+3*cos(f*x+e)^4-4*cos(f*x+e)^2+1)/sin(f*x+e)/(-1+cos(f*x+e))^2/cos(f*x+
e)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1738 vs. $2(147) = 294$.
time = 0.73, size = 1738, normalized size = 11.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*c^2*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^2*cos(6*f*x + 6
*e)^2 + 36*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*a^2*c^2*cos(
2*f*x + 2*e)^2 + (f*x + e)*a^2*c^2*sin(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^
2*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)^2 + 16*(f*x +
e)*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x
 + e)*a^2*c^2 - 4*a^2*c^2*sin(2*f*x + 2*e) - (a^2*c^2*cos(8*f*x + 8*e)^2 +
16*a^2*c^2*cos(6*f*x + 6*e)^2 + 36*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*a^2*c^2*
cos(2*f*x + 2*e)^2 + a^2*c^2*sin(8*f*x + 8*e)^2 + 16*a^2*c^2*sin(6*f*x + 6*
e)^2 + 36*a^2*c^2*sin(4*f*x + 4*e)^2 + 48*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 16*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*a^2*c^2*cos(2*f*x + 2*e) + a^2
*c^2 + 2*(4*a^2*c^2*cos(6*f*x + 6*e) + 6*a^2*c^2*cos(4*f*x + 4*e) + 4*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(8*f*x + 8*e) + 8*(6*a^2*c^2*cos(4*f*x +
4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(6*f*x + 6*e) + 12*(4*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(4*f*x + 4*e) + 4*(2*a^2*c^2*sin(6*f*x +
6*e) + 3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*e))*sin(8*f*x +
 8*e) + 16*(3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(4*(f*x + e
)*a^2*c^2*cos(6*f*x + 6*e) + 6*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e) + 4*(f*x
+ e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 - 2*a^2*c^2*sin(6*f*x + 6
*e) - 2*a^2*c^2*sin(4*f*x + 4*e) - 2*a^2*c^2*sin(2*f*x + 2*e))*cos(8*f*x +
8*e) + 8*(6*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e) + 4*(f*x + e)*a^2*c^2*cos(2*
f*x + 2*e) + (f*x + e)*a^2*c^2 + a^2*c^2*sin(4*f*x + 4*e))*cos(6*f*x + 6*e)
 + 4*(12*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c^2 - 2*a^2*c
^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(2*(f*x + e)*a^2*c^2*sin(6*f*x +
6*e) + 3*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e) + 2*(f*x + e)*a^2*c^2*sin(2*f*x
 + 2*e) + a^2*c^2*cos(6*f*x + 6*e) + a^2*c^2*cos(4*f*x + 4*e) + a^2*c^2*cos
(2*f*x + 2*e))*sin(8*f*x + 8*e) + 4*(12*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)
+ 8*(f*x + e)*a^2*c^2*sin(2*f*x + 2*e) - 2*a^2*c^2*cos(4*f*x + 4*e) - a^2*c
^2)*sin(6*f*x + 6*e) + 4*(12*(f*x + e)*a^2*c^2*sin(2*f*x + 2*e) + 2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*sin(4*f*x + 4*e))*sqrt(a)*sqrt(c)/((2*(4*cos(6
*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)
 + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos
```

```
(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f
*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x
+ 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*
f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e
) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.92, size = 437, normalized size = 2.86

$$\left[\frac{2\sqrt{-ac}\,a^2c^2\cos\left(fx+e\right)^3\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)-\left(3a^2c^2\cos\left(fx+e\right)^2-a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{4f\cos\left(fx+e\right)^3}, \frac{4\sqrt{ac}\,a^2c^2\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)\cos\left(fx+e\right)^3-\left(3a^2c^2\cos\left(fx+e\right)^2-a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{4f\cos\left(fx+e\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*sqrt(-a*c)*a^2*c^2*cos(f*x + e)^3*log(1/2*(a*c*cos(f*x + e)^4 - (co
s(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x +
e)^2) - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
+ e)^3), 1/4*(4*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*
x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)^3 - (3*a^2*c^2*cos(f*x + e)
^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2), x)
```

3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=190

$$\frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac^2(a+a\sec(e+fx))^{3/}}{2f\sqrt{c-c\sec(e}}$$

```
[Out] -1/2*a*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+1/3*c
^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+a^3*c^2*ln(co
s(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^2*c^
2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3994, 3991, 3990, 3556}

$$\frac{a^3c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^3*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) - (a^2*c^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) - (a*c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
2*f*Sqrt[c - c*Sec[e + f*x]]) + (c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
```

```
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rule 3994

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx &= \frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + c\int (a+a\sec(e \\
&= -\frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + \frac{c^2(a+a\sec(}{3f\sqrt{c} \\
&= -\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2(a+a\sec}{2f\sqrt{}} \\
&= -\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac^2(a+a\sec}{2f\sqrt{}} \\
&= \frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{a-}}{f}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.25, size = 149, normalized size = 0.78

$$\frac{a^2c\csc\left(\frac{1}{2}(e+fx)\right)\left(2+6\cos(2(e+fx))+3ifx\cos(3(e+fx))+\cos(e+fx)\left(-6+9ifx-9\log\left(1+e^{2i(e+fx)}\right)\right)-3\cos(3(e+fx))\log\left(1+e^{2i(e+fx)}\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*c*Csc[(e + f*x)/2]*(2 + 6*Cos[2*(e + f*x)] + (3*I)*f*x*Cos[3*(e + f*x)
] + Cos[e + f*x]*(-6 + (9*I)*f*x - 9*Log[1 + E^((2*I)*(e + f*x))]) - 3*Cos[
3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*
Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(24*f)
```

Maple [A]
time = 0.27, size = 199, normalized size = 1.05

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\left(6(\cos^3(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-6(\cos^3(fx+e))\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-6(\cos^3(}{6f\sin(fx+e)\cos(fx+e)(-1+\cos(fx+e))}$
risch	$\frac{a^2c(e^{2i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,x}{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)}-\frac{2a^2c(e^{2i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(fx+}{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)*(6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^3*ln((-cos(f*x+e)+1+s
in(f*x+e))/sin(f*x+e))-6*cos(f*x+e)^3*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x
+e))+7*cos(f*x+e)^3+6*cos(f*x+e)^2-3*cos(f*x+e)-2)/sin(f*x+e)/cos(f*x+e)/(-
1+cos(f*x+e))*a^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(183) = 366$.
time = 0.68, size = 1460, normalized size = 7.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a^2*c*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*cos(4*f*x +
 4*e)^2 + 27*(f*x + e)*a^2*c*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a^2*c*sin(6*f
*x + 6*e)^2 + 27*(f*x + e)*a^2*c*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a^2*c*si
n(2*f*x + 2*e)^2 + 18*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c
- 6*a^2*c*sin(2*f*x + 2*e) - 3*(a^2*c*cos(6*f*x + 6*e)^2 + 9*a^2*c*cos(4*f*
x + 4*e)^2 + 9*a^2*c*cos(2*f*x + 2*e)^2 + a^2*c*sin(6*f*x + 6*e)^2 + 9*a^2*
c*sin(4*f*x + 4*e)^2 + 18*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*c
*sin(2*f*x + 2*e)^2 + 6*a^2*c*cos(2*f*x + 2*e) + a^2*c + 2*(3*a^2*c*cos(4*f
*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(6*f*x + 6*e) + 6*(3*a^2*c
*cos(2*f*x + 2*e) + a^2*c)*cos(4*f*x + 4*e) + 6*(a^2*c*sin(4*f*x + 4*e) + a
^2*c*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1) + 6*(3*(f*x + e)*a^2*c*cos(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*c
os(2*f*x + 2*e) + (f*x + e)*a^2*c - a^2*c*sin(4*f*x + 4*e) - a^2*c*sin(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + (f*x
+ e)*a^2*c)*cos(4*f*x + 4*e) - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*
x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 4*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a
```

```
^2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x
 + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
 e)*a^2*c*sin(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*sin(2*f*x + 2*e) + a^2*c*cos
(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
^2*c*sin(2*f*x + 2*e) - a^2*c)*sin(4*f*x + 4*e) + 6*(a^2*c*cos(6*f*x + 6*e)
 + 3*a^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) + 3*a
^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(a^2*c*cos(6*f*x + 6*e) + 3*a^2*c*c
os(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.85, size = 505, normalized size = 2.66

$$\left[\frac{\left(a^2c\cos\left(fx+e\right)^2-5\,a^2c\cos\left(fx+e\right)-2\,a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+3\left(a^2c\cos\left(fx+e\right)^3+a^2c\cos\left(fx+e\right)^2\right)\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}, \frac{\left(a^2c\cos\left(fx+e\right)^2-5\,a^2c\cos\left(fx+e\right)-2\,a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+6\left(a^2c\cos\left(fx+e\right)^3+a^2c\cos\left(fx+e\right)^2\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
+ e) + 3*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(
a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), 1/6*(
(a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) +
 6*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e
)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2), x)
```

3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=139

$$\frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*a*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+a^3*c*l
n(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-a^
2*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3991, 3990, 3556}

$$\frac{a^3c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^3*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (a^2*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]]) - (a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sq
rt[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3991

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[c, Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + a\int (a+a\sec(e$$

$$= -\frac{a^2c\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+}{2f\sqrt{c-}}$$

$$= -\frac{a^2c\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{ac(a+a\sec(e+}{2f\sqrt{c-}}$$

$$= \frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{a^2c\sqrt{a+a}}{f\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.33, size = 164, normalized size = 1.18

$$\frac{a^2e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\left(i+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(1+ifx+4\cos(e+fx)+\cos(2(e+fx))\left(ifx-\log\left(1+e^{2i(e+fx)}\right)\right)-\log\left(1+e^{2i(e+fx)}\right)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{4\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^2*(1 + E^((2*I)*(e + f*x)))*(I + Cot[(e + f*x)/2])*(1 + I*f*x + 4*Cos[e
+ f*x] + Cos[2*(e + f*x)]*(I*f*x - Log[1 + E^((2*I)*(e + f*x))]) - Log[1 +
E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*
Sec[e + f*x]])/(4*E^(I*(e + f*x))*(1 + E^(I*(e + f*x)))*f)
```

Maple [A]
time = 0.27, size = 179, normalized size = 1.29

method	result
default	$\frac{\left(2\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\left(\cos^2(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-2\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+3(\mathrm{co}}{2f\sin(fx+e)\cos(fx+e)}$
risch	$\frac{a^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2a^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)^2*ln((-cos(f*x+e)+1
+sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f
*x+e))+3*cos(f*x+e)^2+4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*
(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)*a^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(135) = 270$.
time = 0.61, size = 767, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + (
f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + 2*a^2*sin(2*f*x + 2*e) - (a^
2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 +
4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*
cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a^2*cos(2
*f*x + 2*e) + (f*x + e)*a^2 + a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(a
^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*a^2*
sin(2*f*x + 2*e) - a^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2
*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f
*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*
sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [**A**]
time = 3.16, size = 456, normalized size = 3.28

$$\left[\frac{(5a^2\cos(fx+e)+a^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+\left(a^2\cos(fx+e)^2+a^2\cos(fx+e)\right)\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}, \frac{(5a^2\cos(fx+e)+a^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+2\left(a^2\cos(fx+e)^2+a^2\cos(fx+e)\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{ac\cos(fx+e)^2+ac}\right)}{2\left(f\cos(fx+e)^2+f\cos(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + (a^2*cos(f*x + e)^2 +
```

```
a^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3
+ cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos
(f*x + e)^2 + f*cos(f*x + e)), 1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x +
e)^2 + f*cos(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2), x)
```

3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=152

$$\frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^3\sec(e+}{f\sqrt{a+a\sec(e+}}$$

```
[Out] a^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+4*a^3*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)+a^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$\frac{a^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (a^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^2}{x(c-cx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{a^2}{c}-\frac{4a^2}{c(-1+x)}+\frac{a^2}{cx}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4a^3\log(1-\sec(e+fx))\tan}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\,\mathrm{s}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.81, size = 292, normalized size = 1.92

$$\frac{\sqrt{2}e^{\frac{1}{2}i(e+fx)}\sqrt{\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(-ifx+8\log\left(1-e^{i(e+fx)}\right)-3\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{\sec(e+fx)}\,(a(1+\sec(e+fx)))^{5/2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,f(1+\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\sec(e+fx)\sqrt{(1+\cos(e+fx))\sec(e+fx)}\,(a(1+\sec(e+fx)))^{5/2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f(1+\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e
+ f*x)))]*((-I)*f*x + 8*Log[1 - E^(I*(e + f*x))] - 3*Log[1 + E^((2*I)*(e +
f*x))])*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2])
/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*(
1 + Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (Sec[e + f*x]*Sqrt[(1 +
 Cos[e + f*x])*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Tan[e/2 + (f*x)/2
])/(f*(1 + Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.26, size = 183, normalized size = 1.20

method	result
default	$\frac{\left(3\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-8\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+3\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e}{f\sin(fx+e)c}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}-\right.}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f(e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(3*cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))-8*cos(f*x+e)*ln
(-(-1+cos(f*x+e))/sin(f*x+e))+3*cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/si
```

```
n(f*x+e))+cos(f*x+e)*ln(2/(cos(f*x+e)+1))-cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/c
os(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c*a^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2), x)
```

3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}+\frac{a^3\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(co
s(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3995, 3990, 3556}

$$\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) + (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{a^2\int\sqrt{a+a\sec(e+fx)}\,\sqrt{c}}{c^2}$$
$$= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} - \frac{(a^3\tan(e+fx))\int\tan(e}{c\sqrt{a+a\sec(e+fx)}\,\sqrt{c-}}$$
$$= -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(\cos(e+fx))\tan}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.28, size = 111, normalized size = 1.16

$$\frac{a^2\left(-4+ifx-\log\left(1+e^{2i(e+fx)}\right)+\cos(e+fx)\left(-ifx+\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{cf(-1+\cos(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(-4 + I*f*x - Log[1 + E^((2*I)*(e + f*x))] + Cos[e + f*x]*((-I)*f*x +
Log[1 + E^((2*I)*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])
/(c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(88) = 176.
time = 0.25, size = 237, normalized size = 2.47

method	result
default	$-\frac{(-1+\cos(fx+e))\left(\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right.}{f\cos(fx+e)\left(\frac{c(-1+\cos(f}{\cos(fx+}\right.}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)x}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}} - \frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} + \frac{8ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx}}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx}}{e^{2i(fx}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+
cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(cos(f
*x+e)+1))-ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))-ln(-(cos(f*x+e)-1+sin(f
*x+e))/sin(f*x+e))-2*cos(f*x+e)+ln(2/(cos(f*x+e)+1))-2)*(a*(cos(f*x+e)+1)/c
```

```
os(f*x+e))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)
*a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(95) = 190.
time = 3.46, size = 478, normalized size = 4.98

$$\left[\frac{\left(a^{2} c \cos\left(fx+e\right)-a^{2} c\right) \sqrt{-\frac{a}{c}} \log\left(\frac{a \cos\left(fx+e\right)^{4}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)\right) \sqrt{-\frac{a}{c}} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}} \sin\left(fx+e\right)+a}{2 \cos\left(fx+e\right)^{2}}\right) \sin\left(fx+e\right)+4 \left(a^{2} \cos\left(fx+e\right)^{2}+a^{2} \cos\left(fx+e\right)\right) \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2 \left(c^{2} f \cos\left(fx+e\right)-c^{2} f\right) \sin\left(fx+e\right)}, \frac{\left(a^{2} c \cos\left(fx+e\right)-a^{2} c\right) \sqrt{\frac{a}{c}} \arctan\left(\frac{\sqrt{\frac{a}{c}} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}} \cos\left(fx+e\right) \sin\left(fx+e\right)}{a \cos\left(fx+e\right)^{2}+a}\right) \sin\left(fx+e\right)+2 \left(a^{2} \cos\left(fx+e\right)^{2}+a^{2} \cos\left(fx+e\right)\right) \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(c^{2} f \cos\left(fx+e\right)-c^{2} f\right) \sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((a^2*c*cos(f*x + e) - a^2*c)*sqrt(-a/c)*log(1/2*(a*cos(f*x + e)^4 - (
cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a)/cos(f*x +
e)^2)*sin(f*x + e) + 4*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*
f*cos(f*x + e) - c^2*f)*sin(f*x + e)), ((a^2*c*cos(f*x + e) - a^2*c)*sqrt(a
/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*cos(f*x + e)^2 + a))
*sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos
(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: TypeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2), x)
```

3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}+\frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-
cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3995, 3996, 31}

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
```

```
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{a^2\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^2}$$
$$= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{(a^3\tan(e+fx))\,\text{Subst}\left(\int \frac{}{-c}\right.}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{}}$$
$$= -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{a^3\log(1-\cos(e+fx))}{c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.44, size = 155, normalized size = 1.55

$$\frac{a^2\left(4-3ifx+\cos(e+fx)\left(-8+4ifx-8\log\left(1-e^{i(e+fx)}\right)\right)+6\log\left(1-e^{i(e+fx)}\right)+\cos(2(e+fx))\left(-ifx+2\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{2c^2f(-1+\cos(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(4 - (3*I)*f*x + Cos[e + f*x]*(-8 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*
x))]) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 -
 E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(
-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(92) = 184.
time = 0.25, size = 229, normalized size = 2.29

method	result
default	$\frac{(-1+\cos(fx+e))\left(2(\cos^2(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-4(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+3(\cos^2(fx-\right.}{2f\sin(fx+e)\cos(fx+e)^2\left(\right.}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{8ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{3i(fx+e)}-e\right.}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c(}{e}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))*(2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-4*cos(f*x+e)^2*l
n(-(-1+cos(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*cos(f*x+
e)^2+8*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*ln(2/(cos(f*x+e)+1))+2*
cos(f*x+e)-4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1)*(a*(cos(f*x+e)+1)/cos(f*x+e
))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*a^2
```

Maxima [A]
time = 0.52, size = 147, normalized size = 1.47

$$\frac{\frac{4\sqrt{-a}\,a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}}-\frac{2\sqrt{-a}\,a^2\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}}-\frac{\left(\sqrt{-a}\,a^2\sqrt{c}-\frac{2\sqrt{-a}\,a^2\sqrt{c}\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^4}{c^3\sin(fx+e)^4}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/2*(4*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) - 2*sqrt(-
a)*a^2*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) - (sqrt(-a)*a^2
*sqrt(c) - 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos
(f*x + e) + 1)^4/(c^3*sin(f*x + e)^4))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^
2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 2.04, size = 170, normalized size = 1.70

$$-\frac{\frac{2\sqrt{-ac}\,a^3\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c^3|a|}-\frac{2\sqrt{-ac}\,a^3\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right|\right)}{c^3|a|}-\frac{3\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^2\sqrt{-ac}\,a^3+4\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)\sqrt{-ac}\,a^4+2\sqrt{-ac}\,a^5}{a^2c^3|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/2*(2*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^3*abs(a)) - 2*
sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^3*abs(a)) - (3*(a
*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^3 + 4*(a*tan(1/2*f*x + 1/2*e)^2
 - a)*sqrt(-a*c)*a^4 + 2*sqrt(-a*c)*a^5)/(a^2*c^3*abs(a)*tan(1/2*f*x + 1/2*
e)^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2), x)
```

3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}}-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}+\frac{a^3\log}{c^3f\sqrt{a+}}$$

```
[Out] -4/3*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan
(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*
x+e))*tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3995, 3992, 3996, 31}

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^3f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a^3\tan(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7
/2)) - (a^3*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec
[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} + \frac{a^2\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,d}{c^2}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+}{c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+}{c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+}{c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.68, size = 202, normalized size = 1.36

$$\frac{a^2\left(-58+30ifx-3ifx\cos(3(e+fx))+6i\cos(2(e+fx))\left(5i+3fx+6i\log\left(1-e^{i(e+fx)}\right)\right)-60\log\left(1-e^{i(e+fx)}\right)+6\cos(3(e+fx))\log\left(1-e^{i(e+fx)}\right)+9\cos(e+fx)\left(8-5ifx+10\log\left(1-e^{i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{12c^3f(-1+\cos(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(-58 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (6*I)*Cos[2*(e + f*x)
]*(5*I + 3*f*x + (6*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x
))] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + 9*Cos[e + f*x]*(8 - (5*
I)*f*x + 10*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e +
f*x)/2])/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(134) = 268.
time = 0.27, size = 281, normalized size = 1.90

method	result
default	$-\frac{(-1+\cos(fx+e))\left(6(\cos^3(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-3(\cos^3(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-5(\cos^3(fx+e))-18(\cos^2(fx+e))\ln\left(-\right.\right.}{}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(15\,e^{5i(fx+\right.}{3c^3\left(e^{i(fx+e)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(-1+cos(f*x+e))*(6*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3*co
s(f*x+e)^3*ln(2/(cos(f*x+e)+1))-5*cos(f*x+e)^3-18*cos(f*x+e)^2*ln(-(-1+cos(
f*x+e))/sin(f*x+e))+9*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+18*cos(f*x+e)*ln(-(
-1+cos(f*x+e))/sin(f*x+e))-9*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)-6
*ln(-(-1+cos(f*x+e))/sin(f*x+e))+3*ln(2/(cos(f*x+e)+1))-2)*(a*(cos(f*x+e)+1
)/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)/cos(f*x
+e)^3*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4035 vs. $2(144) = 288$.
time = 3.35, size = 4035, normalized size = 27.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(3*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*cos(4*f*x + 4*
e)^2 + 675*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a^2*sin(6*f*x +
6*e)^2 + 675*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*sin(2*f*x
 + 2*e)^2 + 108*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 1200*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 108*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2 - 72*a^2*s
in(2*f*x + 2*e) - 6*(a^2*cos(6*f*x + 6*e)^2 + 225*a^2*cos(4*f*x + 4*e)^2 +
225*a^2*cos(2*f*x + 2*e)^2 + 36*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + 400*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 36*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2
```

```
*sin(6*f*x + 6*e)^2 + 225*a^2*sin(4*f*x + 4*e)^2 + 450*a^2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + 225*a^2*sin(2*f*x + 2*e)^2 + 36*a^2*sin(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + 36*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 30*a^2*cos(2*f*x + 2*e) + a^2 + 2*(15*a^2*cos(4*f*x + 4*e)
 + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 30*(15*a^2*cos(2*f*x +
 2*e) + a^2)*cos(4*f*x + 4*e) - 12*(a^2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x
 + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 20*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 6*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^
2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 6*
a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(a^2*cos(6*f*x + 6*e) + 15*a
^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x +
 2*e))*sin(6*f*x + 6*e) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e
) + 15*a^2*sin(2*f*x + 2*e) - 20*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 6*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^2*sin(6*f*x
+ 6*e) + 15*a^2*sin(4*f*x + 4*e) + 15*a^2*sin(2*f*x + 2*e) - 6*a^2*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e
) + 15*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 6*(15*(f*x + e)*a^2*
cos(4*f*x + 4*e) + 15*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 12*a
^2*sin(4*f*x + 4*e) - 12*a^2*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 90*(15*(f
*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2)*cos(4*f*x + 4*e) - 6*(6*(f*x
+ e)*a^2*cos(6*f*x + 6*e) + 90*(f*x + e)*a^2*cos(4*f*x + 4*e) + 90*(f*x + e
)*a^2*cos(2*f*x + 2*e) - 120*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 36*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 6*(f*x + e)*a^2 + 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*
f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e) + 16*a^2*sin(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 4*(30*(f*x + e)*a^2*cos(6*f*x + 6*e) + 450*(f*x + e)*a^2*cos(4*f*x + 4*
e) + 450*(f*x + e)*a^2*cos(2*f*x + 2*e) - 180*(f*x + e)*a^2*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(f*x + e)*a^2 + 29*a^2*sin(6*f*x
 + 6*e) + 75*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e) - 24*a^2*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 6*(6*(f*x + e)*a^2*cos(6*f*x + 6*e) + 90*(f*x +
 e)*a^2*cos(4*f*x + 4*e) + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 6*(f*x + e)*
a^2 + 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2
*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*(5*(f*x + e)
*a^2*sin(4*f*x + 4*e) + 5*(f*x + e)*a^2*sin(2*f*x + 2*e) + 4*a^2*cos(4*f*x
+ 4*e) + 4*a^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(75*(f*x + e)*a^2*si
```

```
n(2*f*x + 2*e) - 4*a^2)*sin(4*f*x + 4*e) - 6*(6*(f*x + e)*a^2*sin(6*f*x + 6
*e) + 90*(f*x + e)*a^2*sin(4*f*x + 4*e) + 90*(f*x + e)*a^2*sin(2*f*x + 2*e)
 - 120*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 36*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*
a^2*cos(6*f*x + 6*e) - 3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e) - 16
*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*a^2)*sin(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(30*(f*x + e)*a^2*sin(6*f*
x + 6*e) + 450*(f*x + e)*a^2*sin(4*f*x + 4*e) +...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3
 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 3.35, size = 200, normalized size = 1.35

$$-\frac{\frac{6\,\sqrt{-ac}\,a^3\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c^4|a|}-\frac{6\,\sqrt{-ac}\,a^3\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right|\right)}{c^4|a|}-\frac{11\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^3\sqrt{-ac}\,a^3+27\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^2\sqrt{-ac}\,a^4+24\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)\sqrt{-ac}\,a^5+7\,\sqrt{-ac}\,a^6}{a^3c^4|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/6*(6*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^4*abs(a)) - 6*
sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^4*abs(a)) - (11*(
a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*c)*a^3 + 27*(a*tan(1/2*f*x + 1/2*e)
^2 - a)^2*sqrt(-a*c)*a^4 + 24*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^5
 + 7*sqrt(-a*c)*a^6)/(a^3*c^4*abs(a)*tan(1/2*f*x + 1/2*e)^6))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2), x)
```

3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}}-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}-\frac{\quad}{c^3f\sqrt{a+}}$$

```
[Out] -a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*tan
(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan(f*x+e)/
c^3/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*x+e))*ta
n(f*x+e)/c^4/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3995, 3992, 3996, 31}

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^4f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a^3\tan(e+fx)}{c^3f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{a^3\tan(e+fx)}{2c^2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -((a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(9/2)
)) - (a^3*Tan[e + f*x])/(2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) - (a^3*Tan[e + f*x])/(c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^4*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
```

```
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}} + \frac{a^2\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx}{c^2}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+f}{2c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+f}{2c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+f}{2c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

$$= -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+f}{2c^2f\sqrt{a+a\sec(e+fx)}\,(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.77, size = 285, normalized size = 1.47

$$\frac{\sec^{\frac{9}{2}}(e+fx)(a(1+\sec(e+fx)))^{5/2}\left(\frac{16\sqrt{2}\,e^{\frac{1}{2}i(e+fx)}\sqrt{\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\,\left(-ifx+2\log\left(1-e^{i(e+fx)}\right)\right)}{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,f}+\frac{(-54+89\cos(e+fx)-60\cos(2(e+fx))+23\cos(3(e+fx))-6\cos(4(e+fx)))\csc^8\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}{8f}\right)\sin^9\left(\frac{1}{2}(e+fx)\right)}{(1+\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] (Sec[e + f*x]^(9/2)*(a*(1 + Sec[e + f*x]))^(5/2)*((16*Sqrt[2]*E^((I/2)*(e +
 f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*((-I)*f*x +
```

```
2*Log[1 - E^(I*(e + f*x))]))/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1
 + E^((2*I)*(e + f*x)))]*f) + ((-54 + 89*Cos[e + f*x] - 60*Cos[2*(e + f*x)]
 + 23*Cos[3*(e + f*x)] - 6*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]^8*Sec[(e + f*
x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/(8*f))*Sin[(e + f*x)/2]^9)
/((1 + Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(9/2))
```

Maple [A]
time = 0.28, size = 353, normalized size = 1.82

method	result
default	$-\frac{(-1+\cos(fx+e))\left(32(\cos^4(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-16(\cos^4(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-29(\cos^4(fx+e))-128(\cos^3(fx+e))\right.}{}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^4\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^4\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(6\,e^{7i(fx+e)}\right.}{c^4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/f*(-1+cos(f*x+e))*(32*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-16
*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-29*cos(f*x+e)^4-128*cos(f*x+e)^3*ln(-(-1
+cos(f*x+e))/sin(f*x+e))+64*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+20*cos(f*x+e)
^3+192*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-96*cos(f*x+e)^2*ln(2/(c
os(f*x+e)+1))+10*cos(f*x+e)^2-128*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e)
)+64*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-28*cos(f*x+e)+32*ln(-(-1+cos(f*x+e))/s
in(f*x+e))-16*ln(2/(cos(f*x+e)+1))+11)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/
sin(f*x+e)/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(9/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6623 vs. 2(189) = 378.
time = 23.48, size = 6623, normalized size = 34.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 +
 4900*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*cos(2*f*x + 2*e)
^2 + 64*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 3136*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 3136*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 64*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
```

```
^2 + (f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*sin(6*f*x + 6*e)^
2 + 4900*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*sin(2*f*x + 2
*e)^2 + 64*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 3136*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 3136*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 64*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 56*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 46*a^2*sin(2*f*
x + 2*e) - 2*(a^2*cos(8*f*x + 8*e)^2 + 784*a^2*cos(6*f*x + 6*e)^2 + 4900*a^
2*cos(4*f*x + 4*e)^2 + 784*a^2*cos(2*f*x + 2*e)^2 + 64*a^2*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 64*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + a^2*sin(8*f*x + 8*e)^2 + 784*a^2*sin(6*f*x + 6*e)^2 + 4900*a^2*
sin(4*f*x + 4*e)^2 + 3920*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*a^2*s
in(2*f*x + 2*e)^2 + 64*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 3136*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
3136*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*a^2*cos(2*f*x + 2
*e) + a^2 + 2*(28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*a^2*c
os(2*f*x + 2*e) + a^2)*cos(8*f*x + 8*e) + 56*(70*a^2*cos(4*f*x + 4*e) + 28*
a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 140*(28*a^2*cos(2*f*x + 2*e)
 + a^2)*cos(4*f*x + 4*e) - 16*(a^2*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*
e) + 70*a^2*cos(4*f*x + 4*e) + 28*a^2*cos(2*f*x + 2*e) - 56*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + a^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 112*(a^2*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x +
4*e) + 28*a^2*cos(2*f*x + 2*e) - 56*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(a^2*
cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*a
^2*cos(2*f*x + 2*e) - 8*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(a^2
*cos(8*f*x + 8*e) + 28*a^2*cos(6*f*x + 6*e) + 70*a^2*cos(4*f*x + 4*e) + 28*
a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 28*(2*a^2*sin(6*f*x + 6*e) + 5*a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x
 + 2*e))*sin(6*f*x + 6*e) - 16*(a^2*sin(8*f*x + 8*e) + 28*a^2*sin(6*f*x + 6
*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*x + 2*e) - 56*a^2*sin(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*a^2*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112
*(a^2*sin(8*f*x + 8*e) + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e)
+ 28*a^2*sin(2*f*x + 2*e) - 56*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
```

```
sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(a^2*sin(8*f*x +
 8*e) + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*
x + 2*e) - 8*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(a^2*sin(8*f*x + 8*e)
 + 28*a^2*sin(6*f*x + 6*e) + 70*a^2*sin(4*f*x + 4*e) + 28*a^2*sin(2*f*x + 2
*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 2*(28*(f*x + e)*a^2*cos(6*f*x + 6*e) + 70*(f
*x + e)*a^2*cos(4*f*x + 4*e) + 28*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e
)*a^2 - 23*a^2*sin(6*f*x + 6*e) - 66*a^2*sin(4*f*x + 4*e) - 23*a^2*sin(2*f*
x + 2*e))*cos(8*f*x + 8*e) + 28*(140*(f*x + e)*a^2*cos(4*f*x + 4*e) + 56*(f
*x + e)*a^2*cos(2*f*x + 2*e) + 2*(f*x + e)*a^2 - 17*a^2*sin(4*f*x + 4*e))*c
os(6*f*x + 6*e) + 28*(140*(f*x + e)*a^2*cos(2*f*x + 2*e) + 5*(f*x + e)*a^2
+ 17*a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(4*(f*x + e)*a^2*cos(8*f*x
+ 8*e) + 112*(f*x + e)*a^2*cos(6*f*x + 6*e) + 280*(f*x + e)*a^2*cos(4*f*x +
 4*e) + 112*(f*x + e)*a^2*cos(2*f*x + 2*e) - 22...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^5*sec(f*x + e)^5 - 5*c^5*sec(f*x + e)^
4 + 10*c^5*sec(f*x + e)^3 - 10*c^5*sec(f*x + e)^2 + 5*c^5*sec(f*x + e) - c^
5), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.16, size = 230, normalized size = 1.19

$$-\frac{\frac{48\sqrt{-ac}\,a^{3}\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{c^{5}|a|}-\frac{48\sqrt{-ac}\,a^{3}\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right|\right)}{c^{5}|a|}-\frac{100\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{4}\sqrt{-ac}\,a^{3}+352\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{3}\sqrt{-ac}\,a^{4}+480\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{2}\sqrt{-ac}\,a^{5}+292\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)\sqrt{-ac}\,a^{6}+67\sqrt{-ac}\,a^{7}}{a^{4}c^{5}|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] -1/48*(48*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^5*abs(a)) -
48*sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^5*abs(a)) - (1
00*(a*tan(1/2*f*x + 1/2*e)^2 - a)^4*sqrt(-a*c)*a^3 + 352*(a*tan(1/2*f*x + 1
/2*e)^2 - a)^3*sqrt(-a*c)*a^4 + 480*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-
a*c)*a^5 + 292*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^6 + 67*sqrt(-a*c
)*a^7)/(a^4*c^5*abs(a)*tan(1/2*f*x + 1/2*e)^8))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2), x)
```

3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}}-\frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}}-\frac{}{2c^3f\sqrt{a}}$$

```
[Out] -4/5*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)/(a+a*sec(f*x+e))^(1/2)-1/3*a^
3*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*ta
n(f*x+e)/c^3/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-a^3*tan(f*x+e)
/c^4/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)+a^3*ln(1-cos(f*x+e))*t
an(f*x+e)/c^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3995, 3992, 3996, 31}

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^5f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a^3\tan(e+fx)}{c^4f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{a^3\tan(e+fx)}{2c^3f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}-\frac{a^3\tan(e+fx)}{3c^2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

```
[Out] (-4*a^3*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1
1/2)) - (a^3*Tan[e + f*x])/(3*c^2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
 f*x])^(7/2)) - (a^3*Tan[e + f*x])/(2*c^3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c
*Sec[e + f*x])^(5/2)) - (a^3*Tan[e + f*x])/(c^4*f*Sqrt[a + a*Sec[e + f*x]]*
(c - c*Sec[e + f*x])^(3/2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^5
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}} + \frac{a^2\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}}{c^2}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+}{3c^2f\sqrt{a+a\sec(e+fx)}\,(}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+}{3c^2f\sqrt{a+a\sec(e+fx)}\,(}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+}{3c^2f\sqrt{a+a\sec(e+fx)}\,(}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+}{3c^2f\sqrt{a+a\sec(e+fx)}\,(}$$

$$= -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{11/2}} - \frac{a^3\tan(e+}{3c^2f\sqrt{a+a\sec(e+fx)}\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.24, size = 299, normalized size = 1.23

$$\frac{\sec^{\frac{11}{2}}(e+fx)(a(1+\sec(e+fx)))^{5/2}\left(\frac{32i\sqrt{2}\,e^{\frac{1}{2}i(e+fx)}\sqrt{\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\,\left(fx+2i\log\left(1-e^{i(e+fx)}\right)\right)}{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,f} - \frac{(5612\cos(e+fx)-5(625+736\cos(2(e+fx))-367\cos(3(e+fx))+111\cos(4(e+fx))-21\cos(5(e+fx))))\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}{240f}\right)\sin^{11}\left(\frac{1}{2}(e+fx)\right)}{(1+\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]

[Out] (Sec[e + f*x]^(11/2)*(a*(1 + Sec[e + f*x]))^(5/2)*(((32*I)*Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*(f*x + (2*I)*Log[1 - E^(I*(e + f*x))]))/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f) - ((5612*Cos[e + f*x] - 5*(625 + 736*Cos[2*(e + f*x)] - 367*Cos[3*(e + f*x)] + 111*Cos[4*(e + f*x)] - 21*Cos[5*(e + f*x)]))*Csc[(e + f*x)/2]^10*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/(240*f))*Sin[(e + f*x)/2]^11)/((1 + Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(11/2))

Maple [**A**]
time = 0.30, size = 415, normalized size = 1.70

method	result
default	$\frac{(-1+\cos(fx+e))\left(120(\cos^5(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-240(\cos^5(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+233(\cos^5(fx+e))-600(\cos^4(fx+e)\right.}{}$
risch	$\frac{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)x}{c^5(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}}-\frac{2a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)(fx+e)}{c^5(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(105\,e^{9i(fx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE)

[Out] 1/120/f*(-1+cos(f*x+e))*(120*cos(f*x+e)^5*ln(2/(cos(f*x+e)+1))-240*cos(f*x+e)^5*ln(-(-1+cos(f*x+e))/sin(f*x+e))+233*cos(f*x+e)^5-600*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+1200*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-325*cos(f*x+e)^4+1200*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-2400*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))+110*cos(f*x+e)^3-1200*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2400*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+290*cos(f*x+e)^2+600*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-1200*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-295*cos(f*x+e)-120*ln(2/(cos(f*x+e)+1))+240*ln(-(-1+cos(f*x+e))/sin(f*x+e))+83)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)^5/(c*(-1+cos(f*x+e))/cos(f*x+e))^(11/2)*a^2

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9881 vs. 2(234) = 468.
time = 140.96, size = 9881, normalized size = 40.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] -1/15*(15*(f*x + e)*a^2*cos(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*cos(8*f*
x + 8*e)^2 + 661500*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 661500*(f*x + e)*a^2
*cos(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 1500*(f*x +
e)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x
 + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952560*(
f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 21600
0*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 15
00*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1
5*(f*x + e)*a^2*sin(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*sin(8*f*x + 8*e)
^2 + 661500*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 661500*(f*x + e)*a^2*sin(4*f
*x + 4*e)^2 + 30375*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 1500*(f*x + e)*a^2*s
in(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x + e)*a^
2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952560*(f*x + e)
*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x +
 e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1500*(f*x
+ e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1350*(f*x
 + e)*a^2*cos(2*f*x + 2*e) + 15*(f*x + e)*a^2 - 1110*a^2*sin(2*f*x + 2*e) -
 30*(a^2*cos(10*f*x + 10*e)^2 + 2025*a^2*cos(8*f*x + 8*e)^2 + 44100*a^2*cos
(6*f*x + 6*e)^2 + 44100*a^2*cos(4*f*x + 4*e)^2 + 2025*a^2*cos(2*f*x + 2*e)^
2 + 100*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*
a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*a^2*cos(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*a^2*cos(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(10*f*x + 10*e)^2 + 2025*a^2*sin(
8*f*x + 8*e)^2 + 44100*a^2*sin(6*f*x + 6*e)^2 + 44100*a^2*sin(4*f*x + 4*e)^
2 + 18900*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*a^2*sin(2*f*x + 2*e)
^2 + 100*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400
*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*a^2*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*a^2*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*a^2*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*a^2*cos(2*f*x + 2*e) + a^2 + 2*(45*a
^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*e) + 210*a^2*cos(4*f*x + 4*e) +
 45*a^2*cos(2*f*x + 2*e) + a^2)*cos(10*f*x + 10*e) + 90*(210*a^2*cos(6*f*x
+ 6*e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) + a^2)*cos(8*f*
x + 8*e) + 420*(210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) + a^2)*c
os(6*f*x + 6*e) + 420*(45*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 20
*(a^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*
e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) - 120*a^2*cos(7/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*a^2*cos(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 10*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + a^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(a
```

```
^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*e)
+ 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) - 252*a^2*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + a^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 504*(a^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*
x + 6*e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) - 120*a^2*cos
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 240*(a^2*cos(10*f*x + 10*e) + 45*a^2*cos(8*f*x + 8*
e) + 210*a^2*cos(6*f*x + 6*e) + 210*a^2*cos(4*f*x + 4*e) + 45*a^2*cos(2*f*x
 + 2*e) - 10*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2
)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(a^2*cos(10*f*x
 + 10*e) + 45*a^2*cos(8*f*x + 8*e) + 210*a^2*cos(6*f*x + 6*e) + 210*a^2*cos
(4*f*x + 4*e) + 45*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 30*(3*a^2*sin(8*f*x + 8*e) + 14*a^2*sin(6*f*x +
6*e) + 14*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*sin(10*f*x + 10*e)
 + 1350*(14*a^2*sin(6*f*x + 6*e) + 14*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*
x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) - 20*(a^2*sin(10*f*x + 10*e) + 45*a^2*sin(8*f*x
+ 8*e) + 210*a^2*sin(6*f*x + 6*e) + 210*a^2*sin(4*f*x + 4*e) + 45*a^2*sin(2
*f*x + 2*e) - 120*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 252*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*a^2*si
n(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*a^2*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(c^6*sec(f*x + e)^6 - 6*c^6*sec(f*x + e)^5
 + 15*c^6*sec(f*x + e)^4 - 20*c^6*sec(f*x + e)^3 + 15*c^6*sec(f*x + e)^2 -
6*c^6*sec(f*x + e) + c^6), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.78, size = 260, normalized size = 1.07

$$-\frac{\frac{120\,\sqrt{-ac}\,a^{3}\log\left(|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{c^{6}|a|}-\frac{120\,\sqrt{-ac}\,a^{3}\log\left(\left|-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right|\right)}{c^{6}|a|}-\frac{274\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{5}\sqrt{-ac}\,a^{3}+1250\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{4}\sqrt{-ac}\,a^{4}+2320\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{3}\sqrt{-ac}\,a^{5}+2165\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{2}\sqrt{-ac}\,a^{6}+1015\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)\sqrt{-ac}\,a^{7}+191\,\sqrt{-ac}\,a^{8}}{a^{5}c^{6}|a|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] -1/120*(120*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^6*abs(a))
- 120*sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^6*abs(a)) -
 (274*(a*tan(1/2*f*x + 1/2*e)^2 - a)^5*sqrt(-a*c)*a^3 + 1250*(a*tan(1/2*f*x
 + 1/2*e)^2 - a)^4*sqrt(-a*c)*a^4 + 2320*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*s
qrt(-a*c)*a^5 + 2165*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^6 + 1015
*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^7 + 191*sqrt(-a*c)*a^8)/(a^5*c
^6*abs(a)*tan(1/2*f*x + 1/2*e)^10))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2), x)
```

3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=204

$$\frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{8c^4\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\sec(e+}{f\sqrt{a+a\sec(e+}}$$

```
[Out] c^4*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+8*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)-4*c^4*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x
+e))^(1/2)+1/2*c^4*sec(f*x+e)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*se
c(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$\frac{c^4\tan(e+fx)\sec^2(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{8c^4\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{c^4\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (8*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Sec[e + f*x]*Tan[e + f*x])
/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]^
2*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{4c^3}{a}+\frac{c^3}{ax}-\frac{c^3x}{a}-\frac{8c^3}{a(1+x)}\right)\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{8c^4\log(1+\sec(e+fx))\tan(}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\,\text{se}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.00, size = 153, normalized size = 0.75

$$\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\left(-1+ifx+8\cos(e+fx)-16\log\left(1+e^{i(e+fx)}\right)+7\log\left(1+e^{2i(e+fx)}\right)+\cos(2(e+fx))\left(ifx-16\log\left(1+e^{i(e+fx)}\right)+7\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^3*Cot[(e + f*x)/2]*(-1 + I*f*x + 8*Cos[e + f*x] - 16*Log[1 + E^(I*(e + f
*x))] + 7*Log[1 + E^((2*I)*(e + f*x))] + Cos[2*(e + f*x)]*(I*f*x - 16*Log[1
 + E^(I*(e + f*x))] + 7*Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^2*Sqrt[
c - c*Sec[e + f*x]])/(2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.27, size = 189, normalized size = 0.93

method	result
default	$\frac{\left(14\left(\cos^2(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+14\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+2\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+9}{2f\sin(fx+e)(-1+\cos(fx+e))^3a}$
risch	$\frac{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)} - \frac{2c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f} + \frac{2ic^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\left(4\,e^{2i(fx+e)}-e\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(14*cos(f*x+e)^2*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+14*cos(f*x
+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^2*ln(2/(cos(f*
x+e)+1))+9*cos(f*x+e)^2+8*cos(f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2
)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)^2/sin(f*x+e)/(-1+cos(f*x+e
))^3/a
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=151

$$\frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{c^3\sec(e+\ldots}{f\sqrt{a+a\sec(e+\ldots}}$$

```
[Out] c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+4*c^3*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)-c^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$-\frac{c^3\tan(e+fx)\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (c^3*Sec[e + f*x]*Tan[e + f*x])/(
f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^2}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{c^2}{a}+\frac{c^2}{ax}-\frac{4c^2}{a(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4c^3\log(1+\sec(e+fx))\tan(}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\,\mathrm{se}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.01, size = 181, normalized size = 1.20

$$\frac{c^2e^{-3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^3\cos\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\left(1+\cos(e+fx)\left(ifx-8\log\left(1+e^{i(e+fx)}\right)+3\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)}{4\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^2*(1 + E^((2*I)*(e + f*x)))^3*Cos[(e + f*x)/2]*Cot[(e + f*x)/2]*(1 + Cos
[e + f*x]*(I*f*x - 8*Log[1 + E^(I*(e + f*x))] + 3*Log[1 + E^((2*I)*(e + f*x
))]))*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e
+ f*x)/2]))/(4*E^((3*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[
e + f*x])])
```

Maple [A]
time = 0.26, size = 169, normalized size = 1.12

method	result
default	$\frac{\left(3\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+3\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+1\right)}{f\sin(fx+e)(-1+\cos(fx+e))^2a}$
risch	$\frac{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)} - \frac{2c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f} + \frac{2ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+e\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f\left(e^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(3*cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)*ln
(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos
(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*cos(f*x+e)^2*(a*(cos(f*x+e)
+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))^2/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(-c*sec(f*x +
e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=102

$$\frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 78}

$$\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{c-cx}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{c}{ax}-\frac{2c}{a(1+x)}\right)dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\log(1+\sec(e+fx))\tan(}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\text{se}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.32, size = 103, normalized size = 1.01

$$-\frac{c\left(1+e^{i(e+fx)}\right)\left(fx+4i\log\left(1+e^{i(e+fx)}\right)-i\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{\left(-1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*(1 + E^(I*(e + f*x)))*(f*x + (4*I)*Log[1 + E^(I*(e + f*x))] - I*Log[1
+ E^((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/((-1 + E^(I*(e + f*x)))*f
*Sqrt[a*(1 + Sec[e + f*x])]))
```

Maple [A]
time = 0.26, size = 93, normalized size = 0.91

method	result
default	$\frac{\ln\left(-\frac{4\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}(\cos^2(fx+e))\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{f\sin(fx+e)(-1+\cos(fx+e))a}$
risch	$\frac{c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}(e^{i(fx+e)}-1)} - \frac{2c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}(e^{i(fx+e)}-1)f} - \frac{4ic(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,\ln(e}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}(e^{i(fx+e)}-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*ln(-4*cos(f*x+e)/(cos(f*x+e)+1)^2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
*cos(f*x+e)^2*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e)
)/a
```

Maxima [A]
time = 0.59, size = 65, normalized size = 0.64

$$-\frac{\left((fx+e)c + c\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right) - 4\,c\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)\right)\sqrt{c}}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c + c*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*c*arc
tan2(sin(f*x + e), cos(f*x + e) + 1))*sqrt(c)/(sqrt(a)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((-c*sec(f*x + e) + c)^(3/2)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=49

$$\frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] c*ln(1+cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3996, 31}

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}\\ &= \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.99, size = 127, normalized size = 2.59

$$-\frac{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{c\left(-1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)}{\left(-1+e^{i(e+fx)}\right)\sqrt{\frac{a\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -(((1 + E^(I*(e + f*x)))*Sqrt[(c*(-1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e
 + f*x)))]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]))/((-1 + E^(I*(e + f*x)))*
Sqrt[(a*(1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*x)))]*f))
```

Maple [A]
time = 0.24, size = 75, normalized size = 1.53

method	result
default	$\frac{\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)}{f\sin(fx+e)a}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)}-\frac{2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}-\frac{2i\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln\left(e^{i(f\right.}}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2
)*cos(f*x+e)*ln(2/(cos(f*x+e)+1))/sin(f*x+e)/a
```

Maxima [A]
time = 0.50, size = 36, normalized size = 0.73

$$\frac{\sqrt{c}\,\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral(sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [A]
time = 1.22, size = 36, normalized size = 0.73

$$-\frac{\sqrt{-ac}\,c\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{af|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
[Out] -sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*f*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(sin(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3990, 3556}

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{\tan(e+fx)\int \cot(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= \frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.15, size = 104, normalized size = 2.26

$$-\frac{2\left(-1+e^{i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\left(fx+i\log\left(1-e^{2i(e+fx)}\right)\right)\sec(e+fx)}{\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (-2*(-1 + E^(I*(e + f*x)))*Cos[(e + f*x)/2]^2*(f*x + I*Log[1 - E^((2*I)*(e
+ f*x))])*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]
*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.
time = 0.31, size = 101, normalized size = 2.20

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\cos(fx+e)}{f\sin(fx+e)ca}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(ln(-(-1+cos(f*x+e))/sin(f*x+e))-l
n(2/(cos(f*x+e)+1)))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*
x+e)/c/a
```

Maxima [A]
time = 0.58, size = 42, normalized size = 0.91

$$-\frac{fx-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)+e}{\sqrt{a}\,\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -(f*x - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) - 1) + e)/(sqrt(a)*sqrt(
c)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(46) = 92$.
time = 3.45, size = 292, normalized size = 6.35

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(256\cos\left(fx+e\right)^{5}-512\cos\left(fx+e\right)^{3}+175\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-\left(256ac\cos\left(fx+e\right)^{4}-512ac\cos\left(fx+e\right)^{2}+337ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)}{2acf}, -\frac{\sqrt{ac}\arctan\left(\frac{\left(16\cos\left(fx+e\right)^{3}-7\cos\left(fx+e\right)\right)\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(16ac\cos\left(fx+e\right)^{2}-25ac\right)\sin\left(fx+e\right)}\right)}{acf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((256*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos
(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f
*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)
^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), -
sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((16*a
*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Giac [A]
time = 1.29, size = 67, normalized size = 1.46

$$\frac{\frac{\sqrt{-ac}\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{a|c|}-\frac{2\,\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right|\right)}{a|c|}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/2*(sqrt(-a*c)*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a*abs(c)) - 2*sqrt(-a*c
)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*abs(c)))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{\tan(e+fx)}{2cf(1-\cos(e+fx))\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{3\log(1-\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*tan(f*x+e)/c/f/(1-cos(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)+3/4*ln(1-cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+1/4*ln(1+cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c
*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 217, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$-\frac{\tan(e+fx)}{2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{3\tan(e+fx)\log(1-\sec(e+fx))}{4cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\sec(e+fx)+1)}{4cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*c*f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\mathrm{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^2}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\mathrm{Subst}\left(\int \left(\frac{1}{2ac^2(-1+x)^2} - \frac{3}{4ac^2(-1+x)} + \right.\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{3\,\mathrm{lo}}{4cf\sqrt{a-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.50, size = 143, normalized size = 0.85

$$\frac{\left(-1+2ifx-3\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{i(e+fx)}\right)+\cos(e+fx)\left(-2ifx+3\log\left(1-e^{i(e+fx)}\right)+\log\left(1+e^{i(e+fx)}\right)\right)\right)\tan(e+fx)}{2cf(-1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] ((-1 + (2*I)*f*x - 3*Log[1 - E^(I*(e + f*x))] - Log[1 + E^(I*(e + f*x))] +
Cos[e + f*x]*((-2*I)*f*x + 3*Log[1 - E^(I*(e + f*x))] + Log[1 + E^(I*(e + f
*x))]))*Tan[e + f*x])/(2*c*f*(-1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]
*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.27, size = 167, normalized size = 0.99

method	result
default	$-\frac{(-1+\cos(fx+e))\left(6\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-6\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)+4\ln\left(\frac{\ }{\mathrm{co}}\right.\right.}{4f\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)a}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}} - \frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} + \frac{\ }{c\sqrt{\ }}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(-1+cos(f*x+e))*(6*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-4*cos(
f*x+e)*ln(2/(cos(f*x+e)+1))-6*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)+4*
ln(2/(cos(f*x+e)+1))-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(
-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(160) = 320$.
time = 0.59, size = 889, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x +
e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - (cos(2*
f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1
) - 3*(cos(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(4*f*x + 4*(f*x + e)*cos(
2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((c*cos(2*f*x +
 2*e)^2 + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*si
n(2*f*x + 2*e)^2 - 4*c*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*c*cos(2*f*x + 2*e) - 4*(c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*sqrt(a)*sqrt(c)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^2*sec(f*x
+ e)^3 - a*c^2*sec(f*x + e)^2 - a*c^2*sec(f*x + e) + a*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)), x)
```

Giac [A]
time = 2.00, size = 112, normalized size = 0.67

$$-\frac{\frac{3\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{\sqrt{-ac}\,|c|}+\frac{4\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right|\right)}{ac|c|}-\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}{\sqrt{-ac}\,c|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/4*(3*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*abs(c)) + 4*sqrt(-a*
c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*c*abs(c)) - (3*c*tan(1/2*f*x +
 1/2*e)^2 - c)/(sqrt(-a*c)*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=274

$$\frac{\log(\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{7\log(1-\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{\log(1+}{8c^2f\sqrt{a+a\,\mathrm{s}}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+7/8*ln(1-sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+1/8*ln(1+sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))/(a+a*sec(
f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$-\frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{7\tan(e+fx)\log(1-\sec(e+fx))}{8c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\sec(e+fx)+1)}{8c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (7*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*c^2*f*Sqrt[a +
a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*c^2*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^3}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{1}{2ac^3(-1+x)^3}+\frac{3}{4ac^3(-1+x)^2}\right.\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{c^2 f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{7}{8c^2 f\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.01, size = 194, normalized size = 0.71

$$\frac{\left(8-12ifx+21\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(-10+16ifx-28\log\left(1-e^{i(e+fx)}\right)-4\log\left(1+e^{i(e+fx)}\right)\right)+3\log\left(1+e^{i(e+fx)}\right)+\cos(2(e+fx))\left(-4ifx+7\log\left(1-e^{i(e+fx)}\right)+\log\left(1+e^{i(e+fx)}\right)\right)\right)\tan(e+fx)}{8c^2 f(-1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] ((8 - (12*I)*f*x + 21*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-10 + (16*I)
*f*x - 28*Log[1 - E^(I*(e + f*x))] - 4*Log[1 + E^(I*(e + f*x))]) + 3*Log[1
+ E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-4*I)*f*x + 7*Log[1 - E^(I*(e + f*x
))] + Log[1 + E^(I*(e + f*x))]))*Tan[e + f*x])/(8*c^2*f*(-1 + Cos[e + f*x])
^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.28, size = 229, normalized size = 0.84

method	result
default	$-\frac{(-1+\cos(fx+e))\left(28\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-16\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-9\left(\cos^2(fx+e)\right)-56\cos(fx+e)\ln\left(-\right.\right.}{16f\sin(fx+e)\cos(fx}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{2i(fx+e)}+1\right)}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{2i(fx+e)}+1\right)f}+\frac{}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/16/f*(-1+cos(f*x+e))*(28*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-16
*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-9*cos(f*x+e)^2-56*cos(f*x+e)*ln(-(-1+cos
```

```
(f*x+e))/sin(f*x+e))+32*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)+28*ln(
-(-1+cos(f*x+e))/sin(f*x+e))-16*ln(2/(cos(f*x+e)+1))+7)*(a*(cos(f*x+e)+1)/c
os(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/
2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2398 vs. $2(262) = 524$.
time = 0.77, size = 2398, normalized size = 8.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 6
4*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)
*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - (2*(6*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 - 8*(
cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(4*f
*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x +
 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) - 4*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8
*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)
 - 7*(2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36
*cos(2*f*x + 2*e)^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 36*sin(2*f*x + 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e)
 - 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e),
```

```
 cos(2*f*x + 2*e)))^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 1) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + e - 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) + 48*(f*x + e)*cos(2*f*x + 2*e) - 2*(16*f
*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) - 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(
4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos
(2*f*x + 2*e) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 2*(16*(f*x + e)*sin(4*f*x + 4*e) +
 96*(f*x + e)*sin(2*f*x + 2*e) - 64*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*sin(4*f*
x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e) - 5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*e - 1
6*sin(2*f*x + 2*e))/((c^2*cos(4*f*x + 4*e)^2 + 36*c^2*cos(2*f*x + 2*e)^2 +
16*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c^2*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2
 + 12*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c^2*sin(2*f*x + 2*e)^2 + 1
6*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c^2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*c^2*cos(2*f*x + 2*e)
 + c^2 + 2*(6*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 8*(c^2*cos(4*f
*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) - 4*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 8*(c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^2*sin(4*f*x + 4*e) + 6*c^2
*sin(2*f*x + 2*e) - 4*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^2*sin(4*f*
x + 4*e) + 6*c^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)*sqrt(c)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^3*sec(f*x
 + e)^4 - 2*a*c^3*sec(f*x + e)^3 + 2*a*c^3*sec(f*x + e) - a*c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(5/2)), x)
```

Giac [A]
time = 1.76, size = 144, normalized size = 0.53

$$-\frac{\frac{14\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{\sqrt{-ac}\,c|c|}+\frac{16\,\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right|\right)}{ac^{2}|c|}-\frac{21\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}+34\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c+14\,c^{2}}{\sqrt{-ac}\,c^{3}|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/16*(14*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*c*abs(c)) + 16*sqr
t(-a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*c^2*abs(c)) - (21*(c*tan(
1/2*f*x + 1/2*e)^2 - c)^2 + 34*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 14*c^2)/(
sqrt(-a*c)*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{c^4\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c^4\sec(}{af\sqrt{a+a\sec(}}$$

```
[Out] c^4*ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-4*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+c^4*sec(f*x+e)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec
(f*x+e))^(1/2)-8*c^4*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(
c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 90}

$$\frac{c^4\tan(e+fx)\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{8c^4\tan(e+fx)}{af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c^4\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) - (4*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c^4*Sec[e + f*x]*Tan[e + f*x
])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (8*c^4*Tan[e +
 f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{c^3}{a^2}+\frac{c^3}{a^2x}-\frac{8c^3}{a^2(1+x)^2}+\frac{4c^3}{a^2(1+x)}\right)dx, x, \sec(e+f\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\log(1+\sec(e+fx))\,\text{ta}}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.22, size = 204, normalized size = 0.95

$$\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\left(-2+ifx+8\log\left(1+e^{i(e+fx)}\right)+2\cos(e+fx)\left(-9+ifx+8\log\left(1+e^{i(e+fx)}\right)-5\log\left(1+e^{2i(e+fx)}\right)\right)+\cos(2(e+fx))\left(ifx+8\log\left(1+e^{i(e+fx)}\right)-5\log\left(1+e^{2i(e+fx)}\right)\right)-5\log\left(1+e^{2i(e+fx)}\right)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{2af(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^3*Cot[(e + f*x)/2]*(-2 + I*f*x + 8*Log[1 + E^(I*(e + f*x))] + 2*Cos[e +
f*x]*(-9 + I*f*x + 8*Log[1 + E^(I*(e + f*x))] - 5*Log[1 + E^((2*I)*(e + f*x
))]) + Cos[2*(e + f*x)]*(I*f*x + 8*Log[1 + E^(I*(e + f*x))] - 5*Log[1 + E^(
(2*I)*(e + f*x))]) - 5*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]*Sqrt[c -
c*Sec[e + f*x]])/(2*a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.26, size = 275, normalized size = 1.28

method	result
default	$-\frac{\left(\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-5\left(\cos^2(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-5\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos\right.}{}$
risch	$\frac{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)}-\frac{2c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}-\frac{2ic^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(9\,e^{3i(fx+e)}+\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-5*cos(f*x+e)^2*ln((-cos(f*x+e)+1+si
n(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+
e))+cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)^2-5*cos(f*x+e)*ln((-cos(f*
```

```
x+e)+1+sin(f*x+e))/sin(f*x+e))-5*cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/s
in(f*x+e))-6*cos(f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*cos(f*x+e)^
3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e))^2/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2583 vs. $2(215) = 430$.
time = 0.82, size = 2583, normalized size = 12.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*c^3*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + (
f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*sin(2*f*x + 2*e)^2 + 4*(f
*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 4*c^3*sin(2*f*x + 2*e) + 4*(
(f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c
^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 5*(c
^3*cos(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + c^3*sin(4*f*x + 4*e)^2 +
 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3
*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x + 4*e)
)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 8*(c^3*cos(4*f*x + 4*e)
^2 + 4*c^3*cos(2*f*x + 2*e)^2 + 4*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + c^3*sin(4*f*x + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*
x + 4*e) + 4*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e) + 2*c^3*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x
+ 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^3
*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 4*(c^3*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 2*(2*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 2*c^3*sin(
2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*cos(4*f*x + 4*e) + 4*(f
```

```
*x + e)*c^3*cos(2*f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*sin(4*f*x + 4*e) + 1
4*c^3*sin(2*f*x + 2*e) - 10*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e)
+ c^3)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 -
 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 2*(2*(f*x + e)*c^3*cos(4*f*x + 4*e) + 4*(f*x + e)*c^3*cos(2*
f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*sin(4*f*x + 4*e) + 14*c^3*sin(2*f*x +
2*e) - 10*(c^3*cos(4*f*x + 4*e) + 2*c^3*cos(2*f*x + 2*e) + c^3)*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 4*((f*x + e)*c^3*sin(2*f*x + 2*e) + c^3*cos(2*f*x + 2*e))
*sin(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*s
in(2*f*x + 2*e) - 9*c^3*cos(4*f*x + 4*e) - 14*c^3*cos(2*f*x + 2*e) - 9*c^3
- 10*(c^3*sin(4*f*x + 4*e) + 2*c^3*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)
*c^3*sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*sin(2*f*x + 2*e) - 9*c^3*cos(4*f*x
+ 4*e) - 14*c^3*cos(2*f*x + 2*e) - 9*c^3 - 10*(c^3*sin(4*f*x + 4*e) + 2*c^3
*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(c)/((a*cos(4*f*x + 4*e)
^2 + 4*a*cos(2*f*x + 2*e)^2 + 4*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*
f*x + 2*e)^2 + 4*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(2*a*cos(2*
f*x + 2*e) + a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + 4*(a*cos(4*f*x +
4*e) + 2*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(
a*cos(4*f*x + 4*e) + 2*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e) + 2
*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x
+ 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*f
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
```

```
 c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^
2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(co
s(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3995, 3990, 3556}

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\int\sqrt{a+a\sec(e+fx)}\,\sqrt{}}{a^2}$$
$$= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{(c^3\tan(e+fx))\int\tan(}{a\sqrt{a+a\sec(e+fx)}\,\sqrt{c-}}$$
$$= -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(\cos(e+fx))\,\mathrm{t}}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.78, size = 116, normalized size = 1.21

$$\frac{ic^2\cot\left(\frac{1}{2}(e+fx)\right)\left(4i+fx+\cos(e+fx)\left(fx+i\log\left(1+e^{2i(e+fx)}\right)\right)+i\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{af(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*c^2*Cot[(e + f*x)/2]*(4*I + f*x + Cos[e + f*x]*(f*x + I*Log[1 + E^((2*I)
*(e + f*x))]) + I*Log[1 + E^((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(
a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(88) = 176.
time = 0.25, size = 236, normalized size = 2.46

method	result
default	$\frac{\left(\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)+\mathrm{ln}}{f\sin(fx+e)}$
risch	$\frac{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)}-\frac{2c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}-\frac{8ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(f}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,(e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln((-c
os(f*x+e)+1+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f
*x+e)+ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+ln((-cos(f*x+e)+1+sin(f*x+e
))/sin(f*x+e))-ln(2/(cos(f*x+e)+1))+2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

```
*cos(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+
e))/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(95) = 190.
time = 4.20, size = 489, normalized size = 5.09

$$\left[-\frac{4\,c^2\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(ac^2\cos\left(fx+e\right)^2+2\,ac^2\cos\left(fx+e\right)+ac^2\right)\sqrt{-\frac{c}{a}}\log\left(\frac{c\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-\frac{c}{a}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sin(fx+e)+c}{2\cos(fx+e)^2}\right)}{2\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)},\ -\frac{2\,c^2\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(ac^2\cos\left(fx+e\right)^2+2\,ac^2\cos\left(fx+e\right)+ac^2\right)\sqrt{\frac{c}{a}}\arctan\left(\frac{\sqrt{\frac{c}{a}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)}{c\cos(fx+e)^2+c}\right)}{a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
 c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c
^2*cos(f*x + e) + a*c^2)*sqrt(-c/a)*log(1/2*(c*cos(f*x + e)^4 - (cos(f*x +
e)^3 + cos(f*x + e))*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + c)/cos(f*x + e)^2))/(a^
2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqr
t(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(c*cos(f*x + e)^2 +
c)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1+
cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3993, 3996, 31}

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
```

qQ[m + n, 0]

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$
$$= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{(c^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{}{a}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{}}$$
$$= -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.21, size = 114, normalized size = 1.16

$$\frac{ic\cot\left(\frac{1}{2}(e+fx)\right)\left(2i+fx+\cos(e+fx)\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+2i\log\left(1+e^{i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{af(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
[Out] (I*c*Cot[(e + f*x)/2]*(2*I + f*x + Cos[e + f*x]*(f*x + (2*I)*Log[1 + E^(I*(
e + f*x))]) + (2*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a*
f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.24, size = 106, normalized size = 1.08

method	result
default	$-\frac{\left(\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos(fx+e)+\ln\left(\frac{2}{\cos(fx+e)+1}\right)-1\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\left(\cos^2(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{f\sin(fx+e)^3a^2}$
risch	$\frac{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\;x}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\;\left(e^{i(fx+e)}-1\right)} - \frac{2c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\;(fx+e)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\;\left(e^{i(fx+e)}-1\right)f} - \frac{4ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\;e^{i(fx+e}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\;\left(e^{i(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/f*(cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)+ln(2/(cos(f*x+e)+1))-1)*(c
*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(a*(cos(f*x+e)+1)/cos(f*x+e
))^(1/2)/sin(f*x+e)^3/a^2
```

MaximA [A]
time = 0.49, size = 74, normalized size = 0.76

$$\frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}\,a}-\frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}\,a(\cos(fx+e)+1)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a) - c^(3/2
)*sin(f*x + e)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^2*sec(f*x
+ e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.47, size = 73, normalized size = 0.74

$$-\frac{\frac{\sqrt{-ac}\,c^2\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{a^2|c|}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}\,c}{a^2|c|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -(sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)) - (c*t
an(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c/(a^2*abs(c)))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f*
x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3992, 3996, 31}

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]))
 + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
```

qQ[m + n, 0]

Rubi steps

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$
$$= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{1}{a+}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{}}$$
$$= -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.60, size = 106, normalized size = 1.13

$$\frac{i\cot\left(\frac{1}{2}(e+fx)\right)\left(i+fx+\cos(e+fx)\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+2i\log\left(1+e^{i(e+fx)}\right)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a(1+\sec(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (I*Cot[(e + f*x)/2]*(I + f*x + Cos[e + f*x]*(f*x + (2*I)*Log[1 + E^(I*(e +
f*x))]) + (2*I)*Log[1 + E^(I*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f
*x]])/(f*(a*(1 + Sec[e + f*x]))^(3/2))
```

Maple [A]
time = 0.26, size = 119, normalized size = 1.27

method	result
default	$-\frac{\left(2\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos^2(fx+e)-2\cos(fx+e)-2\ln\left(\frac{2}{\cos(fx+e)+1}\right)+1\right)\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sqrt{\frac{a(\cos(fx+e)+}{\cos(fx+e)}}}{2f\sin(fx+e)^3a^2}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)} - \frac{2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f} - \frac{2i\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^2-2*cos(f*x+e)-2*ln(
2/(cos(f*x+e)+1))+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)
/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)^3/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(93) = 186$.
time = 0.54, size = 436, normalized size = 4.64

$$-\frac{\left(\left(fx+e\right)\cos\left(2\,fx+2\,e\right)^{2}+4\left(fx+e\right)\cos\left(fx+e\right)^{2}+\left(fx+e\right)\sin\left(2\,fx+2\,e\right)^{2}+4\left(fx+e\right)\sin\left(fx+e\right)^{2}+fx-2\left(2\left(2\cos\left(fx+e\right)+1\right)\cos\left(2\,fx+2\,e\right)+\cos\left(2\,fx+2\,e\right)^{2}+4\cos\left(fx+e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+4\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+4\sin\left(fx+e\right)^{2}+4\cos\left(fx+e\right)+1\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)+2\left(fx+2\left(fx+e\right)\cos\left(fx+e\right)+e-\sin\left(fx+e\right)\right)\cos\left(2\,fx+2\,e\right)+4\left(fx+e\right)\cos\left(fx+e\right)+2\left(2\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\sin\left(2\,fx+2\,e\right)+e-2\sin\left(fx+e\right)\right)\sqrt{a}\sqrt{c}}{\left(a^{2}\cos\left(2\,fx+2\,e\right)^{2}+4\,a^{2}\cos\left(fx+e\right)^{2}+a^{2}\sin\left(2\,fx+2\,e\right)^{2}+4\,a^{2}\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+4\,a^{2}\sin\left(fx+e\right)^{2}+4\,a^{2}\cos\left(fx+e\right)+a^{2}+2\left(2\,a^{2}\cos\left(fx+e\right)+a^{2}\right)\cos\left(2\,fx+2\,e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin
(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(2*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x + e)^2 + sin(2*f*x +
2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x + e)^2 + 4*cos(f*x + e
) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 2*(f*x + e)*cos(f
*x + e) + e - sin(f*x + e))*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + 2
*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e - 2*sin(f*x
 + e))*sqrt(a)*sqrt(c)/((a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(f*x + e)^2 + a^
2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*sin(f*x
+ e)^2 + 4*a^2*cos(f*x + e) + a^2 + 2*(2*a^2*cos(f*x + e) + a^2)*cos(2*f*x
+ 2*e))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x +
e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.35, size = 83, normalized size = 0.88

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-ac}\,c\log\left(\left|-2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,c\right|\right)}{a^{2}|c|}-\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)\sqrt{-ac}}{a^{2}|c|}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a*c)*c*log(abs(-2*c*tan(1/2*f*x + 1/2*e)^2 -
2*c))/(a^2*abs(c)) - sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2
*abs(c)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2), x)
```

3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=215

$$\frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(1-\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{3\log(1+\text{s}}{4af\sqrt{a+a\sec}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
+1/4*ln(1-sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+3/4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec
(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$-\frac{\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(1-\sec(e+fx))}{4af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{3\tan(e+fx)\log(\sec(e+fx)+1)}{4af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*a*f*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (3*Log[1 + Sec[e + f*x]]*Tan[e + f*x]
)/(4*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/
(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{1}{4a^2c(-1+x)} + \frac{1}{a^2cx} - \frac{1}{2a^2c(1\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log}{4af\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.49, size = 141, normalized size = 0.66

$$\frac{\left(1-2ifx+\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{i(e+fx)}\right)+\cos(e+fx)\left(-2ifx+\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{i(e+fx)}\right)\right)\right)\tan(e+fx)}{2af(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((1 - (2*I)*f*x + Log[1 - E^(I*(e + f*x))] + 3*Log[1 + E^(I*(e + f*x))] + C
os[e + f*x]*((-2*I)*f*x + Log[1 - E^(I*(e + f*x))] + 3*Log[1 + E^(I*(e + f*
x))]))*Tan[e + f*x])/(2*a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*S
qrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.27, size = 159, normalized size = 0.74

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))^2\left(4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+4\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\right.}{4f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sin(fx+e)^3a^2}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{a\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(e^{2i(fx+e)}+1)} - \frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{a\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(e^{2i(fx+e)}+1)f} - \frac{}{a(e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(4*cos(f*x+e)*
ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+4*ln(2/(c
os(f*x+e)+1))+cos(f*x+e)-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1)/(c*(-1+cos(f*
x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/a^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(209) = 418$.
time = 0.58, size = 889, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x +
e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - 3*(cos(
2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1) - (cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) + 2*(4*f*x + 4*(f*x + e)*cos(
2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((a*cos(2*f*x +
 2*e)^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*si
n(2*f*x + 2*e)^2 + 4*a*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*a*cos(2*f*x + 2*e) + 4*(a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*sqrt(c)*f)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c*sec(f*x
 + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e) - a^2*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Giac [A]
time = 2.40, size = 101, normalized size = 0.47

$$\frac{\frac{\sqrt{-ac}\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{a^{2}|c|}-\frac{4\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right|\right)}{a^{2}|c|}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)\sqrt{-ac}}{a^{2}c|c|}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/4*(sqrt(-a*c)*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a^2*abs(c)) - 4*sqrt(-a
*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)) + (c*tan(1/2*f*x +
1/2*e)^2 - c)*sqrt(-a*c)/(a^2*c*abs(c)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(\sin(e+fx))\tan(e+fx)}{acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*cot(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin(f
*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3990, 3554, 3556}

$$\frac{\cot(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Cot[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) +
(Log[Sin[e + f*x]]*Tan[e + f*x])/(a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)\int\cot^3(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{ac\sqrt{}}$$
$$= \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{acf\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.61, size = 121, normalized size = 1.20

$$\frac{\left(1-ifx+\cos(2(e+fx))\left(ifx-\log\left(1-e^{2i(e+fx)}\right)\right)+\log\left(1-e^{2i(e+fx)}\right)\right)\sec^2(e+fx)\tan(e+fx)}{2cf(-1+\sec(e+fx))(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] ((1 - I*f*x + Cos[2*(e + f*x)]*(I*f*x - Log[1 - E^((2*I)*(e + f*x))]) + Log
[1 - E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Tan[e + f*x])/(2*c*f*(-1 + Sec[e
+ f*x])*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.26, size = 175, normalized size = 1.73

method	result
default	$\frac{(-1+\cos(fx+e))^2\left(4(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-4(\cos^2(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-(\cos^2(fx+e))-4\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+\right.}{4f\sin(fx+e)^3\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\cos(fx+e)a^2}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{ac\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{2i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}}-\frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{ac\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{2i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{}{ac}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4/f*(-1+cos(f*x+e))^2*(4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-4*c
os(f*x+e)^2*ln(2/(cos(f*x+e)+1))-cos(f*x+e)^2-4*ln(-(-1+cos(f*x+e))/sin(f*x
+e))+4*ln(2/(cos(f*x+e)+1))-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+
e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(98) = 196$.
time = 0.57, size = 527, normalized size = 5.22

$$-\frac{\left((fx+e)\cos(4fx+4e)^2+4(fx+e)\cos(2fx+2e)^2+(fx+e)\sin(4fx+4e)^2+4(fx+e)\sin(2fx+2e)^2+fx+\left(2(2\cos(2fx+2e)-1)\cos(4fx+4e)-\cos(4fx+4e)^2-4\cos(2fx+2e)^2-\sin(4fx+4e)^2+4\sin(4fx+4e)\sin(2fx+2e)-4\sin(2fx+2e)^2+4\cos(2fx+2e)-1\right)\arctan\left(\sin(2fx+2e),\cos(2fx+2e)-1\right)+2(fx-2(fx+e)\cos(2fx+2e)+e+\sin(2fx+2e))\cos(4fx+4e)-4(fx+e)\cos(2fx+2e)-2(2(fx+e)\sin(2fx+2e)+\cos(2fx+2e))\sin(4fx+4e)+e+2\sin(2fx+2e)\right)\sqrt{a}\sqrt{c}}{\left(a^2c^2\cos(4fx+4e)^2+4a^2c^2\cos(2fx+2e)^2+a^2c^2\sin(4fx+4e)^2-4a^2c^2\sin(4fx+4e)\sin(2fx+2e)+4a^2c^2\sin(2fx+2e)^2-4a^2c^2\cos(2fx+2e)+a^2c^2-2(2a^2c^2\cos(2fx+2e)-a^2c^2)\cos(4fx+4e)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)
*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(2*cos(2*f*
x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)^2
- sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 - 1) + 2*(f*x - 2*(f*x + e)*cos(2*f*x + 2*e) + e + sin(2*f*x + 2*e))*cos(4
*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + e + 2*sin(2*f*x + 2*e))*sqrt(a)*sq
rt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2
*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*c
^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2 - 2*(2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(98) = 196$.
time = 2.34, size = 532, normalized size = 5.27

$$\left[-\frac{9\sqrt{-ac}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{8\left((256\cos(fx+e)^5-512\cos(fx+e)^3+175\cos(fx+e))\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}-(256ac\cos(fx+e)^4-512ac\cos(fx+e)^2+337ac)\sin(fx+e)\right)}{(\cos(fx+e)^2-1)\sin(fx+e)}\right)\sin(fx+e)+(16\cos(fx+e)^3-25\cos(fx+e))\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{18\left(a^2c^2f\cos(fx+e)^2-a^2c^2f\right)\sin(fx+e)},\ -\frac{18\sqrt{ac}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{(16\cos(fx+e)^3-7\cos(fx+e))\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{(16ac\cos(fx+e)^2-25ac)\sin(fx+e)}\right)\sin(fx+e)+(16\cos(fx+e)^3-25\cos(fx+e))\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{18\left(a^2c^2f\cos(fx+e)^2-a^2c^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/18*(9*sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-8*((256*cos(f*x + e)^5 - 512
*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)
^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*
sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(
(a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), -1/18*(18*sqrt(a*c)*(
cos(f*x + e)^2 - 1)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*sin(f*x + e) + (16*cos
(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
```

```
((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f
)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2)),
x)
```

Giac [A]
time = 2.18, size = 153, normalized size = 1.51

$$-\frac{\frac{4\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}\,a|c|}+\frac{8\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2c|c|}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}}{a^2c^2|c|}-\frac{4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}\,ac|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] -1/8*(4*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*abs(c)) + 8*sqrt(-
a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*c*abs(c)) - (c*tan(1/2*f*x
 + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2*c^2*abs(c)) - (4*c*tan(1/2*f*x + 1/2*e)^2
- c)/(sqrt(-a*c)*a*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=347

$$\frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{11\log(1-\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{5\,\mathrm{lo}}{16ac^2f\sqrt{c}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)+11/16*ln(1-sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)+5/16*ln(1+sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^2/f/(1-sec(f*x+e))^2/(a+a*
sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/c^2/f/(1-sec(f*x+
e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^2/f/(1
+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 90}

$$-\frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{8ac^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{8ac^2f(1-\sec(e+fx))^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{11\tan(e+fx)\log(1-\sec(e+fx))}{16ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\log(\sec(e+fx)+1)}{16ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\cos(e+fx))}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (11*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a*c^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (5*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a*c^2*f*(1 - Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a*c^2*f*(1 - Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a*c^2*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
```

```
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)^3}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(-\frac{1}{4a^2c^3(-1+x)^3}+\frac{1}{2a^2c^3(-1+x}\right.\right.}{f\sqrt{a+a\sec}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{}{16ac^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.68, size = 275, normalized size = 0.79

$$\frac{\left(14-16ifx-8ifx\cos(3(e+fx))+22\log\left(1-e^{i(e+fx)}\right)+11\cos(3(e+fx))\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(-12+8ifx-11\log\left(1-e^{i(e+fx)}\right)-5\log\left(1+e^{i(e+fx)}\right)\right)+2\cos(2(e+fx))\left(-5+8ifx-11\log\left(1-e^{i(e+fx)}\right)-5\log\left(1+e^{i(e+fx)}\right)\right)+10\log\left(1+e^{i(e+fx)}\right)+5\cos(3(e+fx))\log\left(1+e^{i(e+fx)}\right)\right)\tan(e+fx)}{32ac^2f(-1+\cos(e+fx))^2(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] ((14 - (16*I)*f*x - (8*I)*f*x*Cos[3*(e + f*x)] + 22*Log[1 - E^(I*(e + f*x))
] + 11*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-12 + (8*I
)*f*x - 11*Log[1 - E^(I*(e + f*x))] - 5*Log[1 + E^(I*(e + f*x))]) + 2*Cos[2
*(e + f*x)]*(-5 + (8*I)*f*x - 11*Log[1 - E^(I*(e + f*x))] - 5*Log[1 + E^(I*
(e + f*x))]) + 10*Log[1 + E^(I*(e + f*x))] + 5*Cos[3*(e + f*x)]*Log[1 + E^(
I*(e + f*x))])*Tan[e + f*x])/(32*a*c^2*f*(-1 + Cos[e + f*x])^2*(1 + Cos[e +
 f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.30, size = 291, normalized size = 0.84

method	result
default	$\frac{(-1+\cos(fx+e))^2\left(44(\cos^3(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-32(\cos^3(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)-13(\cos^3(fx+e))-44(\cos^2(fx+e))\ln\right.}{}$
risch	$\frac{(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)x}{ac^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{2i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}}-\frac{2(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)(fx+e)}{ac^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{2i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/32/f*(-1+cos(f*x+e))^2*(44*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3
2*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-13*cos(f*x+e)^3-44*cos(f*x+e)^2*ln(-(-1
+cos(f*x+e))/sin(f*x+e))+32*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-7*cos(f*x+e)^
2-44*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+32*cos(f*x+e)*ln(2/(cos(f*x
+e)+1))+cos(f*x+e)+44*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*ln(2/(cos(f*x+e)+1
))+11)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^3/cos(f*x+e)^2/(c*(-1
+cos(f*x+e))/cos(f*x+e))^(5/2)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4644 vs. $2(333) = 666$.
time = 3.37, size = 4644, normalized size = 13.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*(
f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x + 4*
e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 8*f*x + 5*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
 - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x + 6*e) - cos(4
*f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(cos(6*f*x + 6*
e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*
f*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - s
in(2*f*x + 2*e)^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*
e) + 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e)
```

```
, cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 - 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) - 2*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 1) + 11*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
- 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(4
*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(6*f*x + 6*e)
 - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x + 6*e) - cos(4*
f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*f
*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - si
n(2*f*x + 2*e)^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e
) + 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 - 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) - 2*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 4*(4*f*x - 4*(f*x + e)*cos(4*f*x + 4*e) - 4*
(f*x + e)*cos(2*f*x + 2*e) + 4*e + 3*sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))
*cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*cos(2*f*x + 2*e) + e)*cos(4*f*x + 4
*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 2*(16*f*x + 16*(f*x + e)*cos(6*f*x +
6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) + 64*(
f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*(f*x + e
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(6*f*x
 + 6*e) + 7*sin(4*f*x + 4*e) + 7*sin(2*f*x + 2*e) - 8*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan...
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c^3*sec(f
*x + e)^5 - a^2*c^3*sec(f*x + e)^4 - 2*a^2*c^3*sec(f*x + e)^3 + 2*a^2*c^3*s
ec(f*x + e)^2 + a^2*c^3*sec(f*x + e) - a^2*c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 1.99, size = 185, normalized size = 0.53

$$-\frac{\frac{22 \log\left(|c| \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2\right)}{\sqrt{-ac}\, ac|c|}+\frac{32 \sqrt{-ac}\, \log\left(\left|c \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2+c\right|\right)}{a^2c^2|c|}-\frac{2 \left(c \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2-c\right) \sqrt{-ac}}{a^2c^3|c|}-\frac{33 \left(c \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2-c\right)^2+56 \left(c \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2-c\right)c+24 c^2}{\sqrt{-ac}\, ac^3|c| \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^4}}{32 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] -1/32*(22*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*c*abs(c)) + 32*s
qrt(-a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^2*c^2*abs(c)) - 2*(c*ta
n(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2*c^3*abs(c)) - (33*(c*tan(1/2*f*x
+ 1/2*e)^2 - c)^2 + 56*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 24*c^2)/(sqrt(-a*
c)*a*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=220

$$\frac{c^4\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{2c^4\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{}{a^2f(1+\sec(}$$

```
[Out] c^4*ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+2*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)-4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^
(1/2)/(c-c*sec(f*x+e))^(1/2)+4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec
(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 90}

$$\frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{4c^4\tan(e+fx)}{a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{2c^4\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{c^4\tan(e+fx)\log(\cos(e+fx))}{a^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^4*Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]) + (2*c^4*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (4*c^4*Tan[e + f*x])/(a^2
*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
+ (4*c^4*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{c^3}{a^3x}-\frac{8c^3}{a^3(1+x)^3}+\frac{4c^3}{a^3(1+x)^2}-\frac{2c^3}{a^3(1+x)}\right)dx,x,\sec(e
\end{aligned}
$$

$$
{}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}
$$

$$
= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^4\log(1+\sec(e+fx))\,t}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c}}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.54, size = 157, normalized size = 0.71

$$
\frac{c^3\cot\left(\frac{1}{2}(e+fx)\right)\left(4\cos(e+fx)\left(-2+ifx-4\log\left(1+e^{i(e+fx)}\right)+\log\left(1+e^{2i(e+fx)}\right)\right)+(3+\cos(2(e+fx)))\left(ifx-4\log\left(1+e^{i(e+fx)}\right)+\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sqrt{c-c\sec(e+fx)}}{2a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cot[(e + f*x)/2]*(4*Cos[e + f*x]*(-2 + I*f*x - 4*Log[1 + E^(I*(e + f*x
))] + Log[1 + E^((2*I)*(e + f*x))]) + (3 + Cos[2*(e + f*x)])*(I*f*x - 4*Log
[1 + E^(I*(e + f*x))] + Log[1 + E^((2*I)*(e + f*x))]))*Sqrt[c - c*Sec[e + f
*x]])/(2*a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.26, size = 335, normalized size = 1.52

method	result
default	$\frac{\left(\left(\cos^2(fx+e)\right)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)+\cos^2(fx}{}$
risch	$\frac{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)} - \frac{2c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f} - \frac{8ic^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{3i(fx+e)}+\right.}{a^2\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(cos(f*x+e)^2*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^2*ln
(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+c
os(f*x+e)^2+2*cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+2*cos(f*
x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(2/(cos(f*x+e
```

```
)+1))-2*cos(f*x+e)+ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+ln(-(cos(f*x+e
)-1+sin(f*x+e))/sin(f*x+e))+ln(2/(cos(f*x+e)+1))+1)*(c*(-1+cos(f*x+e))/cos(
f*x+e))^(7/2)*cos(f*x+e)^4*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^5
/(-1+cos(f*x+e))/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) -
 c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^
3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.50, size = 93, normalized size = 0.42

$$-\frac{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}\sqrt{-ac}\,a^{2}|c|+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)\sqrt{-ac}\,a^{2}c|c|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*abs(c) + 2*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c*abs(c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(
1/2*f*x + 1/2*e))/(a^5*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^3\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(1+
cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3995, 3996, 31}

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]) + (c^3*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3995

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a^2/c^2, Int[Sqrt
[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
```

```
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2}$$
$$= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{(c^3\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{a+}\right.}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{}}$$
$$= -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(1+\cos(e+fx))}{a^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.64, size = 154, normalized size = 1.57

$$\frac{ic^2\cot\left(\frac{1}{2}(e+fx)\right)\left(4i+3fx+\cos(2(e+fx))\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+4\cos(e+fx)\left(2i+fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+6i\log\left(1+e^{i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{2a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*c^2*Cot[(e + f*x)/2]*(4*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Lo
g[1 + E^(I*(e + f*x))]) + 4*Cos[e + f*x]*(2*I + f*x + (2*I)*Log[1 + E^(I*(e
 + f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2
*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.25, size = 144, normalized size = 1.47

method	result
default	$\frac{\left(2(\cos^2(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)+3(\cos^2(fx+e))+4\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)+2\ln\left(\frac{2}{\cos(fx+e)+1}\right)-1\right)\left(\frac{c(-1+\cos(f}{\cos(fx+}\right.}{2f\sin(fx+e)^5a^3}$
risch	$\frac{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)} - \frac{2c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f} - \frac{8ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{3i(fx+e)}+e^{2i}\right.}{a^2\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(2*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+3*cos(f*x+e)^2+4*cos(f*x+e)*ln(2
/(cos(f*x+e)+1))-2*cos(f*x+e)+2*ln(2/(cos(f*x+e)+1))-1)*(c*(-1+cos(f*x+e))/
```

```
cos(f*x+e))^(5/2)*cos(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+
e)^5/a^3
```

Maxima [A]
time = 0.51, size = 108, normalized size = 1.10

$$\frac{\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{\sqrt{-a}\,a^2}+\frac{\frac{2\,\sqrt{-a}\,c^{\frac{5}{2}}\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{\sqrt{-a}\,c^{\frac{5}{2}}\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}{a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/2*(2*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a^2)
+ (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(-a)*c^(5/2
)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(a*sec(f*x + e
) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2
 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 1.76, size = 75, normalized size = 0.77

$$-\frac{\frac{2\,\sqrt{-ac}\,c^3\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{a^3|c|}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}\,|c|}{a^3c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/2*(2*sqrt(-a*c)*c^3*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c))
+ (c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*abs(c)/(a^3*c))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c^2\log(1}{a^2f\sqrt{a+a}}$$

```
[Out] -c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c^2*tan(f*x
+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1+cos(f*x+e))*
tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3993, 3992, 3996, 31}

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
)) - (c^2*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e +
f*x]]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3993

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a/c, Int[Sqrt[a +
 b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\int\frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx}{af(a+a\sec(e+fx))^{3/2}\sqrt{c}}$$

$$= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx}{af(a+a\sec(e+fx))^{3/2}\sqrt{c}}$$

$$= -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx}{af(a+a\sec(e+fx))^{3/2}\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.88, size = 152, normalized size = 1.06

$$\frac{ic\cot\left(\frac{1}{2}(e+fx)\right)\left(4i+3fx+\cos(2(e+fx))\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+\cos(e+fx)\left(6i+4fx+8i\log\left(1+e^{i(e+fx)}\right)\right)+6i\log\left(1+e^{i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{2a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*c*Cot[(e + f*x)/2]*(4*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[
1 + E^(I*(e + f*x))]) + Cos[e + f*x]*(6*I + 4*f*x + (8*I)*Log[1 + E^(I*(e +
 f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2*f
*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.25, size = 152, normalized size = 1.06

method	result
default	$\frac{(-1+\cos(fx+e))\left(4(\cos^2(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)+5(\cos^2(fx+e))+8\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)+4\ln\left(\frac{2}{\cos(fx+e)+}\right.\right.}{4f\sin(fx+e)^5a^3}$
risch	$\frac{c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,x}{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)} - \frac{2c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)f} - \frac{2ic\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(3\,e^{3i(fx+e)}+4\,e}{a^2(e^{i(fx+e)}+1)^3\sqrt{\frac{a(e^{i(fx+e)}+1)}{e^{2i(fx+e)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/4/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+5*cos(f*x+e)^2+8*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)+4*ln(2/(cos(f*x+e)+1))-3)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1937 vs. $2(142) = 284$.
time = 0.77, size = 1937, normalized size = 13.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -((f*x + e)*c*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*c*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*c*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*c*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*c*cos(2*f*x + 2*e) + (f*x + e)*c - 2*(c*cos(4*f*x + 4*e)^2 + 36*c*cos(2*f*x + 2*e)^2 + 16*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(4*f*x + 4*e)^2 + 12*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c*sin(2*f*x + 2*e)^2 + 16*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*c*cos(2*f*x + 2*e) + c)*cos(4*f*x + 4*e) + 12*c*cos(2*f*x + 2*e) + 8*(c*cos(4*f*x + 4*e) + 6*c*cos(2*f*x + 2*e) + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*cos(4*f*x + 4*e) + 6*c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e) + 4*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(

```
2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*arctan2(sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 1) + 2*(6*(f*x + e)*c*cos(2*f*x + 2*e) + (f*x + e)*c - 4*c*sin
(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(4*(f*x + e)*c*cos(4*f*x + 4*e) + 24*(f
*x + e)*c*cos(2*f*x + 2*e) + 16*(f*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*(f*x + e)*c + 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*
x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(4*(f*x
+ e)*c*cos(4*f*x + 4*e) + 24*(f*x + e)*c*cos(2*f*x + 2*e) + 4*(f*x + e)*c +
 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*c*sin(2*f*x + 2*e) + 2*c*cos(2*f*x
 + 2*e))*sin(4*f*x + 4*e) - 8*c*sin(2*f*x + 2*e) + 2*(4*(f*x + e)*c*sin(4*f
*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) + 16*(f*x + e)*c*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*c*cos(4*f*x + 4*e) - 2*c*cos(2*f
*x + 2*e) - 3*c)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(
4*(f*x + e)*c*sin(4*f*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) - 3*c*cos(
4*f*x + 4*e) - 2*c*cos(2*f*x + 2*e) - 3*c)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a^3*cos(4*f*x + 4*e)^2 + 36*a^3*cos
(2*f*x + 2*e)^2 + 16*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 16*a^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^3*
sin(4*f*x + 4*e)^2 + 12*a^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a^3*sin(
2*f*x + 2*e)^2 + 16*a^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 16*a^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*a^
3*cos(2*f*x + 2*e) + a^3 + 2*(6*a^3*cos(2*f*x + 2*e) + a^3)*cos(4*f*x + 4*e
) + 8*(a^3*cos(4*f*x + 4*e) + 6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^3)*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 8*(a^3*cos(4*f*x + 4*e) + 6*a^3*cos(2*f*x + 2*e)
+ a^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^3*sin(4*
f*x + 4*e) + 6*a^3*sin(2*f*x + 2*e) + 4*a^3*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 8*(a^3*sin(4*f*x + 4*e) + 6*a^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^3*sec(f*x
+ e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.68, size = 111, normalized size = 0.77

$$-\frac{\frac{4\sqrt{-ac}\,c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}\,a^3|c|-2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}\,a^3c|c|}{a^6c^2}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/4*(4*sqrt(-a*c)*c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c))
+ ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*abs(c) - 2*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c*abs(c))/(a^6*c^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{c\log(1}{a^2f\sqrt{a+a}}$$

```
[Out] -1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c*tan(f*x
+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f*x+e))*ta
n(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3992, 3996, 31}

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]) - (c*Tan[e + f*x])/(a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*
x]]) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3992

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/c, Int[Sqrt[a + b*Cs
c[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(b + a*x)^(m - 1/2)*((d + c
*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && E
qQ[m + n, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+}{af(a+a\sec(e+fx))^{3/2}\sqrt{}}$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+}{af(a+a\sec(e+fx))^{3/2}\sqrt{}}$$

$$= -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+}{af(a+a\sec(e+fx))^{3/2}\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.69, size = 151, normalized size = 1.08

$$\frac{i\cot\left(\frac{1}{2}(e+fx)\right)\left(3i+3fx+\cos(2(e+fx))\left(fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+4\cos(e+fx)\left(i+fx+2i\log\left(1+e^{i(e+fx)}\right)\right)+6i\log\left(1+e^{i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{2a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((I/2)*Cot[(e + f*x)/2]*(3*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[1
+ E^(I*(e + f*x))]) + 4*Cos[e + f*x]*(I + f*x + (2*I)*Log[1 + E^(I*(e + f*x
))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2*f*(1
+ Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 0.31, size = 152, normalized size = 1.09

method	result
default	$\frac{(-1+\cos(fx+e))^2\left(8(\cos^2(fx+e))\ln\left(\frac{2}{\cos(fx+e)+1}\right)+7(\cos^2(fx+e))+16\cos(fx+e)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2\cos(fx+e)+8\ln\left(\frac{2}{\cos(fx+e}\right.\right.}{8f\sin(fx+e)^5a^3}$
risch	$\frac{(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,x}{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)} - \frac{2(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{a^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)f} - \frac{2i\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(2e^{3i(fx+e)}+3e^{2i}}{a^2(e^{i(fx+e)}+1)^3\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(-1+cos(f*x+e))^2*(8*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+7*cos(f*x+e)^2
+16*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f*x+e)+8*ln(2/(cos(f*x+e)+1))-5)*
(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*co
s(f*x+e)/sin(f*x+e)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(136) = 272$.
time = 0.73, size = 1280, normalized size = 9.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
+ e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)
^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(4*cos(3*f*x + 3*e) + 6*cos(2
*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8
*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x
+ 3*e)^2 + 12*(4*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2
 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f
*x + e))*sin(4*f*x + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2
*sin(f*x + e))*sin(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*
e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e
) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 4*(f*x + e)*cos(3
*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e -
 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*cos(4*f*x + 4*e)
 + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e)*co
s(3*f*x + 3*e) + 12*(f*x + 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) +
 8*(f*x + e)*cos(f*x + e) + 2*(4*(f*x + e)*sin(3*f*x + 3*e) + 6*(f*x + e)*s
in(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) + 3*cos(2*f
*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) + 4*(12*(f*x + e)*sin(2*f*x +
2*e) + 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) + 6*(8*(f*x + e)*sin(
f*x + e) - 1)*sin(2*f*x + 2*e) + e - 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*
cos(4*f*x + 4*e)^2 + 16*a^3*cos(3*f*x + 3*e)^2 + 36*a^3*cos(2*f*x + 2*e)^2
+ 16*a^3*cos(f*x + e)^2 + a^3*sin(4*f*x + 4*e)^2 + 16*a^3*sin(3*f*x + 3*e)^
2 + 36*a^3*sin(2*f*x + 2*e)^2 + 48*a^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*a
^3*sin(f*x + e)^2 + 8*a^3*cos(f*x + e) + a^3 + 2*(4*a^3*cos(3*f*x + 3*e) +
```

```
6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(4*f*x + 4*e) + 8*(6*
a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(3*f*x + 3*e) + 12*(4*a
^3*cos(f*x + e) + a^3)*cos(2*f*x + 2*e) + 4*(2*a^3*sin(3*f*x + 3*e) + 3*a^3
*sin(2*f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*sin(2*
f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x +
e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.58, size = 124, normalized size = 0.89

$$-\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-ac}\,c\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c\right|\right)}{a^{3}|c|}+\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}\sqrt{-ac}\,a^{3}c|c|-4\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)\sqrt{-ac}\,a^{3}c^{2}|c|}{a^{6}c^{4}}\right)}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] -1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a*c)*c*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c)
)/(a^3*abs(c)) + (sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*c
*abs(c) - 4*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c^2*abs(c
))/(a^6*c^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2), x)
```

3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=270

$$\frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(1-\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{7\log(}{8a^2f\sqrt{a+}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)+1/8*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+7/8*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(
f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 84}

$$-\frac{3\tan(e+fx)}{4a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(1-\sec(e+fx))}{8a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{7\tan(e+fx)\log(\sec(e+fx)+1)}{8a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\cos(e+fx))}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c
*Sec[e + f*x]]) + (Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(8*a^2*f*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (7*Log[1 + Sec[e + f*x]]*Tan[e +
f*x])/(8*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e +
 f*x]/(4*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]]) - (3*Tan[e + f*x])/(4*a^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)}\,dx, x, \sec(e+fx\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int\left(-\frac{1}{8a^3c(-1+x)}+\frac{1}{a^3cx}-\frac{1}{2a^3c(1+\right.\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c}}$$

$$= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log}{8a^2f\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.74, size = 195, normalized size = 0.72

$$\frac{\left(8-12ifx+3\log\left(1-e^{i(e+fx)}\right)+21\log\left(1+e^{i(e+fx)}\right)+\cos(2(e+fx))\left(-4ifx+\log\left(1-e^{i(e+fx)}\right)+7\log\left(1+e^{i(e+fx)}\right)\right)+2\cos(e+fx)\left(5-8ifx+2\log\left(1-e^{i(e+fx)}\right)+14\log\left(1+e^{i(e+fx)}\right)\right)\right)\tan(e+fx)}{8a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((8 - (12*I)*f*x + 3*Log[1 - E^(I*(e + f*x))] + 21*Log[1 + E^(I*(e + f*x))]
 + Cos[2*(e + f*x)]*((-4*I)*f*x + Log[1 - E^(I*(e + f*x))] + 7*Log[1 + E^(I
*(e + f*x))]) + 2*Cos[e + f*x]*(5 - (8*I)*f*x + 2*Log[1 - E^(I*(e + f*x))]
+ 14*Log[1 + E^(I*(e + f*x))]))*Tan[e + f*x])/(8*a^2*f*(1 + Cos[e + f*x])^2
*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.29, size = 223, normalized size = 0.83

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))^3\left(16\left(\cos^2(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-4\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+9\left(\cos^2(fx+e)\right)+3\right.}{16f\sqrt{\frac{c(-1+\cos(fx+e}{\cos(fx+e)}}}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/16/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^3*(16*cos(f*x+e)
^2*ln(2/(cos(f*x+e)+1))-4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+9*co
```

```
s(f*x+e)^2+32*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*cos(f*x+e)*ln(-(-1+cos(f*x+
e))/sin(f*x+e))-2*cos(f*x+e)+16*ln(2/(cos(f*x+e)+1))-4*ln(-(-1+cos(f*x+e))/
sin(f*x+e))-7)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2398 vs. $2(262) = 524$.
time = 0.77, size = 2398, normalized size = 8.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 6
4*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)
*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - 7*(2*(6*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 8
*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(4
*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*sin(2*f*x
 + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) + 4*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
1) - (2*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36
*cos(2*f*x + 2*e)^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) + 36*sin(2*f*x + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e)
 + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*a
```

```
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 1) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + e - 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) + 48*(f*x + e)*cos(2*f*x + 2*e) + 2*(16*f
*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos(2*f*x + 2*e) + 64*(f*
x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(
4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 2*(16*f*x + 16*(f*x + e)*cos(4*f*x + 4*e) + 96*(f*x + e)*cos
(2*f*x + 2*e) + 16*e + 5*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 2*(16*(f*x + e)*sin(4*f*x + 4*e) +
 96*(f*x + e)*sin(2*f*x + 2*e) + 64*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) - 5)*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(16*(f*x + e)*sin(4*f*
x + 4*e) + 96*(f*x + e)*sin(2*f*x + 2*e) - 5*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e) - 5)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*e - 1
6*sin(2*f*x + 2*e))/((a^2*cos(4*f*x + 4*e)^2 + 36*a^2*cos(2*f*x + 2*e)^2 +
16*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2
 + 12*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a^2*sin(2*f*x + 2*e)^2 + 1
6*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a^2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*a^2*cos(2*f*x + 2*e)
 + a^2 + 2*(6*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 8*(a^2*cos(4*f
*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 4*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 8*(a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^2*sin(4*f*x + 4*e) + 6*a^2
*sin(2*f*x + 2*e) + 4*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a^2*sin(4*f*
x + 4*e) + 6*a^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)*sqrt(c)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c*sec(f*x
 + e)^4 + 2*a^3*c*sec(f*x + e)^3 - 2*a^3*c*sec(f*x + e) - a^3*c), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Giac [A]
time = 1.75, size = 144, normalized size = 0.53

$$\frac{\frac{2\sqrt{-ac}\,\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^3|c|}-\frac{16\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^3|c|}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}\,a^3c^2|c|-6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}\,a^3c^3|c|}{a^6c^6}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/16*(2*sqrt(-a*c)*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a^3*abs(c)) - 16*sqr
t(-a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*abs(c)) - ((c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*c^2*abs(c) - 6*(c*tan(1/2*f*x + 1/2*e)^
2 - c)*sqrt(-a*c)*a^3*c^3*abs(c))/(a^6*c^6))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{5\log(1-\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{11\,\mathrm{l}}{16a^2cf\sqrt{}}$$

```
[Out] ln(cos(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)+5/16*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*s
ec(f*x+e))^(1/2)+11/16*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))
^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1-sec(f*x+e))/(a+a*se
c(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1+sec(f*x+e)
)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a^2/c/f/(1
+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3997, 90}

$$-\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2a^2cf(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{8a^2cf(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\log(1-\sec(e+fx))}{16a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{11\tan(e+fx)\log(\sec(e+fx)+1)}{16a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\cos(e+fx))}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]) + (5*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(16*a^2*c*f*Sqrt
[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (11*Log[1 + Sec[e + f*x]]*
Tan[e + f*x])/(16*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Tan[e + f*x]/(8*a^2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(8*a^2*c*f*(1 + Sec[e + f*x])^2*Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*a^2*c*f*(1
+ Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
```

```
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)^2}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} \\ &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(\frac{1}{8a^3c^2(-1+x)^2} - \frac{5}{16a^3c^2(-1+x)}\right.\right.}{f\sqrt{a+a\sec(}} \\ &= \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{}{16a^2c}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.44, size = 275, normalized size = 0.80

$$\frac{\left(-14+16ifx-8ifx\cos(3(e+fx))-10\log\left(1-e^{i(e+fx)}\right)+5\cos(3(e+fx))\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(-12+8ifx-5\log\left(1-e^{i(e+fx)}\right)-11\log\left(1+e^{i(e+fx)}\right)\right)-22\log\left(1+e^{i(e+fx)}\right)+11\cos(3(e+fx))\log\left(1+e^{i(e+fx)}\right)+2\cos(2(e+fx))\left(5-8ifx+5\log\left(1-e^{i(e+fx)}\right)+11\log\left(1+e^{i(e+fx)}\right)\right)\right)\tan(e+fx)}{32a^2cf(-1+\cos(e+fx))(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] ((-14 + (16*I)*f*x - (8*I)*f*x*Cos[3*(e + f*x)] - 10*Log[1 - E^(I*(e + f*x)
)] + 5*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-12 + (8*I
)*f*x - 5*Log[1 - E^(I*(e + f*x))] - 11*Log[1 + E^(I*(e + f*x))]) - 22*Log[
1 + E^(I*(e + f*x))] + 11*Cos[3*(e + f*x)]*Log[1 + E^(I*(e + f*x))] + 2*Cos
[2*(e + f*x)]*(5 - (8*I)*f*x + 5*Log[1 - E^(I*(e + f*x))] + 11*Log[1 + E^(I
*(e + f*x))]))*Tan[e + f*x])/(32*a^2*c*f*(-1 + Cos[e + f*x])*(1 + Cos[e + f
*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.27, size = 284, normalized size = 0.82

method	result
default	$-\frac{\left(20\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-32\left(\cos^3(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-13\left(\cos^3(fx+e)\right)+20\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e}{\sin(fx+e)}\right.\right.}{}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2c\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}} - \frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2c\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/32/f*(20*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*cos(f*x+e)^3*ln
(2/(cos(f*x+e)+1))-13*cos(f*x+e)^3+20*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(
f*x+e))-32*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+7*cos(f*x+e)^2-20*cos(f*x+e)*l
n(-(-1+cos(f*x+e))/sin(f*x+e))+32*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e
)-20*ln(-(-1+cos(f*x+e))/sin(f*x+e))+32*ln(2/(cos(f*x+e)+1))-11)*(a*(cos(f*
x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/c^3/sin(f*x+e)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4644 vs. $2(333) = 666$.
time = 3.36, size = 4644, normalized size = 13.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*(
f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x + 4*
e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 8*f*x + 11*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e
) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos
(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos(6*f*x + 6*
e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*x + 6*e) - cos(
4*f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 4*(cos(6*f*x + 6
*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6
*f*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
sin(2*f*x + 2*e)^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2
*e) - 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e
```

```
), cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) + 2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) + 5*(2*(cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
- 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e) - 1)*cos(4
*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos(6*f*x + 6*e)
 - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*x + 6*e) - cos(4*
f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 4*(cos(6*f*x + 6*e
) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - sin(6*f
*x + 6*e)^2 - sin(4*f*x + 4*e)^2 - 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - si
n(2*f*x + 2*e)^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e
) - 4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - 4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 8*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e) + 2*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 4*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) - 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 1) + 4*(4*f*x - 4*(f*x + e)*cos(4*f*x + 4*e) - 4*
(f*x + e)*cos(2*f*x + 2*e) + 4*e + 3*sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))
*cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*cos(2*f*x + 2*e) + e)*cos(4*f*x + 4
*e) - 16*(f*x + e)*cos(2*f*x + 2*e) + 2*(16*f*x + 16*(f*x + e)*cos(6*f*x +
6*e) - 16*(f*x + e)*cos(4*f*x + 4*e) - 16*(f*x + e)*cos(2*f*x + 2*e) - 64*(
f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(f*x + e
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*e + 5*sin(6*f*x
 + 6*e) + 7*sin(4*f*x + 4*e) + 7*sin(2*f*x + 2*e) + 8*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan...
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c^2*sec(f*
x + e)^5 + a^3*c^2*sec(f*x + e)^4 - 2*a^3*c^2*sec(f*x + e)^3 - 2*a^3*c^2*se
c(f*x + e)^2 + a^3*c^2*sec(f*x + e) + a^3*c^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**A**]
time = 2.02, size = 193, normalized size = 0.56

$$-\frac{\frac{10\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{\sqrt{-ac}\,a^{2}|c|}+\frac{32\,\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right|\right)}{a^{3}c|c|}-\frac{2\,\left(5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)}{\sqrt{-ac}\,a^{2}c|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}\sqrt{-ac}\,a^{3}c^{3}|c|-8\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)\sqrt{-ac}\,a^{3}c^{4}|c|}{a^{6}c^{8}}}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] -1/32*(10*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*abs(c)) + 32*s
qrt(-a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*c*abs(c)) - 2*(5*c*ta
n(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a^2*c*abs(c)*tan(1/2*f*x + 1/2*e)^2)
+ ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*c^3*abs(c) - 8*(c*tan(1/
2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^3*c^4*abs(c))/(a^6*c^8))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=151

$$\frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{}{a^2c^2f\,\sqrt{}}$$

```
[Out] 1/2*cot(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*
cot(f*x+e)^3/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin
(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3990, 3554, 3556}

$$-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\cot(e+fx)}{2a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] Cot[e + f*x]/(2*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
) - Cot[e + f*x]^3/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
 f*x]]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(a^2*c^2*f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3990

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[Cot[e + f*x]^(2*m), x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^5(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{}{a^2c}$$
$$= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{}{4a^2c}$$
$$= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{}{4a^2c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.26, size = 149, normalized size = 0.99

$$\frac{\csc^3(e+fx)\left(2-3ifx+\cos(2(e+fx))\left(-4+4ifx-4\log\left(1-e^{2i(e+fx)}\right)\right)+3\log\left(1-e^{2i(e+fx)}\right)+\cos(4(e+fx))\left(-ifx+\log\left(1-e^{2i(e+fx)}\right)\right)\right)\sec(e+fx)}{8a^2c^2f\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Csc[e + f*x]^3*(2 - (3*I)*f*x + Cos[2*(e + f*x)]*(-4 + (4*I)*f*x - 4*Log[1
 - E^((2*I)*(e + f*x))]) + 3*Log[1 - E^((2*I)*(e + f*x))] + Cos[4*(e + f*x)
]*((-I)*f*x + Log[1 - E^((2*I)*(e + f*x))]))*Sec[e + f*x])/(8*a^2*c^2*f*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 0.29, size = 237, normalized size = 1.57

method	result
default	$-\frac{(-1+\cos(fx+e))^3\left(32\left(\cos^4(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-32\left(\cos^4(fx+e)\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-13\left(\cos^4(fx+e)\right)-64\left(\cos^2(fx+e)\right)\ln}{32f\sin(fx+e)^5\,\mathrm{co}}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/32/f*(-1+cos(f*x+e))^3*(32*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-
32*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-13*cos(f*x+e)^4-64*cos(f*x+e)^2*ln(-(-
1+cos(f*x+e))/sin(f*x+e))+64*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)
```

```
^2+32*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*ln(2/(cos(f*x+e)+1))+11)*(a*(cos(f
*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^5/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos
(f*x+e))^(5/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1505 vs. $2(145) = 290$.
time = 0.79, size = 1505, normalized size = 9.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] -((f*x + e)*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*cos(6*f*x + 6*e)^2 + 36*(f*x
 + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin(8
*f*x + 8*e)^2 + 16*(f*x + e)*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*sin(4*f*x +
4*e)^2 + 16*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(4*cos(6*f*x + 6*e) - 6
*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x +
8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) -
 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 36*
cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2 +
16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*f*
x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) - 1) + 2*(f*x - 4*(f*x + e)*cos(6*f*x + 6*e) + 6*(f*x + e)
*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + 2*sin(6*f*x + 6*e) -
 2*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) - 8*(f*x + 6*(f*
x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + sin(4*f*x + 4*
e))*cos(6*f*x + 6*e) + 4*(3*f*x - 12*(f*x + e)*cos(2*f*x + 2*e) + 3*e + 2*s
in(2*f*x + 2*e))*cos(4*f*x + 4*e) - 8*(f*x + e)*cos(2*f*x + 2*e) - 4*(2*(f*
x + e)*sin(6*f*x + 6*e) - 3*(f*x + e)*sin(4*f*x + 4*e) + 2*(f*x + e)*sin(2*
f*x + 2*e) + cos(6*f*x + 6*e) - cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*sin(8*
f*x + 8*e) - 4*(12*(f*x + e)*sin(4*f*x + 4*e) - 8*(f*x + e)*sin(2*f*x + 2*e
) - 2*cos(4*f*x + 4*e) - 1)*sin(6*f*x + 6*e) - 4*(12*(f*x + e)*sin(2*f*x +
2*e) + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + e + 4*sin(2*f*x + 2*e))*s
qrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e)^2
 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c^3*
sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*x +
 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(2*f
*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*f*x
 + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3
)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x +
2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3
```

```
)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x +
4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(4*f
*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A]
time = 5.30, size = 610, normalized size = 4.04

$$\left[-\frac{162\,\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-ac}\,\log\left(-\frac{8\,\left(\left(256\,\cos\left(fx+e\right)^{5}-512\,\cos\left(fx+e\right)^{3}+175\,\cos\left(fx+e\right)\right)\sqrt{-ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-\left(256\,ac\cos\left(fx+e\right)^{4}-512\,ac\cos\left(fx+e\right)^{2}+337\,ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\left(832\,\cos\left(fx+e\right)^{5}-1988\,\cos\left(fx+e\right)^{3}+1075\,\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{324\,\left(a^{3}c^{3}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{3}f\cos\left(fx+e\right)^{2}+a^{3}c^{3}f\right)\sin\left(fx+e\right)},\,-\frac{324\,\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{ac}\,\arctan\left(\frac{\left(16\,\cos\left(fx+e\right)^{3}-7\,\cos\left(fx+e\right)\right)\sqrt{ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(16\,ac\cos\left(fx+e\right)^{2}-25\,ac\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\left(832\,\cos\left(fx+e\right)^{5}-1988\,\cos\left(fx+e\right)^{3}+1075\,\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{324\,\left(a^{3}c^{3}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{3}f\cos\left(fx+e\right)^{2}+a^{3}c^{3}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [-1/324*(162*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-8*((25
6*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) -
 (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/
((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^5 - 1
988*cos(f*x + e)^3 + 1075*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 -
2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)), -1/324*(324*(cos(f*x
 + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos
(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*
sin(f*x + e) + (832*cos(f*x + e)^5 - 1988*cos(f*x + e)^3 + 1075*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*
f)*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 2.04, size = 225, normalized size = 1.49

$$-\frac{\frac{32\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{\sqrt{-ac}\,a^{2}c|c|}+\frac{64\,\sqrt{-ac}\,\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+c\right|\right)}{a^{3}c^{2}|c|}-\frac{48\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}+84\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c+37\,c^{2}}{\sqrt{-ac}\,a^{2}c^{3}|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}\sqrt{-ac}\,a^{3}c^{4}|c|-10\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)\sqrt{-ac}\,a^{3}c^{5}|c|}{a^{6}c^{10}}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] -1/64*(32*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*c*abs(c)) + 64
*sqrt(-a*c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a^3*c^2*abs(c)) - (48*(
c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 84*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 37*
c^2)/(sqrt(-a*c)*a^2*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4) + ((c*tan(1/2*f*x +
 1/2*e)^2 - c)^2*sqrt(-a*c)*a^3*c^4*abs(c) - 10*(c*tan(1/2*f*x + 1/2*e)^2 -
 c)*sqrt(-a*c)*a^3*c^5*abs(c))/(a^6*c^10))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.131 $\int (1+\sec(e+fx))^m (c-c\sec(e+fx))^n \, dx$

Optimal. Leaf size=92

$$\frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2}+n;\frac{1}{2}-m,1;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(c-c\sec(e+fx))^n \tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2+n,1,1/2-m,3/2+n,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(c-
c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3997, 141}

$$\frac{2^{m+\frac{1}{2}} \tan(e+fx)(c-c\sec(e+fx))^n F_1\left(n+\frac{1}{2};\frac{1}{2}-m,1;n+\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - Sec[e + f*x])/2, 1
 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 +
 Sec[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+\right.}{f\sqrt{1+\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}+n;\frac{1}{2}-m,1;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx)),1-\sec(\right.}{f(1+2n)\sqrt{1+\sec(e+}}$$

Mathematica [F]
time = 1.10, size = 0, normalized size = 0.00

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (1+\sec(fx+e))^m (c-c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
[Out] int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(\sec (e+fx)-1))^n (\sec (e+fx)+1)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**n*(sec(e + f*x) + 1)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos \left(e+f\,x\right)}+1\right)}^m\,{\left(c-\frac{c}{\cos \left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n, x)
```

3.132 $\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=109

$$\frac{2^{\frac{1}{2}+n}cF_1\left(\frac{1}{2}+m;\frac{1}{2}-n,1;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+f}{f(1+2m)}$$

```
[Out] 2^(1/2+n)*c*AppellF1(1/2+m,1,1/2-n,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f/(1+2*m)
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$,
Rules used = {3997, 142, 141}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}F_1\left(m+\frac{1}{2};\frac{1}{2}-n,1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + Sec[e + f*x])/2,
 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c -
 c*Sec[e + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e}{f\sqrt{a+a\,s}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{1}{2}+m;\frac{1}{2}-n,1;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\text{se}}{}$$

Mathematica [**F**]
time = 0.46, size = 0, normalized size = 0.00

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [**F**]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m (-c(\sec(e+fx)-1))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m \left(c-\frac{c}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n, x)
```

3.133 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=101

$$\frac{2^{\frac{1}{2}+n}cF_1\left(\frac{7}{2};\frac{1}{2}-n,1;\frac{9}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{-1+n}\tan(e+fx)}{7f}$$

```
[Out] 1/7*2^(1/2+n)*c*AppellF1(7/2,1,1/2-n,9/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3997, 142, 141}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^3(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{7}{2};\frac{1}{2}-n,1;\frac{9}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[7/2, 1/2 - n, 1, 9/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^3*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(7*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(\right.}{f\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{7}{2};\frac{1}{2}-n,1;\frac{9}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx\right.}{}$$

Mathematica [F]
time = 3.41, size = 0, normalized size = 0.00

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(fx+e\right)\right)^{3}\left(c-c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3(-c\sec(e+fx)+c)^{n}\sec(e+fx)\,dx+\int 3(-c\sec(e+fx)+c)^{n}\sec^{2}(e+fx)\,dx+\int(-c\sec(e+fx)+c)^{n}\sec^{3}(e+fx)\,dx+\int(-c\sec(e+fx)+c)^{n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**n,x)
```

```
[Out] a**3*(Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral(3*(-c
*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)*
*n*sec(e + f*x)**3, x) + Integral((-c*sec(e + f*x) + c)**n, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n, x)
```

3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=101

$$\frac{2^{\frac{1}{2}+n}cF_1\left(\frac{5}{2};\frac{1}{2}-n,1;\frac{7}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx))^2(c-c}{5f}$$

```
[Out] 1/5*2^(1/2+n)*c*AppellF1(5/2,1,1/2-n,7/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3997, 142, 141}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^2(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{5}{2};\frac{1}{2}-n,1;\frac{7}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[5/2, 1/2 - n, 1, 7/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^2*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(5*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e\right.}{f\sqrt{a+a\sec}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{5}{2};\frac{1}{2}-n,1;\frac{7}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)}{}$$

Mathematica [**F**]
time = 1.57, size = 0, normalized size = 0.00

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n, x]
```

Maple [**F**]
time = 0.15, size = 0, normalized size = 0.00

$$\int (a+a\sec(fx+e))^2(c-c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(-c*sec(f*x + e) +
 c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2(-c\sec(e+fx)+c)^{n}\sec(e+fx)\,dx+\int(-c\sec(e+fx)+c)^{n}\sec^{2}(e+fx)\,dx+\int(-c\sec(e+fx)+c)^{n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**n,x)
```

```
[Out] a**2*(Integral(2*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*s
ec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)**n
, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n, x)
```

3.135 $\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=99

$$\frac{2^{\frac{1}{2}+n}cF_1\left(\frac{3}{2};\frac{1}{2}-n,1;\frac{5}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx))(c-c\sec(e+fx))^{n-1}\tan(e+fx)}{3f}$$

```
[Out] 1/3*2^(1/2+n)*c*AppellF1(3/2,1,1/2-n,5/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(
1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f
```

Rubi [A]

time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3997, 142, 141}

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(\frac{3}{2};\frac{1}{2}-n,1;\frac{5}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(3*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+ax}\,(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e}{f\sqrt{a+a\,s}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{3}{2};\frac{1}{2}-n,1;\frac{5}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)}{}$$

Mathematica [F]
time = 1.71, size = 0, normalized size = 0.00

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (a+a\sec(fx+e))\,(c-c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(-c\sec\left(e+fx\right)+c\right)^{n}\sec\left(e+fx\right)dx+\int\left(-c\sec\left(e+fx\right)+c\right)^{n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))**n,x)
```

```
[Out] a*(Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e
+ f*x) + c)**n, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n, x)
```

3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=99

$$-\frac{2^{\frac{1}{2}+n}cF_1\left(-\frac{1}{2};\frac{1}{2}-n,1;\frac{1}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{-1+}}{f(a+a\sec(e+fx))}$$

```
[Out] -2^(1/2+n)*c*AppellF1(-1/2,1,1/2-n,1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-
sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]

time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3997, 142, 141}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(-\frac{1}{2};\frac{1}{2}-n,1;\frac{1}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + n)*c*AppellF1[-1/2, 1/2 - n, 1, 1/2, (1 + Sec[e + f*x])/2, 1 +
Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*Ta
n[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int \frac{(}{x}\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}cF_1\left(-\frac{1}{2};\frac{1}{2}-n,1;\frac{1}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx}{f(a+a\sec(e+fx))}$$

Mathematica [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(fx+e))^n}{a+a\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(-c\sec(e+fx)+c)^n}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

```
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{2^{\frac{1}{2}+n}cF_1\left(-\frac{3}{2};\frac{1}{2}-n,1;-\frac{1}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{-1+}}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -1/3*2^(1/2+n)*c*AppellF1(-3/2,1,1/2-n,-1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e)
)*(1-sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*x+
e))^2
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3997, 142, 141}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}F_1\left(-\frac{3}{2};\frac{1}{2}-n,1;-\frac{1}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c\sec(e+fx))^{n-1}}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(2^(1/2 + n)*c*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2,
1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n
)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^2)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\Big(\int$$
$$f\sqrt{a+a\sec(e+fx)}$$

$$= -\frac{2^{\frac{1}{2}+n}cF_1\left(-\frac{3}{2};\frac{1}{2}-n,1;-\frac{1}{2};\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+}{3f(a+a\sec(e+fx))^2}$$

Mathematica [F]
time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(fx+e))^n}{(a+a\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) +
 a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(-c\sec{(e+fx)}+c)^n}{\sec^2{(e+fx)}+2\sec{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=172

$$\frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^3\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+$$

```
[Out] 6*a^3*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^3*
hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/
(1+2*n)/(a+a*sec(f*x+e))^(1/2)-2*a^3*(c-c*sec(f*x+e))^(1+n)*tan(f*x+e)/c/f/
(3+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3997, 90, 67}

$$\frac{2a^3\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{6a^3\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{2a^3\tan(e+fx)(c-c\sec(e+fx))^{n+1}}{cf(2n+3)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (6*a^3*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^3*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
- (2*a^3*(c - c*Sec[e + f*x])^(1 + n)*Tan[e + f*x])/(c*f*(3 + 2*n)*Sqrt[a +
 a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^2(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \left(3a^2(c-cx)^{-\frac{1}{2}+n}+\frac{a^2(c-cx)^{-\frac{1}{2}}}{x}\right.\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c}}$$

$$= \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-c\sec(e+f}{cf(3+2n)\sqrt{a-}}$$

$$= \frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}\right.}{}$$

Mathematica [F]
time = 3.52, size = 0, normalized size = 0.00

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (a+a\sec(fx+e))^{\frac{5}{2}}(c-c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n, x)
```

3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=119

$$\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^2*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*
hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/
(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3994, 3997, 67}

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*a^2*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
 f*x]]) + (2*a^2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(
c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3994

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a, Int[Sqrt[a + b
*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LeQ[n, -2^(-1)]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
```

```
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + a\int \sqrt{a+a\sec(e$$

$$= \frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(a^2c\tan(e+fx))}{f\sqrt{a+a\mathrm{s}}}$$

$$= \frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\,{}_2F_1\left(1,\frac{1}{2}+n;\right.}{}$$

Mathematica [F]
time = 1.82, size = 0, normalized size = 0.00

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (a+a\sec(fx+e))^{\frac{3}{2}}(c-c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**n,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n, x)
```

3.140 $\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=68

$$\frac{2a\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e
)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3997, 67}

$$\frac{2a\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] (2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^n\,dx &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(e+fx)\right)(c-c\sec(e+fx}{f(1+2n)\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sec\left(fx+e\right)}\,\left(c-c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
[Out] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,(-c(\sec\left(e+fx\right)-1))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n, x)
```

3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1-\sec(e\right.}{f(1+2n)}$$

```
[Out] -hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*
x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n],[3/2+n],1-sec(
f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3997, 88, 67, 70}

$$\frac{2\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[
e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])) + (2*Hype
rgeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n
*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] && !IntegerQ[p]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{(ac\tan(e+fx))\text{S}}{f\sqrt{a+a\,\text{s}}}$$

$$= -\frac{{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} -$$

Mathematica [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
[Out] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]], x]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(fx+e))^n}{\sqrt{a+a\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec{(e+fx)}-1))^n}{\sqrt{a(\sec{(e+fx)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((-c*(sec(e + f*x) - 1))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^n}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{(5-2n)\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{4af(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;1\right.}{af}$$

```
[Out] -1/4*(5-2*n)*hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+
e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n],
[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec(f*
x+e))^(1/2)-1/2*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f
*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3997, 105, 162, 67, 70}

$$-\frac{(5-2n)\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{4af(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)(c-c\sec(e+fx))^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};1-\sec(e+fx)\right)}{af(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)(c-c\sec(e+fx))^n}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/4*((5 - 2*n)*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2
]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*
x]]) + (2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*S
ec[e + f*x])^n*Tan[e + f*x])/(a*f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]]) - ((c
 - c*Sec[e + f*x])^n*Tan[e + f*x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec
[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c-cx)^{-}}{}\right.}{2af\sqrt{a+a\sec(e}}$$

$$= -\frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{(c\tan(e+fx))\text{Subst}\left(\int \frac{(c-c}{}\right.}{af\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(5-2n)\,{}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(}{4af(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [F]
time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2), x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(c - c\sec(fx + e))^n}{(a + a\sec(fx + e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)
^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e + fx) - 1))^n}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}-\frac{\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f-arctan(1/2*
a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)*a^(1/2)/c/f
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {21, 3861, 3859, 209, 3880}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(c*f)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx &= \frac{a\int \frac{1}{\sqrt{a+a\sec(e+fx)}}\,dx}{c}\\
&= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{c} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c}\\
&= -\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx\right.}{}\\
&= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{\sqrt{2}\,\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+}\right.}{cf}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.69, size = 133, normalized size = 1.46

$$-\frac{i\sqrt{1+e^{2i(e+fx)}}\left(\sinh^{-1}\left(e^{i(e+fx)}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{-1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)-\tanh^{-1}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{a(1+\sec(e+fx))}}{c\left(1+e^{i(e+fx)}\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

```
[Out] ((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]*(ArcSinh[E^(I*(e + f*x))] - Sqrt[2]*Arc
Tanh[(-1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] - ArcT
anh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*(1 + E^(
I*(e + f*x)))*f)
```

Maple [A]
time = 0.47, size = 141, normalized size = 1.55

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(\ln\left(\frac{\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cos(fx+e)+1}{\sin(fx+e)}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\cdots}{2\cos\cdots}\right.\right.}{cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*(ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin
(f*x+e))+2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)/cos(f*x+e)*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) + c), x)
```

Fricas [A]
time = 2.78, size = 316, normalized size = 3.47

$$\left[\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3a\cos(fx+e)^2+2a\cos(fx+e)-a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{2cf},\ \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{cf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x +
e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*log((2*a*cos(f*
x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), (sqrt(2)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)) - 2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a\sec\left(e+fx\right)+a}}{\sec\left(e+fx\right)+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) + 1), x)/c
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(76) = 152.
time = 1.10, size = 189, normalized size = 2.08

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}\left|a\right|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}\left|a\right|-6a\right|}\right)}{c\left|a\right|}+\frac{\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}\right)}{c}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/(c*abs(a)) + sqrt(-a)*log((sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2)/c)*sgn(cos(f*x + e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{c+\frac{c}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)), x)
```

3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=231

$$-\frac{2c\cot(e+fx)\Pi\left(\frac{c}{c+d};\mathrm{ArcSin}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}}{a\sqrt{c+d}\,f}$$

```
[Out] -2*c*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-d
)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)
*(d*(1+sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1/2)-(c-d)*EllipticE(
tan(f*x+e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+
d*sec(f*x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4012, 3865, 4053}

$$-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\Pi\left(\frac{c}{c+d};\mathrm{ArcSin}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{(c-d)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}\,E\left(\mathrm{ArcSin}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*c*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[
e + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f
*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x
]))/(a*Sqrt[c + d]*f) - ((c - d)*EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e +
 f*x])], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e +
f*x]])/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] +
```

```
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))])))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{c\int \sqrt{c+d\sec(e+fx)}\,dx}{a} + (-c+d)\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,d$$

$$= -\frac{2c\cot(e+fx)\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(}{c+d\sec(}}}{a\sqrt{c+d}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 810 vs. 2(231) = 462.
time = 33.95, size = 810, normalized size = 3.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(2*Sec[(e + f*x)/2]*(-(c*S
in[(e + f*x)/2]) + d*Sin[(e + f*x)/2]) - 2*(-c + d)*Sin[e + f*x]))/(f*(d +
c*Cos[e + f*x])*(a + a*Sec[e + f*x])) + (2*Cos[e/2 + (f*x)/2]^2*(c + d*Sec[
e + f*x])^(3/2)*(c^2*Tan[(e + f*x)/2] - d^2*Tan[(e + f*x)/2] - 2*c^2*Tan[(e
 + f*x)/2]^3 + 2*c*d*Tan[(e + f*x)/2]^3 + c^2*Tan[(e + f*x)/2]^5 - 2*c*d*Ta
n[(e + f*x)/2]^5 + d^2*Tan[(e + f*x)/2]^5 - 4*c^2*EllipticPi[-1, ArcSin[Tan
[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d -
 c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] - 4*c^2*EllipticPi[-
1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Tan[(e + f*x)/2]^2*Sqrt[1 - T
```

```
an[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^
2)/(c + d)] + (c^2 - d^2)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c +
d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*T
an[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 2*c*(c - d)*EllipticF[
ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1
+ Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/
2]^2)/(c + d)]))/(f*(d + c*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*(a + a*Se
c[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)*(
1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e
 + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [A]
time = 0.53, size = 295, normalized size = 1.28

method	result
default	$-\frac{\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{d+c\cos(fx+e)}{(\cos(fx+e)+1)(c+d)}}(\cos(fx+e)+1)^2\left(2\,\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{c-d}{c+d}}\right)c^2-2c\,\mathrm{E}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*(cos(f*x+e)+1)^2*(2*Ellipti
cF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c^2-2*c*EllipticF((-1+co
s(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+EllipticE((-1+cos(f*x+e))/sin(f
*x+e),((c-d)/(c+d))^(1/2))*c^2-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/
(c+d))^(1/2))*d^2-4*c^2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+
d))^(1/2)))*(-1+cos(f*x+e))/(d+c*cos(f*x+e))/sin(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{c+d\sec{(e+fx)}}}{\sec{(e+fx)}+1}\,dx + \int \frac{d\sqrt{c+d\sec{(e+fx)}}\sec{(e+fx)}}{\sec{(e+fx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x) + Integral(d*sq
rt(c + d*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)),x)
```

```
[Out] int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)), x)
```

3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=225

$$-\frac{2\cot(e+fx)\Pi\left(\frac{c}{c+d};\text{ArcSin}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}}{a\sqrt{c+d}\,f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-d)/
(c+d))^(1/2))*(c+d*sec(f*x+e))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)*(
d*(1+sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1/2)-EllipticE(tan(f*x+
e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*
x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4010, 3865, 4053}

$$-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\Pi\left(\frac{c}{c+d};\text{ArcSin}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}\,E\left(\text{ArcSin}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[e
+ f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f*x
]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*x])
)/(a*Sqrt[c + d]*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (
c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*
f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4010

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b
^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{\int \sqrt{c+d\sec(e+fx)}\,dx}{a} - \int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$$

$$= -\frac{2\cot(e+fx)\Pi\left(\frac{c}{c+d};\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right)|\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e\cdots}{c+d\sec(e\cdots}}}{a\sqrt{c+d}\,f}$$

Mathematica [A]
time = 8.35, size = 178, normalized size = 0.79

$$-\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\left((c+d)E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)-4c\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c+d)f(1+\cos(e+fx))^2\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^4*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)
/(c + d)] + 2*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]
- 4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sqrt[(1 +
Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c + d)*f*(1 + Cos[e + f*x
])^2*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))])
```

Maple [A]
time = 0.25, size = 285, normalized size = 1.27

method	result
default	$-\frac{\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{d+c\cos(fx+e)}{(\cos(fx+e)+1)(c+d)}}(\cos(fx+e)+1)^2\left(2\,\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{c-d}{c+d}}\right)c-2\,\mathrm{Elli}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*(cos(f*x+e)+1)^2*(2*Ellipti
cF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c-2*EllipticF((-1+cos(f*
x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE((-1+cos(f*x+e))/sin(f*x
+e),((c-d)/(c+d))^(1/2))+d*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d
))^(1/2))-4*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+d))^(1/2))
)*(-1+cos(f*x+e))/(d+c*cos(f*x+e))/sin(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}{a+\frac{a}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)),x)
[Out] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)), x)
```

3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=319

$$\frac{2\sqrt{c+d}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right)|\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx)}{c-d}}}{a(c-d)f}$$

```
[Out] 2*cot(f*x+e)*EllipticF((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2),((c+d)/(c-d))^(1/
2))*(c+d)^(1/2)*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(-d*(1+sec(f*x+e))/(c-d))^(1
/2)/a/(c-d)/f-2*cot(f*x+e)*EllipticPi((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2),(c
+d)/c,((c+d)/(c-d))^(1/2))*(c+d)^(1/2)*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(-d*(
1+sec(f*x+e))/(c-d))^(1/2)/a/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((c-d)
/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/(c-d)/f/((
c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4014, 4006, 3869, 3917, 4053}

$$\frac{2\sqrt{c+d}\,\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right)|\frac{c+d}{c-d}\right)}{af(c-d)}-\frac{2\sqrt{c+d}\,\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\Pi\left(\frac{c+d}{c};\text{ArcSin}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right)|\frac{c+d}{c-d}\right)}{acf}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}\,E\left(\text{ArcSin}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Sqrt[c + d]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[
c + d]], (c + d)/(c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1
 + Sec[e + f*x]))/(c - d))])/(a*(c - d)*f) - (2*Sqrt[c + d]*Cot[e + f*x]*El
lipticPi[(c + d)/c, ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(
c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))/
(c - d))])/(a*c*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (c
 - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(
c - d)*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4014

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*C
sc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), I
nt[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] ||
EqQ[c^2 - d^2, 0])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{\int \frac{-ac+ad-ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}\,dx}{a^2(c-d)} + \frac{a\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+}}{a+a\sec(e+fx)}}{-ac+ad}$$

$$= -\frac{E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\,\mathrm{se}}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

$$= \frac{2\sqrt{c+d}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right)|\frac{c+d}{c-d}\right.}{a(c-d)f}$$

Mathematica [A]

time = 12.32, size = 187, normalized size = 0.59

$$\frac{2\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left((c+d)E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-2d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+4(-c+d)\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)\right)\sec(e+fx)}{a(-c+d)f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(d + c*Cos
[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*((c + d)*EllipticE[ArcSin[Tan[(e +
 f*x)/2]], (c - d)/(c + d)] + 2*(c - 2*d)*EllipticF[ArcSin[Tan[(e + f*x)/2]
], (c - d)/(c + d)] + 4*(-c + d)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (
c - d)/(c + d)])*Sec[e + f*x])/(a*(-c + d)*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A]

time = 0.31, size = 327, normalized size = 1.03

method	result
default	$-\frac{\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{d+c\cos(fx+e)}{(\cos(fx+e)+1)(c+d)}}(\cos(fx+e)+1)^2(-1+\cos(fx+e))\left(2\,\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*
```

```
x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c-4*Elli
pticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE((-1+cos
(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE((-1+cos(f*x+e))/sin(f*
x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-
d)/(c+d))^(1/2))+4*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+d))^(
1/2))*d)/(d+c*cos(f*x+e))/sin(f*x+e)^2/(c-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*d*sec(f*x + e)^2 + a*c + (a*c + a*d)*s
ec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sec(e+fx)}\sec(e+fx)+\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sec(e + f*x))*sec(e + f*x) + sqrt(c + d*sec(e + f*x)
)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)), x)
```

3.147 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=271

$$\frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^4\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{2d^2(6c^2+$$

```
[Out] 2*a*d*(2*c+d)*(2*c^2+2*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2
*(6*c^2+8*c*d+3*d^2)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2
/5*d^3*(4*c+3*d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2
/7*d^4*(a-a*sec(f*x+e))^3*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)
*c^4*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 90, 65, 212}

$$\frac{2a^{3/2}c^4\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{2d^4\tan(e+fx)(a-a\sec(e+fx))^3}{7a^2f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\left(6c^2+8cd+3d^2\right)\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}+\frac{2d^3(4c+3d)\tan(e+fx)(a-a\sec(e+fx))^2}{5af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (2*a*d*(2*c + d)*(2*c^2 + 2*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e +
f*x]]) + (2*a^(3/2)*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f
*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(6*c^2
+ 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e +
 f*x]]) + (2*d^3*(4*c + 3*d)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sq
rt[a + a*Sec[e + f*x]]) - (2*d^4*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(7*a^
2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^4\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(c+dx)^4}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \left(\frac{d(2c+d)(2c^2+2cd+d^2)}{\sqrt{a-ax}} + \frac{c^4}{x\sqrt{a-a}}\right.\right.}{f\sqrt{a-a}}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(6c^2+8cd}{}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(6c^2+8cd}{}$$

$$= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^4\tanh^{-1}}{f\sqrt{a-a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 14.46, size = 587, normalized size = 2.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (Cos[e + f*x]^4*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e +
f*x])^4*((8*d*(105*c^3 + 105*c^2*d + 56*c*d^2 + 12*d^3)*Sin[(e + f*x)/2])/1
05 + (2*d^4*Sec[e + f*x]^3*Sin[(e + f*x)/2])/7 + (4*Sec[e + f*x]^2*(14*c*d^
3*Sin[(e + f*x)/2] + 3*d^4*Sin[(e + f*x)/2]))/35 + (4*Sec[e + f*x]*(105*c^2
*d^2*Sin[(e + f*x)/2] + 56*c*d^3*Sin[(e + f*x)/2] + 12*d^4*Sin[(e + f*x)/2]
))/105))/(f*(d + c*Cos[e + f*x])^4) - (8*(-3 - 2*Sqrt[2])*c^4*Cos[(e + f*x)
/4]^4*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e
+ f*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[
(e + f*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Cos[e + f*x]
^3*(EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
] - 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(e + f*x)/2])*
Sec[(e + f*x)/4]^2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[
e + f*x])^4*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(f*(d + c*Cos[e + f*x
])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(247) = 494$.
time = 0.30, size = 546, normalized size = 2.01

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-105\left(\cos^3(fx+e)\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}\,\sin(fx+e)\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/840/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-105*cos(f*x+e)^3*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*sin(f*x+e)*c^4-315*cos(f*x+e)^2*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*sin(f*x+e)*c^4-315*cos(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*sin(f*x+e)*c^4-105*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(7/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^4*sin(f*x+e)+6720*cos(f*x+e)
^4*c^3*d+6720*cos(f*x+e)^4*c^2*d^2+3584*cos(f*x+e)^4*c*d^3+768*cos(f*x+e)^4
*d^4-6720*cos(f*x+e)^3*c^3*d-3360*cos(f*x+e)^3*c^2*d^2-1792*cos(f*x+e)^3*c*
d^3-384*cos(f*x+e)^3*d^4-3360*cos(f*x+e)^2*c^2*d^2-448*cos(f*x+e)^2*c*d^3-9
```

```
6*cos(f*x+e)^2*d^4-1344*cos(f*x+e)*c*d^3-48*cos(f*x+e)*d^4-240*d^4)/cos(f*x
+e)^3/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/210*(16*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(1/4)*(7*(15*c^3*d*sin(6*f*x + 6*e) + 5*(9*c^3*d + 3*c^2*d^2 + 4*c*d^3)*
sin(4*f*x + 4*e) + (45*c^3*d + 30*c^2*d^2 + 28*c*d^3 + 6*d^4)*sin(2*f*x + 2
*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (105*c^3*d*
cos(6*f*x + 6*e) + 105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4 + 35*(9*c^3*
d + 3*c^2*d^2 + 4*c*d^3)*cos(4*f*x + 4*e) + 7*(45*c^3*d + 30*c^2*d^2 + 28*c
*d^3 + 6*d^4)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))*sqrt(a) + 105*((c^4*cos(2*f*x + 2*e)^4 + c^4*sin(2*f*x + 2*e
)^4 + 4*c^4*cos(2*f*x + 2*e)^3 + 6*c^4*cos(2*f*x + 2*e)^2 + 4*c^4*cos(2*f*x
 + 2*e) + c^4 + 2*(c^4*cos(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)*s
in(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^4*
cos(2*f*x + 2*e)^4 + c^4*sin(2*f*x + 2*e)^4 + 4*c^4*cos(2*f*x + 2*e)^3 + 6*
c^4*cos(2*f*x + 2*e)^2 + 4*c^4*cos(2*f*x + 2*e) + c^4 + 2*(c^4*cos(2*f*x +
2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^4*f*cos(2*f*x + 2*e)^4 + c^4*f*sin(
2*f*x + 2*e)^4 + 4*c^4*f*cos(2*f*x + 2*e)^3 + 6*c^4*f*cos(2*f*x + 2*e)^2 +
4*c^4*f*cos(2*f*x + 2*e) + c^4*f + 2*(c^4*f*cos(2*f*x + 2*e)^2 + 2*c^4*f*co
s(2*f*x + 2*e) + c^4*f)*sin(2*f*x + 2*e)^2)*integrate((((cos(10*f*x + 10*e)
*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e
)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)
^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2
*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x
+ 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x
 + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*
x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*ar
```

```
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*
f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(
6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin
(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*
cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e
)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*
e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(
6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) +
cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f
*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e
) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f
*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*
(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x +
 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) +
4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*
e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin
(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e)
 + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6
*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*
cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x +
 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*
cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x
 + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(1
0*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x +
2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + s
in(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*...
```

Fricas [A]

time = 3.05, size = 503, normalized size = 1.86

$$\left[\frac{105 \left(c^4 \cos\left(fx+e\right)^4 + c^4 \cos\left(fx+e\right)^3\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(fx+e)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e) + a\cos(fx+e) - a}{\cos(fx+e)+1}\right) + 2\left(15\,d^4 + 4\left(105\,c^3 d + 105\,c^2 d^2 + 56\,c d^3 + 12\,d^4\right)\cos\left(fx+e\right)^3 + 2\left(105\,c^2 d^2 + 56\,c d^3 + 12\,d^4\right)\cos\left(fx+e\right)^2 + 6\left(14\,c d^3 + 3\,d^4\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{105\left(f\cos\left(fx+e\right)^4 + f\cos\left(fx+e\right)^3\right)}, -\frac{2\left(105\left(c^4\cos\left(fx+e\right)^4 + c^4\cos\left(fx+e\right)^3\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) - \left(15\,d^4 + 4\left(105\,c^3 d + 105\,c^2 d^2 + 56\,c d^3 + 12\,d^4\right)\cos\left(fx+e\right)^3 + 2\left(105\,c^2 d^2 + 56\,c d^3 + 12\,d^4\right)\cos\left(fx+e\right)^2 + 6\left(14\,c d^3 + 3\,d^4\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)\right)}{105\left(f\cos\left(fx+e\right)^4 + f\cos\left(fx+e\right)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*d^4 + 4*(
```

105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^3 + 2*(105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*d^4 + 4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^3 + 2*(105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**4, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(250) = 500.
time = 1.53, size = 535, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] -1/105*(105*sqrt(-a)*a*c^4*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(420*sqrt(2)*a^4*c^3*d*sgn(cos(f*x + e)) + 630*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x + e)) + 420*sqrt(2)*a^4*c*d^3*sgn(cos(f*x + e)) + 105*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)) - (1260*sqrt(2)*a^4*c^3*d*sgn(cos(f*x + e)) + 1470*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x + e)) + 700*sqrt(2)*a^4*c*d^3*sgn(cos(f*x + e)) + 105*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)) - (1260*sqrt(2)*a^4*c^3*d*sgn(cos(f*x + e)) + 1050*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x + e)) + 476*sqrt(2)*a^4*c*d^3*sgn(cos(f*x + e)) + 147*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)) - (420*sqrt(2)*a^4*c^3*d*sgn(cos(f*x + e)) + 210*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x + e)) + 196*sqrt(2)*a^4*c*d^3*sgn(cos(f*x + e)) + 27*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4, x)
```

3.148 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=205

$$\frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{2d^2(3c+2d)(a-a}{3f\sqrt{a+}}$$

```
[Out] 2*a*d*(3*c^2+3*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(3*c+2*
d)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*d^3*(a-a*sec(f*
x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^3*arctanh((a-a*se
c(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e
))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 90, 65, 212}

$$\frac{2a^{3/2}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2(3c+2d)\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2d^3\tan(e+fx)(a-a\sec(e+fx))^2}{5af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a*d*(3*c^2 + 3*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(3/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(3*c + 2*d)*(a -
a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{d(3c^2+3cd+d^2)}{\sqrt{a-ax}} + \frac{c^3}{x\sqrt{a-ax}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\,\text{se}}{3f\sqrt{a+a}}$$

$$= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(3c+2d)(a-a\,\text{se}}{3f\sqrt{a+a}}$$

$$= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^3\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{f\sqrt{a-a\sec(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 14.33, size = 517, normalized size = 2.52

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e +
f*x])^3*((2*d*(45*c^2 + 30*c*d + 8*d^2)*Sin[(e + f*x)/2])/15 + (2*d^3*Sec[e
 + f*x]^2*Sin[(e + f*x)/2])/5 + (2*Sec[e + f*x]*(15*c*d^2*Sin[(e + f*x)/2]
+ 4*d^3*Sin[(e + f*x)/2]))/15))/(f*(d + c*Cos[e + f*x])^3) - (8*(-3 - 2*Sqr
t[2])*c^3*Cos[(e + f*x)/4]^4*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(e
+ f*x)/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[
(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(e
+ f*x)/2])*Cos[e + f*x]^2*(EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqr
t[2]]], 17 - 12*Sqrt[2]] - 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x
)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt
[2])*Cos[(e + f*x)/2])*Sec[(e + f*x)/4]^2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec
[e + f*x])]*(c + d*Sec[e + f*x])^3*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]
)/(f*(d + c*Cos[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(185) = 370$.
time = 0.25, size = 389, normalized size = 1.90

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(15\sin(fx+e)\left(\cos^2(fx+e)\right)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}\sqrt{2}\,c^3+3\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(15*sin(f*x+e)*cos(f*x+e)^2*arc
tanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2)
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*2^(1/2)*c^3+30*sin(f*x+e)*cos(f*x+e)
*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(
1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*2^(1/2)*c^3+15*2^(1/2)*arctanh(1
/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*c^3*sin(f*x+e)+360*cos(f*x+e)^3*c^2*d+240*
cos(f*x+e)^3*c*d^2+64*cos(f*x+e)^3*d^3-360*cos(f*x+e)^2*c^2*d-120*cos(f*x+e
)^2*c*d^2-32*cos(f*x+e)^2*d^3-120*cos(f*x+e)*c*d^2-8*cos(f*x+e)*d^3-24*d^3)
/cos(f*x+e)^2/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f
*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^3*co
s(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) + c^3)*a
rctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*f*x + 2*e)
^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f)*integrate
((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8
*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x +
2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8
*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x +
4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x +
2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
 + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2
*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x + 6*e) + 3*co
s(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 +
6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x +
6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3*cos(6*f*x + 6*e) + 3*cos
(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 6
*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 9*cos(6*f*x + 6
*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 +
```

```
6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 6*((c^3 + 2*c^2*d + 4*c*d^2
)*f*cos(2*f*x + 2*e)^2 + (c^3 + 2*c^2*d + 4*c*d^2)*f*sin(2*f*x + 2*e)^2 + 2
*(c^3 + 2*c^2*d + 4*c*d^2)*f*cos(2*f*x + 2*e) + (c^3 + 2*c^2*d + 4*c*d^2)*f
)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2
*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + si
n(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*c
os(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((c
os(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*
cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*c
os(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2
*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(
2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(3*cos(6*f*x +
6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*c...
```

Fricas [A]
time = 3.08, size = 421, normalized size = 2.05

$$\left[\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(3\,d^3+\left(45\,c^2d+30\,cd^2+8\,d^3\right)\cos\left(fx+e\right)^2+\left(15\,cd^2+4\,d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)},\ -\frac{2\left(15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,d^3+\left(45\,c^2d+30\,cd^2+8\,d^3\right)\cos\left(fx+e\right)^2+\left(15\,cd^2+4\,d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{15\left(f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*d^3 + (45*c^
2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*d^2 + 4*d^3)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a
)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(
f*x + e))) - (3*d^3 + (45*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*
d^2 + 4*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3, x)
```

Giac [A]
time = 1.37, size = 365, normalized size = 1.78

$$-\frac{\frac{15\sqrt{-a}\,ac^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}\left|a\right|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}\left|a\right|-6a\right|}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{\left|a\right|}-\frac{2\left(\left(\sqrt{2}\left(45a^{3}c^{2}d\operatorname{sgn}\left(\cos\left(fx+e\right)\right)+15a^{3}cd^{2}\operatorname{sgn}\left(\cos\left(fx+e\right)\right)+7a^{3}d^{3}\operatorname{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-10\sqrt{2}\left(9a^{3}c^{2}d\operatorname{sgn}\left(\cos\left(fx+e\right)\right)+6a^{3}cd^{2}\operatorname{sgn}\left(\cos\left(fx+e\right)\right)+a^{3}d^{3}\operatorname{sgn}\left(\cos\left(fx+e\right)\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+15\sqrt{2}\left(3a^{3}c^{2}d\operatorname{sgn}\left(\cos\left(fx+e\right)\right)+3a^{3}cd^{2}\operatorname{sgn}\left(\cos\left(fx+e\right)\right)+a^{3}d^{3}\operatorname{sgn}\left(\cos\left(fx+e\right)\right)\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*a*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*((sqrt(2)*(45*a^3*c^2*d*sgn(cos(f
*x + e)) + 15*a^3*c*d^2*sgn(cos(f*x + e)) + 7*a^3*d^3*sgn(cos(f*x + e)))*ta
n(1/2*f*x + 1/2*e)^2 - 10*sqrt(2)*(9*a^3*c^2*d*sgn(cos(f*x + e)) + 6*a^3*c*
d^2*sgn(cos(f*x + e)) + a^3*d^3*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2
+ 15*sqrt(2)*(3*a^3*c^2*d*sgn(cos(f*x + e)) + 3*a^3*c*d^2*sgn(cos(f*x + e))
 + a^3*d^3*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e
)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3, x)
```

3.149 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=144

$$\frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*d*(2*c+d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(a-a*sec(f*x+e))*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^2*arctanh((a-a*sec(f*x+e))^
(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 90, 65, 212}

$$\frac{2a^{3/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\tan(e+fx)(a-a\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*d*(2*c + d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(3/2)*c^
2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec
[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a - a*Sec[e + f*x])*Tan[e +
f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \left(\frac{d(2c+d)}{\sqrt{a-ax}}+\frac{c^2}{x\sqrt{a-ax}}-\cdots\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\,\mathrm{se}\cdots}}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e\cdots}{3f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e\cdots}{3f\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e\cdots}}{\sqrt{a}}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.64, size = 444, normalized size = 3.08

$$\frac{\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\,(c+d\sec(e+fx))^2\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\left(256\,{}_3F_2\left(\frac{3}{2},2,\frac{7}{2};1,\frac{9}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sin^6\left(\frac{1}{2}(e+fx)\right)\left(c+d-2c\sin^2\left(\frac{1}{2}(e+fx)\right)\right)^2+1024\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sin^6\left(\frac{1}{2}(e+fx)\right)\left(d^2+cd\left(2-3\sin^2\left(\frac{1}{2}(e+fx)\right)\right)+c^2\left(1-3\sin^2\left(\frac{1}{2}(e+fx)\right)+2\sin^4\left(\frac{1}{2}(e+fx)\right)\right)\right)-\frac{7\sqrt{2}\left(-3\operatorname{ArcSin}\left(\sqrt{2}\sqrt{\sin^2\left(\frac{1}{2}(e+fx)\right)}\right)+\sqrt{2}\sqrt{\sin^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\left(3+4\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\left(15d^2+10cd\left(3-2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)+c^2\left(15-20\sin^2\left(\frac{1}{2}(e+fx)\right)+12\sin^4\left(\frac{1}{2}(e+fx)\right)\right)\right)}{\sqrt{\sin^2\left(\frac{1}{2}(e+fx)\right)}}\right)}{672f(d+c\cos(e+fx))^2\sec^{\frac{5}{2}}(e+fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

[Out] (Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^2*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(256*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(c + d - 2*c*Sin[(e + f*x)/2]^2)^2 + 1024*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(d^2 + c*d*(2 - 3*Sin[(e + f*x)/2]^2) + c^2*(1 - 3*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]^4)) - (7*Sqrt[2]*(-3*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]] + Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(3 + 4*Sin[(e + f*x)/2]^2))*(15*d^2 + 10*c*d*(3 - 2*Sin[(e + f*x)/2]^2) + c^2*(15 - 20*Sin[(e + f*x)/2]^2 + 12*Sin[(e + f*x)/2]^4)))/Sqrt[Sin[(e + f*x)/2]^2]))/(672*f*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2))

Maple [A]
time = 0.21, size = 248, normalized size = 1.72

method	result
default	$\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(3\sin(fx+e)\cos(fx+e)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}c^2+3\sqrt{2}\ a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*c^2+3*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*c^2*sin(f*x+e)-24*cos(f*x+e)^2*c*d-8*cos(f*x+e)^2*d^2+24*cos(f*x+e)*c*d+4*cos(f*x+e)*d^2+4*d^2)/sin(f*x+e)/cos(f*x+e)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/6*(8*(3*c*d*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(2*f*x + 2*e) - (3*c*d*cos(2*f*x + 2*e) + 3*c*d + d^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 3*((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x + 2*e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan

```
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)) + 1) - (c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x + 2*
e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - 1) - 2*(c^2*f*cos(2*f*x + 2*e)^2 + c^2*f*sin(2*f*x + 2*e)^2 + 2*c^2
*f*cos(2*f*x + 2*e) + c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*
e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2
*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*
x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6
*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))
*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 4*((c^2 + 2*c*d + 2*d^2)*f*cos(2*f*x +
 2*e)^2 + (c^2 + 2*c*d + 2*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(c^2 + 2*c*d + 2*d
^2)*f*cos(2*f*x + 2*e) + (c^2 + 2*c*d + 2*d^2)*f)*integrate((((cos(6*f*x +
6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2
*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6
*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*
e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
```

```
cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*
e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e
) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*
f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x
+ 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*co
s(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e
) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x +
 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1...
```

Fricas [A]
time = 3.61, size = 347, normalized size = 2.41

$$\left[\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(d^2+2\left(3\,cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)},-\frac{2\left(3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)}{\sqrt{a}\,\sin(fx+e)}\right)-\left(d^2+2\left(3\,cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(d^2 + 2*(3*c*d + d
^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x
 + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(sqrt(a)*sin(f*x + e))) - (d^2 + 2*(3*c*d + d^2)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e
))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(129) = 258$.
time = 1.26, size = 268, normalized size = 1.86

$$-\frac{\frac{3\sqrt{-a}\,ac^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(6\sqrt{2}\,a^2cd\operatorname{sgn}(\cos(fx+e))+3\sqrt{2}\,a^2d^2\operatorname{sgn}(\cos(fx+e))-\left(6\sqrt{2}\,a^2cd\operatorname{sgn}(\cos(fx+e))+\sqrt{2}\,a^2d^2\operatorname{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(6*sqrt(2)*a^2*c*d*sgn(cos(f*x + e)
) + 3*sqrt(2)*a^2*d^2*sgn(cos(f*x + e)) - (6*sqrt(2)*a^2*c*d*sgn(cos(f*x +
e)) + sqrt(2)*a^2*d^2*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*
x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
 a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2, x)
```

3.150 $\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}\,c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f+2*a*d*tan(f
*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4000, 3859, 209, 3877}

$$\frac{2\sqrt{a}\,c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a*d*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))\,dx = c\int \sqrt{a+a\sec(e+fx)}\,dx + d\int \sec(e+fx)\sqrt{a+a\sec\cdots}$$

$$= \frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(2ac)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{\cdots}{\sqrt{a\cdots}}\right.}{f}$$

$$= \frac{2\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2ad\tan(e+f\cdots}{f\sqrt{a+a\sec(e+\cdots}}$$

Mathematica [A]
time = 0.34, size = 76, normalized size = 1.15

$$\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(\sqrt{2}\,c\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\cos(e+fx)}+2d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[
(e + f*x)/2]]*Sqrt[Cos[e + f*x]] + 2*d*Sin[(e + f*x)/2]))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.
time = 0.17, size = 118, normalized size = 1.79

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)c\sin(fx+e)+2d\cos(fx+e)\cdots\right.}{f\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c*sin(f*x+e)+2*d*cos(f*x+e)-2*d)/sin(f*x+e)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(62) = 124$.
time = 0.54, size = 159, normalized size = 2.41

$$\frac{\sqrt{a}\,c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)+\sin\left(fx+e\right),\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)+\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e))/f

Fricas [A]
time = 2.36, size = 256, normalized size = 3.88

$$\left[\frac{\left(c\cos\left(fx+e\right)+c\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)+2\,d\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{f\cos\left(fx+e\right)+f},-\frac{2\left(\left(c\cos\left(fx+e\right)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\sqrt{a}\sin\left(fx+e\right)}\right)-d\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)\right)}{f\cos\left(fx+e\right)+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e) + f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(58) = 116.
time = 1.15, size = 193, normalized size = 2.92

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,ad\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a}+\frac{\sqrt{-a}\,ac\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a*d*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)/(a*tan(1/2*f*x + 1/2*e)^2 - a) + sqrt(-a)*a*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a))/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)

3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}-\frac{2\sqrt{a}\,\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{c+d}\,f}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/c/f-2*arctan(a^
(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)*d^(1/2
)/c/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4010, 3859, 209, 4052, 211}

$$\frac{2\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2\sqrt{a}\,\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) -
 (2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4010

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b
^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{c}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{(2ad)\text{Subst}\left(\int \frac{1}{ac+ad+d}\right.}{}$$

$$= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\sqrt{a}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e}{\sqrt{c+d}\,\sqrt{a+a\sec}}\right.}{c\sqrt{c+d}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 26.11, size = 2650, normalized size = 25.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-4*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e +
 f*x)/2])/(1 + Cos[(e + f*x)/2])]*(d + c*Cos[e + f*x])*(c*EllipticF[ArcSin[
Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*Ellipti
cPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*S
qrt[2]] + d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[
```

```
c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqr
t[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c -
 d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))
*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(d + c*Cos[e +
f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(2*(d + c*Cos[e + f*x]))
)*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x
)/4]^2])/(c*(c + d)*f*(c + d*Sec[e + f*x])*((Sqrt[2]*Sqrt[(-1 + Sqrt[2] - (
-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*EllipticF[ArcSin
[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*Ellipt
icPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] + d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt
[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sq
rt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c
- d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])
)*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4])/(c*(c + d)*Sqrt[3 - 2*Sqrt[2] - Tan[
(e + f*x)/4]^2]) + (2*Sqrt[2]*Cos[(e + f*x)/4]*Sqrt[(-1 + Sqrt[2] - (-2 + S
qrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*EllipticF[ArcSin[Tan[(
e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-
3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2
]] + d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c
- d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]
 + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
+ d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt
[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(
c*(c + d)) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*(c*EllipticF[ArcSin[Tan[(e + f*x
)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*S
qrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*
(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] -
 d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + Elli
pticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), A
rcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e
+ f*x]]*(((-2 + Sqrt[2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])) + ((-
1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Sin[(e + f*x)/2])/(2*(1 + Co
s[(e + f*x)/2])^2))*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c*(c + d)*Sq
rt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]
) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(
e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*EllipticF[ArcSin[Tan[(e + f*x)/4]/S
qrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*Sqrt[2]
, ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(Ellip
ticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)),
ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi
[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[
Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2
)*Sin[e + f*x]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c*(c + d)) - (4*S
qrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)
```

```
/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[Sec[e + f*x]]*Sqrt[3 - 2*Sqrt[2] - Tan[(e
 + f*x)/4]^2]*((c*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(
e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]
^2)/(3 - 2*Sqrt[2])]) - ((c + d)*Sec[(e + f*x)/4]^2)/(2*Sqrt[3 - 2*Sqrt[2]]
*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*T
an[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]
^2)/(3 - 2*Sqrt[2]))) + d*(Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1
 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e +
 f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 + ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4
]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)))) + Sec[(e + f
*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])
]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-
3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[
2]*Sqrt[c*(c - d)] + d)))))))/(c*(c + d))))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(85) = 170$.
time = 2.89, size = 501, normalized size = 4.77

method	result
default	$-\frac{\left(2\sqrt{\frac{d}{c-d}}\,\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)+d\ln\left(-\frac{2\left(\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{-\frac{2\cos(f}{\cos(fx+}}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+d*ln(-2*(2^(1/2)*(d/(c-d
))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d)
)^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/
2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))
^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))-d*ln(2*(2^(1/2)*(d/(c-d))
^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^
(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(
f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(
1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*2^(1/2)/(d/(c-d))^(1/2)/c/((
c+d)*(c-d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)

Fricas [A]
time = 2.59, size = 715, normalized size = 6.81

$$\left[\frac{\sqrt{-\frac{ad}{c+d}}\log\left(\frac{2\,(c+d)\sqrt{-\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}\right)+\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{cf},\ -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{-\frac{ad}{c+d}}\log\left(\frac{2\,(c+d)\sqrt{-\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}\right)}{cf},\ \frac{2\sqrt{\frac{ad}{c+d}}\arctan\left(\frac{(c+d)\sqrt{\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)+\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{cf},\ -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\sqrt{\frac{ad}{c+d}}\arctan\left(\frac{(c+d)\sqrt{\frac{ad}{c+d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)\right)}{cf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] [(sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), (2*sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*(sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/(c*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(85) = 170.

time = 1.19, size = 277, normalized size = 2.64

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-a}\,d\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac+3ad\right)}{4\sqrt{-cd-d^2}\,a}\right)}{\sqrt{-cd-d^2}\,c}+\frac{\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c|a|}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*sqrt(-a)*d*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt
(-c*d - d^2)*a))/(sqrt(-c*d - d^2)*c) + sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(
-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(
2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
 + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(c*abs(a)))*sgn(cos(f*x + e)
)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{c+\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=219

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{a^{3/2}\sqrt{d}\,(3c+2d)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)\text{ta}}{c^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+}}$$

```
[Out] -a*d*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)
*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(3/2)*(3*c+2*d)*arctanh(d^(1/2)*(a-a*sec(f*x
+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f*x+e)/c^2/(c+d)^(3/2)/f/(a-a*s
ec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 105, 162, 65, 212, 214}

$$-\frac{a^{3/2}\sqrt{d}\,(3c+2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)}{c^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2a^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(3*c +
 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan
[e + f*x])/(c^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) - (a*d*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*S
ec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} - \frac{(a\tan(e+fx))\text{Subst}\Big(\int}{c(c+d)f\sqrt{a-a\,\text{s}}}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} - \frac{(a^2\tan(e+fx))\,\text{Subst}\Big(}{c^2f\sqrt{a-a\sec(e}}$$

$$= -\frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} + \frac{(2a\tan(e+fx))\text{Subst}\Big(}{c^2f\sqrt{a-a\sec(e}}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}\,(3c+2d)\tanh^{-1}}{c^2(c+d)^{3/2}f\sqrt{a-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 28.73, size = 2907, normalized size = 13.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e +
 f*x])]*(-((d*Sin[(e + f*x)/2])/(c^2*(c + d))) + (d^2*Sin[(e + f*x)/2])/(c^
2*(c + d)*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e + f*x])^2) - (2*Sqrt[2]*C
os[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1
+ Cos[(e + f*x)/2])]*(d + c*Cos[e + f*x])^2*(c*(2*c + d)*EllipticF[ArcSin[T
an[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*Ellipt
icPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] + d*(3*c + 2*d)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*
Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2
]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]]))*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(c
 + d)*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(2*
(c + d)*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])
```

/(2*c*(c + d)*(d + c*Cos[e + f*x])))*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x
])]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c^2*(c + d)^2*f*(c + d*Sec[e
 + f*x])^2*((Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos
[(e + f*x)/2])]*(c*(2*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*S
qrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[
Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(3*c + 2*d)*(El
lipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)
), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + Ellipti
cPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcS
in[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f
*x]]*Tan[(e + f*x)/4])/(Sqrt[2]*c^2*(c + d)^2*Sqrt[3 - 2*Sqrt[2] - Tan[(e +
 f*x)/4]^2]) + (Sqrt[2]*Cos[(e + f*x)/4]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2]
)*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(c*(2*c + d)*EllipticF[ArcSin[T
an[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*Ellipt
icPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] + d*(3*c + 2*d)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*
Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2
]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[3 - 2*Sqrt[2] - Tan[
(e + f*x)/4]^2])/(c^2*(c + d)^2) - (Sqrt[2]*Cos[(e + f*x)/4]^2*(c*(2*c + d)
*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] -
 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*
Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(3*c + 2*d)*(EllipticPi[-(((-3 + 2*Sqrt[2])
*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/S
qrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d
))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*(((-2 + Sqrt[2])*Sin[(e
 + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])) + ((-1 + Sqrt[2] - (-2 + Sqrt[2])*Co
s[(e + f*x)/2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])^2))*Sqrt[3 - 2*
Sqrt[2] - Tan[(e + f*x)/4]^2])/(c^2*(c + d)^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sq
rt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]) - (Sqrt[2]*Cos[(e + f*x)/
4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*
x)/2])]*(c*(2*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e +
 f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(3*c + 2*d)*(EllipticPi
[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSi
n[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3
 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(
e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin
[e + f*x]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(c^2*(c + d)^2) - (2*Sq
rt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/
2])/(1 + Cos[(e + f*x)/2])]*Sqrt[Sec[e + f*x]]*Sqrt[3 - 2*Sqrt[2] - Tan[(e
+ f*x)/4]^2]*((c*(2*c + d)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[
1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e

```
+ f*x)/4]^2)/(3 - 2*Sqrt[2])]) - ((c + d)^2*Sec[(e + f*x)/4]^2)/(Sqrt[3 - 2
*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*S
qrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(e
+ f*x)/4]^2)/(3 - 2*Sqrt[2]))) + d*(3*c + 2*d)*(Sec[(e + f*x)/4]^2/(4*Sqrt[
3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 -
 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 + ((-3 + 2*Sqrt[2])*(c
 + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)]
 - d)))) + Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqr...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 97142 vs. $2(189) = 378$.
time = 0.92, size = 97143, normalized size = 443.58

method	result	size
default	Expression too large to display	97143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^2, x)
```

Fricas [**A**]
time = 7.84, size = 1499, normalized size = 6.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d + 2*
d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a
*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c
 + d)*cos(f*x + e) + d)) - 2*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2
 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)
```

```
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(
f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 +
 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -1/2*(2*c*d*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 4*((c^2 + c
*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d
 + 2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
+ 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c
^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -(c*d*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((3*c^2 + 2*c*d)*cos(
f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(a*d
/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) - ((c^2 + c*d)*cos(f*x + e)^2 + c*
d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^
2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x +
 e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -(
c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(
(c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*
sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2
 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(
c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x +
 e))))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*
x + e) + (c^3*d + c^2*d^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(189) = 378.
time = 1.39, size = 617, normalized size = 2.82

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\left(3\sqrt{-a}\,acd+2\sqrt{-a}\,ad^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}d+ac+3ad\right)}{4\sqrt{-cd-d^{2}}\,a}\right)}{\left(c^{3}+c^{2}d\right)\sqrt{-cd-d^{2}}\,a}+\frac{\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}\left|a\right|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}\left|a\right|-6a\right|}\right)}{c^{2}\left|a\right|}-\frac{4\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}\sqrt{-a}\,acd+3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}\sqrt{-a}\,ad^{2}+\sqrt{-a}\,a^{2}cd-\sqrt{-a}\,a^{2}d^{2}\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}d+2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}ac+6\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}ad+a^{2}c-a^{2}d\right)\left(c^{3}-cd^{2}\right)}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*(3*sqrt(-a)*a*c*d + 2*sqrt(-a)*a*d^2)*arctan(1/4*sqrt
(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^
2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))
^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((c^3 + c^2*d)*sqrt(-c*d - d^2)*a
) + sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))/(c^2*abs(a)) - 4*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a*c*d + 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
 - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a*d^2 + sqrt(-a)*a^2*c*d
 - sqrt(-a)*a^2*d^2)/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
 + 1/2*e)^2 + a))^4*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*
x + 1/2*e)^2 + a))^4*d + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2
*f*x + 1/2*e)^2 + a))^2*a*c + 6*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*a*d + a^2*c - a^2*d)*(c^3 - c*d^2)))*sgn(cos(f
*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2, x)
```

3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=287

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{a^{3/2}\sqrt{d}\left(15c^2+20cd+8d^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e}{\sqrt{a}\sqrt{c+}}\right.}{4c^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,s}}$$

```
[Out] -1/2*a*d*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/4
*a*d*(7*c+4*d)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(
1/2)+2*a^(3/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-
a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a^(3/2)*(15*c^2+20*c*d+8*d^2
)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f
*x+e)/c^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {4025, 105, 156, 162, 65, 212, 214}

$$\frac{2a^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^{3/2}\sqrt{d}\left(15c^2+20cd+8d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{ad(7c+4d)\tan(e+fx)}{4c^2f(c+d)^2\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}-\frac{ad\tan(e+fx)}{2cf(c+d)\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(3/2)*Sqrt[d]*(15*c^
2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqr
t[c + d])]*Tan[e + f*x])/(4*c^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (a*d*Tan[e + f*x])/(2*c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x])^2) - (a*d*(7*c + 4*d)*Tan[e + f*x])/(4*c^2*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} - \frac{(a\tan(e+fx))\text{Subs}}{2c(c+d)f\sqrt{a}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} - \frac{ad(7c}{4c^2(c+d)^2f\sqrt{a+a}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} - \frac{ad(7c}{4c^2(c+d)^2f\sqrt{a+a}}$$

$$= -\frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} - \frac{ad(7c}{4c^2(c+d)^2f\sqrt{a+a}}$$

$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}\,(15c^2+20cd+}{4c^3(c+d)^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.46, size = 3070, normalized size = 10.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e +
 f*x])]*((-3*d*(3*c + 2*d)*Sin[(e + f*x)/2])/(4*c^3*(c + d)^2) - (d^3*Sin[(
e + f*x)/2])/(2*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (11*c*d^2*Sin[(e + f*
x)/2] + 8*d^3*Sin[(e + f*x)/2])/(4*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f
*(c + d*Sec[e + f*x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*C
os[e + f*x])^3*(c*(8*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]
/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c + d)^3*EllipticPi[-3 + 2*Sq
rt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(
15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*
Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2
```

```
]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]]))*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(c
 + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*
c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]
])/(2*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e
+ f*x]])/(c*(c + d)^2*(d + c*Cos[e + f*x])) + (d^2*Cos[(3*(e + f*x))/2]*Sqr
t[Sec[e + f*x]])/(2*c^2*(c + d)^2*(d + c*Cos[e + f*x])))*Sec[e + f*x]^3*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[
1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(2*c^3*(c + d)^3*f*(c + d*Sec[e +
f*x])^3*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])*(c*(8*c^2 + 9*c*d + 4*d^2)*El
lipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16
*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sq
rt[2]]], 17 - 12*Sqrt[2]] + d*(15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(((-3
+ 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e
 + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqr
t[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)
/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*
x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*c^3*(c + d)^3*Sqrt[
1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-3 + 2*Sqr
t[2])*(c*(8*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], Ar
cSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(15*c^2 +
20*c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*S
qrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12
*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*
(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2
]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e +
f*x)/4]^2])/(8*c^3*(c + d)^3*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])
 + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]*(c*(8*c^2 + 9*c*d + 4*d^2)*Ellipti
cF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c +
 d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]] + d*(15*c^2 + 20*c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*S
qrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*
x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])
*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/S
qrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]
*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan
[(e + f*x)/4]^2])/(4*c^3*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]
^2*(c*(8*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*
Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcSi
n[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(15*c^2 + 20*
c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt
[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sq
rt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c
- d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])
```

```
)*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4
]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^3*(c + d)^3) - (Sqr
t[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqrt[Sec[e + f*x]]*Sqrt[1 + (-3 + 2*Sqr
t[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*((c*
(8*c^2 + 9*c*d + 4*d^2)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 -
 Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f
*x)/4]^2)/(3 - 2*Sqrt[2])]) - (4*(c + d)^3*Sec[(e + f*x)/4]^2)/(Sqrt[3 - 2*
Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sq
rt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan[(e +
 f*x)/4]^2)/(3 - 2*Sqrt[2]))) + d*(15*c^2 + 20*c*d + 8*d^2)*(Sec[(e + f*x)/
4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 330371 vs. $2(249) = 498$.
time = 3.27, size = 330372, normalized size = 1151.12

method	result	size
default	Expression too large to display	330372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(263) = 526$.
time = 13.97, size = 2470, normalized size = 8.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/8*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(
f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (
30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d))*
```

```
log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)
*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*(c^2*d^2
+ 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^3*d
 + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d
^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)) - 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*
d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^
5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^
3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(c^2*d
^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^
3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3
+ d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (1
5*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*
d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)
*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*
cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d
)*cos(f*x + e) + d)) + 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d
^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
 + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5
*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3
*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), 1/4*((15*c^2*d^2
 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*
c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2
*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqr
t(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*si
n(f*x + e))) + 4*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f
*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d
 + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^
2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - (3*(3*c^3*d + 2*c^2*d^2)*
cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3
 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*
c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*
d^4)*f), -1/4*(8*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f
*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d
 + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2
+ 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c
^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*
```

```
d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(a*d/(c + d))*arctan((c + d)*sqrt
(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin
(f*x + e))) + (3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^
3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^
7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^
4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos
(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1381 vs. $2(249) = 498$.
time = 1.80, size = 1381, normalized size = 4.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(sqrt(2)*(15*sqrt(-a)*a*c^2*d + 20*sqrt(-a)*a*c*d^2 + 8*sqrt(-
a)*a*d^3)*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(
1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((c^5 +
2*c^4*d + c^3*d^2)*sqrt(-c*d - d^2)*a) + 4*sqrt(2)*sqrt(-a)*a*log(abs(2*(sq
rt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sq
rt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*
f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(c^3*abs(a)) - 4*(9*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)
*a*c^4*d + 25*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)
^2 + a))^6*sqrt(-a)*a*c^3*d^2 - 33*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a*c^2*d^3 - 13*(sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a*c*d^4 + 12*
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqr
t(-a)*a*d^5 + 27*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^4*sqrt(-a)*a^2*c^4*d + 87*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*c^3*d^2 + 149*(sqrt(-a)*tan
```

```
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*c^2
*d^3 - 59*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
 a))^4*sqrt(-a)*a^2*c*d^4 - 76*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*d^5 + 27*(sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^3*c^4*d + 43*(sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a
)*a^3*c^3*d^2 - 99*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^2*sqrt(-a)*a^3*c^2*d^3 - 7*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - s
qrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^3*c*d^4 + 36*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^3*d^5
 + 9*sqrt(-a)*a^4*c^4*d - 19*sqrt(-a)*a^4*c^3*d^2 + 7*sqrt(-a)*a^4*c^2*d^3
+ 7*sqrt(-a)*a^4*c*d^4 - 4*sqrt(-a)*a^4*d^5)/((c^6 - 2*c^4*d^2 + c^2*d^4)*(
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*c -
 (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*d
+ 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
*a*c + 6*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
a))^2*a*d + a^2*c - a^2*d)^2))*sgn(cos(f*x + e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3, x)
```

3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=241

$$\frac{2a^{5/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(}{}$$

```
[Out] 2/35*a^2*(6*c+13*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+
2/7*a^2*(c+d*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/105*a^2*(7
2*c^3+486*c^2*d+378*c*d^2+104*d^3+d*(24*c^2+111*c*d+52*d^2)*sec(f*x+e))*tan
(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(5/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/
2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 158, 152, 65, 212}

$$\frac{2a^{5/2}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)\left(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3+243c^2d+189cd^2+52d^3)\right)}{105f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c+d\sec(e+fx))^3}{7f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(6c+13d)\tan(e+fx)(c+d\sec(e+fx))^2}{35f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(6*c + 13*d)*(c
+ d*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^
2*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 111*c*d + 52*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
```

```
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}} + \frac{(2a\tan(e+fx))\mathrm{S}}{7f\sqrt{}}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+}{7}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+}{7}$$

$$= \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+}{7}$$

$$= \frac{2a^{5/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{2}{}$$

Mathematica [A]
time = 3.91, size = 219, normalized size = 0.91

$$\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}\,c^3\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos^{\frac{7}{2}}(e+fx)+2(210c^2d+378cd^2+164d^3+9(35c^3+175c^2d+154cd^2+52d^3)\cos(e+fx)+2d(105c^2+189cd+52d^2)\cos(2(e+fx))+105c^3\cos(3(e+fx))+525c^2d\cos(3(e+fx))+378cd^2\cos(3(e+fx))+104d^3\cos(3(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{420f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2]*
c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 2*(210*c^2*d + 37
8*c*d^2 + 164*d^3 + 9*(35*c^3 + 175*c^2*d + 154*c*d^2 + 52*d^3)*Cos[e + f*x
] + 2*d*(105*c^2 + 189*c*d + 52*d^2)*Cos[2*(e + f*x)] + 105*c^3*Cos[3*(e +
f*x)] + 525*c^2*d*Cos[3*(e + f*x)] + 378*c*d^2*Cos[3*(e + f*x)] + 104*d^3*C
os[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(216) = 432$.
time = 3.30, size = 539, normalized size = 2.24

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(105\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^3(fx+e))\sin(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}\sqrt{2}\,c^3+315\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/840/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(105*arctanh(1/2*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^3*sin(f*x+
e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*2^(1/2)*c^3+315*arctanh(1/2*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*s
in(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*2^(1/2)*c^3+315*arctanh(1/2*
(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x
+e)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*2^(1/2)*c^3+105*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(7/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3*sin(f*x+e)-1680*cos(f*x+e)^4
*c^3-8400*cos(f*x+e)^4*c^2*d-6048*cos(f*x+e)^4*c*d^2-1664*cos(f*x+e)^4*d^3+
1680*cos(f*x+e)^3*c^3+6720*cos(f*x+e)^3*c^2*d+3024*cos(f*x+e)^3*c*d^2+832*c
os(f*x+e)^3*d^3+1680*cos(f*x+e)^2*c^2*d+2016*cos(f*x+e)^2*c*d^2+208*cos(f*x
+e)^2*d^3+1008*cos(f*x+e)*c*d^2+384*cos(f*x+e)*d^3+240*d^3)/cos(f*x+e)^3/si
n(f*x+e)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/210*(4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(7*(15*(a*c^3 + 3*a*c^2*d)*sin(6*f*x + 6*e) + 5*(9*a*c^3 + 33*a*c^2
*d + 18*a*c*d^2 + 4*a*d^3)*sin(4*f*x + 4*e) + (45*a*c^3 + 195*a*c^2*d + 144
*a*c*d^2 + 52*a*d^3)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)) - (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3 +
 105*(a*c^3 + 3*a*c^2*d)*cos(6*f*x + 6*e) + 35*(9*a*c^3 + 33*a*c^2*d + 18*a
*c*d^2 + 4*a*d^3)*cos(4*f*x + 4*e) + 7*(45*a*c^3 + 195*a*c^2*d + 144*a*c*d^
2 + 52*a*d^3)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))*sqrt(a) + 105*((a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*sin(2*f*x +
 2*e)^4 + 4*a*c^3*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2 + 4*a*c^3
*cos(2*f*x + 2*e) + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*x
 + 2*e) + a*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
```

x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) + 1) - (a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*sin(2*f*x + 2*e)^4 + 4*a*c^3
*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2 + 4*a*c^3*cos(2*f*x + 2*e)
 + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*x + 2*e) + a*c^3)*
sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a
*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x + 2*e)^4 + 4*a*c^3*f*cos(2*f*
x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4*a*c^3*f*cos(2*f*x + 2*e) + a*
c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*
f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos
(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(
2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin
(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*c
os(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin
(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*si
n(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin
(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*si
n(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin
(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*si
n(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos
(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*co
s(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin
(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*si
n(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
+ 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2
+ (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(c
os(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos

```
(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(
2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3
 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x +
 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
+ 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)...
```

Fricas [A]
time = 3.41, size = 513, normalized size = 2.13

$$\left[\frac{105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{-a}\,\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(15ad^3+\left(105ac^3+525ac^2d+378acd^2+104ad^3\right)\cos(fx+e)^3+\left(105ac^2d+189acd^2+52ad^3\right)\cos(fx+e)^2+3\left(21acd^2+13ad^3\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{105\left(f\cos(fx+e)^4+f\cos(fx+e)^3\right)},\ -\frac{2\left(105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15ad^3+\left(105ac^3+525ac^2d+378acd^2+104ad^3\right)\cos(fx+e)^3+\left(105ac^2d+189acd^2+52ad^3\right)\cos(fx+e)^2+3\left(21acd^2+13ad^3\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{105\left(f\cos(fx+e)^4+f\cos(fx+e)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a*d^3
 + (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (10
5*a*c^2*d + 189*a*c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d
^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c
^3*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a*d^3 + (105*a*c^3 + 525*a*c^2*d
+ 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (105*a*c^2*d + 189*a*c*d^2 + 52
*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x +
 e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^{\frac{3}{2}}\,(c+d\sec(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(216) = 432.

time = 1.87, size = 525, normalized size = 2.18

$$-\frac{\frac{105\,\sqrt{-a}\,a^2c^3\log\left(\frac{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{|a|}+\frac{2\,\left(105\,\sqrt{2}\,a^5c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+630\,\sqrt{2}\,a^5c^2d\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+630\,\sqrt{2}\,a^5cd^2\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+210\,\sqrt{2}\,a^5d^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(315\,\sqrt{2}\,a^5c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+1680\,\sqrt{2}\,a^5c^2d\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+1260\,\sqrt{2}\,a^5cd^2\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+280\,\sqrt{2}\,a^5d^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(315\,\sqrt{2}\,a^5c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+1470\,\sqrt{2}\,a^5c^2d\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+882\,\sqrt{2}\,a^5cd^2\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+266\,\sqrt{2}\,a^5d^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(105\,\sqrt{2}\,a^5c^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+420\,\sqrt{2}\,a^5c^2d\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+252\,\sqrt{2}\,a^5cd^2\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+76\,\sqrt{2}\,a^5d^3\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*a^2*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqr
t(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(105*sqrt(2)*a^5*c^3*sgn(cos(
f*x + e)) + 630*sqrt(2)*a^5*c^2*d*sgn(cos(f*x + e)) + 630*sqrt(2)*a^5*c*d^2
*sgn(cos(f*x + e)) + 210*sqrt(2)*a^5*d^3*sgn(cos(f*x + e)) - (315*sqrt(2)*a
^5*c^3*sgn(cos(f*x + e)) + 1680*sqrt(2)*a^5*c^2*d*sgn(cos(f*x + e)) + 1260*
sqrt(2)*a^5*c*d^2*sgn(cos(f*x + e)) + 280*sqrt(2)*a^5*d^3*sgn(cos(f*x + e))
 - (315*sqrt(2)*a^5*c^3*sgn(cos(f*x + e)) + 1470*sqrt(2)*a^5*c^2*d*sgn(cos(
f*x + e)) + 882*sqrt(2)*a^5*c*d^2*sgn(cos(f*x + e)) + 266*sqrt(2)*a^5*d^3*s
gn(cos(f*x + e)) - (105*sqrt(2)*a^5*c^3*sgn(cos(f*x + e)) + 420*sqrt(2)*a^5
*c^2*d*sgn(cos(f*x + e)) + 252*sqrt(2)*a^5*c*d^2*sgn(cos(f*x + e)) + 76*sqr
t(2)*a^5*d^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e
)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^
2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3, x)
```

3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=176

$$\frac{2a^{5/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(2(6c^2+25c}{}$$

```
[Out] 2/5*a^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(12
*c^2+50*c*d+18*d^2+d*(4*c+9*d)*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1
/2)+2*a^(5/2)*c^2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a
*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 158, 152, 65, 212}

$$\frac{2a^{5/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)\left(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx)\right)}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c+d\sec(e+fx))^2}{5f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(c + d*Sec[e + f
*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(2*(6*c^2 + 25
*c*d + 9*d^2) + d*(4*c + 9*d)*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[a + a*
Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
```

```
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)(c+dx)^2}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{(2a\tan(e+fx))\text{Su}}{5f\sqrt{a}}$$

$$= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(6c^2+25cd+}{}$$

$$= \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(2(6c^2+25cd+}{}$$

$$= \frac{2a^{5/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{2a^2}{}$$

Mathematica [A]
time = 1.38, size = 145, normalized size = 0.82

$$\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}\,c^2\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos^{\frac{5}{2}}(e+fx)+2(15c^2+50cd+24d^2+2d(10c+9d)\cos(e+fx)+(15c^2+50cd+18d^2)\cos(2(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*c
^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(15*c^2 + 50*c*d
 + 24*d^2 + 2*d*(10*c + 9*d)*Cos[e + f*x] + (15*c^2 + 50*c*d + 18*d^2)*Cos[
2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(155) = 310$.
time = 2.81, size = 382, normalized size = 2.17

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(15\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)\sqrt{2}\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}c^2+3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(15*arctanh(1/2*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*sin(f*x+e
)*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*c^2+30*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)*sin(f
*x+e)*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*c^2+15*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^2*sin(f*x+e)+120*cos(f*x+e)^3*c^2+400*co
s(f*x+e)^3*c*d+144*cos(f*x+e)^3*d^2-120*cos(f*x+e)^2*c^2-320*cos(f*x+e)^2*c
*d-72*cos(f*x+e)^2*d^2-80*cos(f*x+e)*c*d-48*cos(f*x+e)*d^2-24*d^2)/cos(f*x+
e)^2/sin(f*x+e)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*c
os(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) -
 (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2*f*x +
 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(a*c^2*
f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2
*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x +
2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2
```

```
*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*
e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e
)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x +
 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2
*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*
e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 +
 (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2
*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*
x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)
*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*
x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*
e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(
4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2),
x) - 2*((5*a*c^2 + 12*a*c*d + 4*a*d^2)*f*cos(2*f*x + 2*e)^2 + (5*a*c^2 + 12
*a*c*d + 4*a*d^2)*f*sin(2*f*x + 2*e)^2 + 2*(5*a*c^2 + 12*a*c*d + 4*a*d^2)*f
*cos(2*f*x + 2*e) + (5*a*c^2 + 12*a*c*d + 4*a*d^2)*f)*integrate((cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x
+ 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x
```

```
+ 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*...
```

Fricas [A]
time = 2.99, size = 427, normalized size = 2.43

$$\left[\frac{15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(3ad^2+\left(15ac^2+50acd+18ad^2\right)\cos(fx+e)^2+\left(10acd+9ad^2\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}, -\frac{2\left(15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3ad^2+\left(15ac^2+50acd+18ad^2\right)\cos(fx+e)^2+\left(10acd+9ad^2\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a*d^2 +
(15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
+ e)^3 + f*cos(f*x + e)^2), -2/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x
 + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))) - (3*a*d^2 + (15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos
(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\sec(e+fx)+1)\right)^{\frac{3}{2}}\left(c+d\sec(e+fx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(155) = 310.
time = 1.56, size = 359, normalized size = 2.04

$$-\frac{\frac{15\sqrt{-a}\,a^2c^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(\left(\sqrt{2}\left(15a^4c^2\mathrm{sgn}(\cos(fx+e))+40a^4cd\,\mathrm{sgn}(\cos(fx+e))+12a^4d^2\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-10\sqrt{2}\left(3a^4c^2\mathrm{sgn}(\cos(fx+e))+10a^4cd\,\mathrm{sgn}(\cos(fx+e))+3a^4d^2\mathrm{sgn}(\cos(fx+e))\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+15\sqrt{2}\left(a^4c^2\mathrm{sgn}(\cos(fx+e))+4a^4cd\,\mathrm{sgn}(\cos(fx+e))+2a^4d^2\mathrm{sgn}(\cos(fx+e))\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*a^2*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
```

```
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*
abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*((sqrt(2)*(15*a^4*c^2*sgn(cos(f
*x + e)) + 40*a^4*c*d*sgn(cos(f*x + e)) + 12*a^4*d^2*sgn(cos(f*x + e)))*tan
(1/2*f*x + 1/2*e)^2 - 10*sqrt(2)*(3*a^4*c^2*sgn(cos(f*x + e)) + 10*a^4*c*d*
sgn(cos(f*x + e)) + 3*a^4*d^2*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 +
15*sqrt(2)*(a^4*c^2*sgn(cos(f*x + e)) + 4*a^4*c*d*sgn(cos(f*x + e)) + 2*a^4
*d^2*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 -
a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2, x)
```

3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=105

$$\frac{2a^{3/2}c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f}$$

```
[Out] 2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/3*a^2*(3*
c+4*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a*d*(a+a*sec(f*x+e))^(1/2)*t
an(f*x+e)/f
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4002, 4000, 3859, 209, 3877}

$$\frac{2a^{3/2}c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^2*(3*c + 4*d)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*Sqr
t[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \frac{2ad\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{3f}+\frac{2}{3}\int\sqrt{a+a\sec(e\cdots$$

$$=\frac{2ad\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{3f}+(ac)\int\sqrt{a+a\sec(\cdots$$

$$=\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{2ad\sqrt{a+a\sec(e+fx)}\,\tan(e\cdots}{3f}$$

$$=\frac{2a^{3/2}c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e\cdots}{3f\sqrt{a+a\sec(e\cdots}}$$

Mathematica [A]
time = 0.60, size = 102, normalized size = 0.97

$$\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\left(3\sqrt{2}\,c\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos^{\frac{3}{2}}(e+fx)+2(d+(3c+5d)\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(3*Sqrt[2]*c*Ar
cSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(3/2) + 2*(d + (3*c + 5*d)*Cos[
e + f*x])*Sin[(e + f*x)/2]))/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(91) = 182$.
time = 2.91, size = 237, normalized size = 2.26

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-3\cos(fx+e)\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}c-3(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-3*cos(f*x+e)*sin(f*x+e)*arctan
h(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2
^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*c-3*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(3/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+
e)/cos(f*x+e)*2^(1/2))*c*sin(f*x+e)+12*cos(f*x+e)^2*c+20*cos(f*x+e)^2*d-12*
c*cos(f*x+e)-16*d*cos(f*x+e)-4*d)/sin(f*x+e)/cos(f*x+e)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(97) = 194$.
time = 0.61, size = 1076, normalized size = 10.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
```

```
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)
```

Fricas [A]
time = 1.99, size = 343, normalized size = 3.27

$$\left[\frac{3\left(ac\cos\left(fx+e\right)^{2}+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(ad+\left(3\,ac+5\,ad\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^{2}+f\cos\left(fx+e\right)\right)},-\frac{2\left(3\left(ac\cos\left(fx+e\right)^{2}+ac\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(ad+\left(3\,ac+5\,ad\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)\right)}{3\left(f\cos\left(fx+e\right)^{2}+f\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
 e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(a*d + (3*a*c + 5*a
*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*
cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x
 + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(sqrt(a)*sin(f*x + e))) - (a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e
))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(91) = 182.
time = 1.32, size = 263, normalized size = 2.50

$$-\frac{\frac{3\sqrt{-a}a^2c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(3\sqrt{2}a^3c\,\mathrm{sgn}(\cos(fx+e))+6\sqrt{2}a^3d\,\mathrm{sgn}(\cos(fx+e))-\left(3\sqrt{2}a^3c\,\mathrm{sgn}(\cos(fx+e))+4\sqrt{2}a^3d\,\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a^2*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(3*sqrt(2)*a^3*c*sgn(cos(f*x + e))
+ 6*sqrt(2)*a^3*d*sgn(cos(f*x + e)) - (3*sqrt(2)*a^3*c*sgn(cos(f*x + e)) +
4*sqrt(2)*a^3*d*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/
2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/
f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=110

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2a^{3/2}(c-d)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{d}\sqrt{c+d}f}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+2*a^(3/2)*(
c-d)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))/
c/f/d^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4012, 3859, 209, 4052, 211}

$$\frac{2a^{3/2}(c-d)\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}}+\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f) +
 (2*a^(3/2)*(c - d)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sec[e + f*x]])])/(c*Sqrt[d]*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{a\int \sqrt{a+a\sec(e+fx)}\,dx}{c} + \frac{(ac-ad)\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{c}$$

$$= -\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{(2a^2(c-d))\,\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2a^{3/2}(c-d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\,\mathrm{t}\ldots}{\sqrt{c+d}\sqrt{a+a\ldots}}\right.}{c\sqrt{d}\sqrt{c+d}\,f}$$

Mathematica [A]
time = 0.52, size = 135, normalized size = 1.23

$$\frac{\sqrt{2}\,a\left(\sqrt{d}\sqrt{c+d}\,\mathrm{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(c-d)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\right)\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{c\sqrt{d}\sqrt{c+d}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*a*(Sqrt[d]*Sqrt[c + d]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (c - d)*
ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]])]
)*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(c*Sqrt[d
]*Sqrt[c + d]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(90) = 180$.
time = 4.65, size = 868, normalized size = 7.89

method	result
default	$-\frac{\left(2\sqrt{\frac{d}{c-d}}\,\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)+\ln\left(-\frac{2\left(\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)-}}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+ln(-2*(2^(1/2)*(d/(c-d))
^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^
(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(
f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*
cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))*c-ln(-2*(2^(1/2)*(d/(c-d))^
(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(
1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f
*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*c
os(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))*d-ln(2*(-2^(1/2)*(d/(c-d))^(
1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))^(1
/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*c
os(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/
2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*c+ln(2*(-2^(1/2)*(d/(c-d))^(1
/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))^(1/
2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*co
s(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2
)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*d)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*2^(1/2)*a/((c+d)*(c-d))^(1/2)
/c/(d/(c-d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Fricas [**A**]
time = 4.01, size = 777, normalized size = 7.06

$$\left[-\frac{(ac-ad)\sqrt{-\frac{a}{cd+d^2}}\log\left(\frac{2\,(cd+d^2)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}\right)-\sqrt{-a}\,a\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{cf},\ -\frac{2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+(ac-ad)\sqrt{-\frac{a}{cd+d^2}}\log\left(\frac{2\,(cd+d^2)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac+2\,ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}{c\cos(fx+e)^2+(c+d)\cos(fx+e)+d}\right)}{cf},\ -\frac{2\,(ac-ad)\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{(c+d)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)-\sqrt{-a}\,a\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{cf},\ -\frac{2\left((ac-ad)\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{(c+d)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)+a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\right)}{cf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-((a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*c
os(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*a^(3/2)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a*c -
a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*co
s(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*
cos(f*x + e) + d)))/(c*f), -(2*(a*c - a*d)*sqrt(a/(c*d + d^2))*arctan((c +
d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(a*sin(f*x + e))) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e
) - a)/(cos(f*x + e) + 1)))/(c*f), -2*((a*c - a*d)*sqrt(a/(c*d + d^2))*arct
an((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(a*sin(f*x + e))) + a^(3/2)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{c+d\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(90) = 180.
time = 1.19, size = 285, normalized size = 2.59

$$\frac{\sqrt{2}\sqrt{-a}\,a^3\left(\frac{2\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac+3\,ad\right)}{4\sqrt{-cd-d^2}\,a}\right)}{\sqrt{-cd-d^2}\,a^2c}-\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{ac|a|}\right)\mathrm{sgn}(\cos(fx+e))}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a)*a^3*(2*sqrt(2)*(c - d)*arctan(1/4*sqrt(2)*((sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*
tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a
*d)/(sqrt(-c*d - d^2)*a))/(sqrt(-c*d - d^2)*a^2*c) - sqrt(2)*log(abs(2*(sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqr
t(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f
*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(a*c*abs(a)))*sgn(cos(f*x
+ e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{a^{5/2}(c^2-3cd-2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right.}{c^2\sqrt{d}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}$$

```
[Out] a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a^
(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*sec(f*x
+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(5/2)*(c^2-3*c*d-2*d^2)*arctanh(d^(1/2)
*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c+d)^(3/2)/f/d
^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 156, 162, 65, 212, 214}

$$\frac{a^{5/2}(c^2-3cd-2d^2)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2\sqrt{d}f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2(c-d)\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(c^2 - 3*c*d -
 2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*T
an[e + f*x])/(c^2*Sqrt[d]*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[
e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} - \frac{(a\tan(e+fx))\text{Subst}\Big(}{c(c+d)f\sqrt{a-a\text{s}}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} - \frac{(a^3\tan(e+fx))\,\text{Subst}}{c^2f\sqrt{a-a\sec(e}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} + \frac{(2a^2\tan(e+fx))\,\text{Subs}}{c^2f\sqrt{a-a\sec}}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(c^2-3cd-2d^2)\,\text{ta}}{c^2\sqrt{d}\,(c+d)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.92, size = 2862, normalized size = 12.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^3*Sec[e + f*x]*(a*(1 + Sec[e + f*x
]))^(3/2)*(((c - d)*Sin[(e + f*x)/2])/(2*c^2*(c + d)) + (-(c*d*Sin[(e + f*x
)/2]) + d^2*Sin[(e + f*x)/2])/(2*c^2*(c + d)*(d + c*Cos[e + f*x]))))/(f*(c
+ d*Sec[e + f*x])^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e
 + f*x])^2*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2
]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(
e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*
(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] -
 d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + Elli
pticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), A
rcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[(e + f*
x)/2]^3*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(c + d)*(d + c*Cos[e + f*
x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*(c + d)*(d + c*Cos[e +
f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c*(c + d)*(d + c*Co
```

```
s[e + f*x])))*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[1 + (-3 + 2*Sq
rt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(c
^2*(c + d)^2*f*(c + d*Sec[e + f*x])^2*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])
*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4
]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*(EllipticP
i[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcS
in[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-
3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[
(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Ta
n[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^2*(c + d
)^2*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-
3 + 2*Sqrt[2])*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sq
rt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[T
an[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d
^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d
)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] +
EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d
), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Se
c[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/
(4*c^2*(c + d)^2*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) + (Sqrt[3 -
 2*Sqrt[2]]*Cos[(e + f*x)/4]*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]
/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqr
t[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2
 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]
*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[
c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt
[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e
 + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(2*c^2*(c + d)^
2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(c*(3*c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*Ellip
ticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12
*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/
(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*
Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c +
 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]
]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[1 + (-3 + 2*Sqr
t[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(2*
c^2*(c + d)^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqrt[Sec[e + f*x]]
*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan
[(e + f*x)/4]^2]*((c*(3*c + d)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*S
qrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan
[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - ((c + d)^2*Sec[(e + f*x)/4]^2)/(Sqrt[3
 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 -
```

```
12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*Tan
[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) - (c^2 - 3*c*d - 2*d^2)*(Sec[(e + f*x)/4
]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqr
t[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 + ((-3 + 2
*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]*Sqr
t[c*(c - d)] - d)))) + Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - T
an[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x
)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62282 vs. $2(199) = 398$.
time = 9.26, size = 62283, normalized size = 271.98

method	result	size
default	Expression too large to display	62283

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 16.61, size = 1726, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*
c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x +
 e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*co
s(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*
cos(f*x + e) + d)) + 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a
```

```
*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2
+ (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), 1/2*(2*(
a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e) - 4*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*
d + a*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3
+ (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a
*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqr
t(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(
c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x + e
)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), ((a*
c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(
f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(a
/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + (a*c*d + a*d^2 + (a*c^2 + a*
c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 +
c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d
 + c^2*d^2)*f), ((a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2
*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*
cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 2*(a*c*d
 + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f
*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e))))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d
 + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(199) = 398.

time = 1.66, size = 688, normalized size = 3.00

$$-\frac{\frac{\sqrt{-a}\,a^{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{c^{2}|a|}-\frac{\sqrt{2}\left(\sqrt{-a}\,a^{2}c^{2}\mathrm{sgn}(\cos(fx+e))-3\sqrt{-a}\,a^{2}cd\,\mathrm{sgn}(\cos(fx+e))-2\sqrt{-a}\,a^{2}d^{2}\mathrm{sgn}(\cos(fx+e))\right)\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}d+ac+3ad\right)}{4\sqrt{-cd-d^{2}}\,a}\right)}{\left(\sqrt{2}c^{3}+\sqrt{2}c^{2}d\right)\sqrt{-cd-d^{2}}\,a}+\frac{4\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{2}c\,\mathrm{sgn}(\cos(fx+e))+3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{2}d\,\mathrm{sgn}(\cos(fx+e))+\sqrt{-a}\,a^{3}c\,\mathrm{sgn}(\cos(fx+e))-\sqrt{-a}\,a^{3}d\,\mathrm{sgn}(\cos(fx+e))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}d+2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}ac+6\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}ad+a^{2}c-a^{2}d\right)\left(\sqrt{2}c^{2}+\sqrt{2}cd\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -(sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f
*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x
 + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a
))*sgn(cos(f*x + e))/(c^2*abs(a)) - sqrt(2)*(sqrt(-a)*a^2*c^2*sgn(cos(f*x +
 e)) - 3*sqrt(-a)*a^2*c*d*sgn(cos(f*x + e)) - 2*sqrt(-a)*a^2*d^2*sgn(cos(f*
x + e)))*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(
1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((sqrt(2)
*c^3 + sqrt(2)*c^2*d)*sqrt(-c*d - d^2)*a) + 4*((sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^2*c*sgn(cos(f*x + e)
) + 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))
^2*sqrt(-a)*a^2*d*sgn(cos(f*x + e)) + sqrt(-a)*a^3*c*sgn(cos(f*x + e)) - sq
rt(-a)*a^3*d*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^4*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^4*d + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*c + 6*(sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*d + a^2*c - a^2*d)*(sqrt(2)*c^2 +
 sqrt(2)*c*d)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2, x)
```

3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=310

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\operatorname{se}}}{\sqrt{a}\sqrt{c}}\right.}{4c^3\sqrt{d}\,(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

```
[Out] 1/2*a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2
)+1/4*a^2*(3*c^2-7*c*d-4*d^2)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(a+
a*sec(f*x+e))^(1/2)+2*a^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f
*x+e)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+1/4*a^(5/2)*(3*c^
3-15*c^2*d-20*c*d^2-8*d^3)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(
c+d)^(1/2))*tan(f*x+e)/c^3/(c+d)^(5/2)/f/d^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 156, 162, 65, 212, 214}

$$\frac{2a^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4c^3\sqrt{d}f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2f(c+d)^2\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}+\frac{a^2(c-d)\tan(e+fx)}{2cf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(3*c^3 - 15*c^
2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]
*Sqrt[c + d])]*Tan[e + f*x])/(4*c^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a - a*Sec[
e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(2*c*(c +
d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (a^2*(3*c^2 - 7*c*d
 - 4*d^2)*Tan[e + f*x])/(4*c^2*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*
Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} - \frac{(a\tan(e+fx))\text{Subst}\Big(}{2c(c+d)f\sqrt{a-}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd}{4c^2(c+d)^2f\sqrt{a+a\,\text{se}}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd}{4c^2(c+d)^2f\sqrt{a+a\,\text{se}}}$$

$$= \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} + \frac{a^2(3c^2-7cd}{4c^2(c+d)^2f\sqrt{a+a\,\text{se}}}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(3c^3-15c^2d-20cd}{4c^3\sqrt{d}\,(c+d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.62, size = 3166, normalized size = 10.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^3*Sec[e + f*x]^2*(a*(1 + Sec[e + f
*x]))^(3/2)*(-1/8*((-5*c^2 + 7*c*d + 6*d^2)*Sin[(e + f*x)/2])/(c^3*(c + d)^
2) + (c*d^2*Sin[(e + f*x)/2] - d^3*Sin[(e + f*x)/2])/(4*c^3*(c + d)*(d + c*
Cos[e + f*x])^2) + (-7*c^2*d*Sin[(e + f*x)/2] + 7*c*d^2*Sin[(e + f*x)/2] +
8*d^3*Sin[(e + f*x)/2])/(8*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*
Sec[e + f*x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e + f
*x])^3*(c*(11*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3
 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2],
ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (3*c^3 - 1
5*c^2*d - 20*c*d^2 - 8*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c +
 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2
```

```
]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqr
t[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17
 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]^3*((7*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]]
)/(16*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*
x]])/(16*c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec
[e + f*x]])/(4*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sq
rt[Sec[e + f*x]])/(2*c*(c + d)^2*(d + c*Cos[e + f*x])) + (d^2*Cos[(3*(e + f
*x))/2]*Sqrt[Sec[e + f*x]])/(4*c^2*(c + d)^2*(d + c*Cos[e + f*x])))*Sec[e +
 f*x]^2*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x
)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^3*(c + d)^3*f*(c
 + d*Sec[e + f*x])^3*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])*(c*(11*c^2 + 9*c
*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12
*Sqrt[2]] - 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]
/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*
d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c -
d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] +
 EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] +
d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[S
ec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]
)/(16*c^3*(c + d)^3*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3
 - 2*Sqrt[2]]*(-3 + 2*Sqrt[2])*(c*(11*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin
[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c + d)^3*Ell
ipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 -
12*Sqrt[2]] - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*(EllipticPi[-(((-3 + 2*
Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f
*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2]
)*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/
Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4
]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(16*c^3*(c + d)^3*Sqrt[1 +
(-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/
4]*(c*(11*c^2 + 9*c*d + 4*d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2
*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcS
in[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (3*c^3 - 15*c^
2*d - 20*c*d^2 - 8*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*S
qrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]],
 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]
*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 1
2*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*
Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*c^3*(c
 + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(c*(11*c^2 + 9*c*d + 4*d
^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
] - 16*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3
- 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*(El
lipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)
), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + Ellipti
```

```
cPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcS
in[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(
3/2)*Sin[e + f*x]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3
 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*c^3*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]
*Cos[(e + f*x)/4]^2*Sqrt[Sec[e + f*x]]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f
*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*((c*(11*c^2 + 9*c*d
+ 4*d^2)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/
4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 -
2*Sqrt[2])]) - (4*(c + d)^3*Sec[(e + f*x)/4]^2)/(Sqrt[3 - 2*Sqrt[2]]*Sqrt[1
 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e +
 f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 234362 vs. $2(272) = 544$.
time = 11.86, size = 234363, normalized size = 756.01

method	result	size
default	Expression too large to display	234363

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(286) = 572$.
time = 31.32, size = 2831, normalized size = 9.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a
*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d
- 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 2
```

7*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - 2*((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 4*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - ((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4

```
*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*
d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*
a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c
^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d
^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)/(a*sin(f*x + e))) + 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a
*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2
*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a
*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - ((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)
*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)
*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^
2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 +
 2*c^4*d^3 + c^3*d^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(c+d\sec(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1592 vs. 2(272) = 544.
time = 2.37, size = 1592, normalized size = 5.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) - sqrt(2)*(3*sqrt(-a)*a^2*c^3*sgn(c
os(f*x + e)) - 15*sqrt(-a)*a^2*c^2*d*sgn(cos(f*x + e)) - 20*sqrt(-a)*a^2*c*
d^2*sgn(cos(f*x + e)) - 8*sqrt(-a)*a^2*d^3*sgn(cos(f*x + e)))*arctan(1/4*sq
rt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)
)^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a
))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((sqrt(2)*c^5 + 2*sqrt(2)*c^4*d
```

```
 + sqrt(2)*c^3*d^2)*sqrt(-c*d - d^2)*a) + 4*(5*(sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2*c^4*sgn(cos(f*x +
e)) + 13*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
a))^6*sqrt(-a)*a^2*c^3*d*sgn(cos(f*x + e)) - 29*(sqrt(-a)*tan(1/2*f*x + 1/2
*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2*c^2*d^2*sgn(cos(f
*x + e)) - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a))^6*sqrt(-a)*a^2*c*d^3*sgn(cos(f*x + e)) + 12*(sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2*d^4*sgn(cos(f*x
 + e)) + 15*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2
 + a))^4*sqrt(-a)*a^3*c^4*sgn(cos(f*x + e)) + 35*(sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^3*c^3*d*sgn(cos(f*
x + e)) + 33*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a))^4*sqrt(-a)*a^3*c^2*d^2*sgn(cos(f*x + e)) - 135*(sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^3*c*d^3*sgn(
cos(f*x + e)) - 76*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^4*sqrt(-a)*a^3*d^4*sgn(cos(f*x + e)) + 15*(sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^4*c^4*sgn(c
os(f*x + e)) + 7*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^2*sqrt(-a)*a^4*c^3*d*sgn(cos(f*x + e)) - 87*(sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^4*c^2*d^2*s
gn(cos(f*x + e)) + 29*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
+ 1/2*e)^2 + a))^2*sqrt(-a)*a^4*c*d^3*sgn(cos(f*x + e)) + 36*(sqrt(-a)*tan(
1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^4*d^4*
sgn(cos(f*x + e)) + 5*sqrt(-a)*a^5*c^4*sgn(cos(f*x + e)) - 15*sqrt(-a)*a^5*
c^3*d*sgn(cos(f*x + e)) + 11*sqrt(-a)*a^5*c^2*d^2*sgn(cos(f*x + e)) + 3*sqr
t(-a)*a^5*c*d^3*sgn(cos(f*x + e)) - 4*sqrt(-a)*a^5*d^4*sgn(cos(f*x + e)))/(
(sqrt(2)*c^5 + sqrt(2)*c^4*d - sqrt(2)*c^3*d^2 - sqrt(2)*c^2*d^3)*((sqrt(-a
)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*c - (sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*d + 2*(sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*c + 6
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*
d + a^2*c - a^2*d)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3, x)
```

3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=336

$$\frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{7/2}c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}+\frac{2ad(3c^2$$

```
[Out] 2*a^3*(3*c^3+12*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2
/5*a*d*(3*c^2+15*c*d+13*d^2)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e
))^(1/2)-6/7*d^2*(c+2*d)*(a-a*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(
1/2)+2/9*d^3*(a-a*sec(f*x+e))^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2/3*(
c^3+12*c^2*d+24*c*d^2+12*d^3)*(a^3-a^3*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*
x+e))^(1/2)+2*a^(7/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e
)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 186, 65, 212}

$$\frac{2a^{7/2}c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{a-a\sec(e+fx)}}-\frac{2(c^3+12c^2d+24cd^2+12d^3)\tan(e+fx)\left(a^3-a^3\sec(e+fx)\right)}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}+\frac{2ad(3c^2+15cd+13d^2)\tan(e+fx)(a-a\sec(e+fx))^2}{5f\sqrt{a\sec(e+fx)+a}}-\frac{6d^2(c+2d)\tan(e+fx)(a-a\sec(e+fx))^3}{7f\sqrt{a\sec(e+fx)+a}}+\frac{2d^3\tan(e+fx)(a-a\sec(e+fx))^4}{9af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^3*(3*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^(7/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan
[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(
3*c^2 + 15*c*d + 13*d^2)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a +
 a*Sec[e + f*x]]) - (6*d^2*(c + 2*d)*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(
7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(a - a*Sec[e + f*x])^4*Tan[e + f*x])
/(9*a*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^3 + 12*c^2*d + 24*c*d^2 + 12*d^3)
*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
```

```
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2(c+dx)^3}{x\sqrt{a-ax}}\,dx, x, \sec(e+f\ldots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{a^2\left(3c^3+12c^2d+12cd^2+4d^3\right)}{\sqrt{a-ax}}+\frac{\ldots}{x\sqrt{\ldots}}\right.\right.}{\ldots}$$

$$= \frac{2a^3\left(3c^3+12c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\left(3c^2+\ldots\right.}{\ldots}$$

$$= \frac{2a^3\left(3c^3+12c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad\left(3c^2+\ldots\right.}{\ldots}$$

$$= \frac{2a^3\left(3c^3+12c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^3\tan\ldots}{f\sqrt{a\ldots}}$$

Mathematica [A]

time = 6.38, size = 286, normalized size = 0.85

$$\frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2520\sqrt{2}\,c^3\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos^{\frac{9}{2}}(e+fx)+2(2520c^3+8883c^2d+8370cd^2+2908d^3+(630c^3+5292c^2d+7290cd^2+2792d^3)\cos(e+fx)+4(840c^3+2898c^2d+2610cd^2+803d^3)\cos(2(e+fx))+210c^3\cos(3(e+fx))+1764c^2d\cos(3(e+fx))+2070cd^2\cos(3(e+fx))+584d^3\cos(3(e+fx))+840c^3\cos(4(e+fx))+2709c^2d\cos(4(e+fx))+2070cd^2\cos(4(e+fx))+584d^3\cos(4(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2520f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*(2520*Sqrt[
2]*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(9/2) + 2*(2520*c^3 +
8883*c^2*d + 8370*c*d^2 + 2908*d^3 + (630*c^3 + 5292*c^2*d + 7290*c*d^2 + 2
792*d^3)*Cos[e + f*x] + 4*(840*c^3 + 2898*c^2*d + 2610*c*d^2 + 803*d^3)*Cos
[2*(e + f*x)] + 210*c^3*Cos[3*(e + f*x)] + 1764*c^2*d*Cos[3*(e + f*x)] + 20
70*c*d^2*Cos[3*(e + f*x)] + 584*d^3*Cos[3*(e + f*x)] + 840*c^3*Cos[4*(e + f
*x)] + 2709*c^2*d*Cos[4*(e + f*x)] + 2070*c*d^2*Cos[4*(e + f*x)] + 584*d^3*
Cos[4*(e + f*x)])*Sin[(e + f*x)/2]))/(2520*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(308) = 616$.
time = 1.56, size = 677, normalized size = 2.01

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(315\sin(fx+e)\sqrt{2}\,(\cos^4(fx+e))\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}}c^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5040/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(315*sin(f*x+e)*2^(1/2)*cos(f
*x+e)^4*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x
+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*c^3+1260*sin(f*x+e)*2^(1/
2)*cos(f*x+e)^3*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*c^3+1890*sin(f*x+
e)*2^(1/2)*cos(f*x+e)^2*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*c^3+1260*
sin(f*x+e)*2^(1/2)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*c^3
+315*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*arctanh(1/2*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3*sin(f*x+e)+26880
*cos(f*x+e)^5*c^3+86688*cos(f*x+e)^5*c^2*d+66240*cos(f*x+e)^5*c*d^2+18688*c
os(f*x+e)^5*d^3-23520*cos(f*x+e)^4*c^3-58464*cos(f*x+e)^4*c^2*d-33120*cos(f
*x+e)^4*c*d^2-9344*cos(f*x+e)^4*d^3-3360*cos(f*x+e)^3*c^3-22176*cos(f*x+e)^
3*c^2*d-15840*cos(f*x+e)^3*c*d^2-2336*cos(f*x+e)^3*d^3-6048*cos(f*x+e)^2*c^
2*d-12960*cos(f*x+e)^2*c*d^2-2848*cos(f*x+e)^2*d^3-4320*cos(f*x+e)*c*d^2-30
40*cos(f*x+e)*d^3-1120*d^3)/cos(f*x+e)^4/sin(f*x+e)*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 3.17, size = 653, normalized size = 1.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(35*a
^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*cos(
f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)*c
os(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e)^
2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(3
15*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (35
*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*co
s(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)
*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e
)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3, x)
```

Giac [**A**]
time = 1.75, size = 613, normalized size = 1.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/315*(315*sqrt(-a)*a^3*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqr
t(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(945*sqrt(2)*a^7*c^3*sgn(cos(
f*x + e)) + 3780*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) + 3780*sqrt(2)*a^7*c*d
^2*sgn(cos(f*x + e)) + 1260*sqrt(2)*a^7*d^3*sgn(cos(f*x + e)) - (3570*sqrt(
2)*a^7*c^3*sgn(cos(f*x + e)) + 12600*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) +
10080*sqrt(2)*a^7*c*d^2*sgn(cos(f*x + e)) + 2520*sqrt(2)*a^7*d^3*sgn(cos(f*
x + e)) - (5040*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)) + 15876*sqrt(2)*a^7*c^2*d
*sgn(cos(f*x + e)) + 11340*sqrt(2)*a^7*c*d^2*sgn(cos(f*x + e)) + 3276*sqrt(
2)*a^7*d^3*sgn(cos(f*x + e)) - (3150*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)) + 90
72*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) + 6480*sqrt(2)*a^7*c*d^2*sgn(cos(f*x
 + e)) + 1872*sqrt(2)*a^7*d^3*sgn(cos(f*x + e)) - (735*sqrt(2)*a^7*c^3*sgn(
cos(f*x + e)) + 2016*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) + 1440*sqrt(2)*a^7
*c*d^2*sgn(cos(f*x + e)) + 416*sqrt(2)*a^7*d^3*sgn(cos(f*x + e)))*tan(1/2*f
*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x
+ 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^4*sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos \left(e+f\,x\right)}\right)^{5/2}\,\left(c+\frac{d}{\cos \left(e+f\,x\right)}\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3, x)
```

3.161 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=258

$$\frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{7/2}c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-}{5f\sqrt{a}}$$

```
[Out] 2*a^3*(c+2*d)*(3*c+2*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*a*d*(2*c+5*
d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/7*d^2*(a-a*sec(
f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*(c^2+8*c*d+8*d^2)*(a^3-a^
3*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*c^2*arctanh((a-
a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 186, 65, 212}

$$\frac{2a^{7/2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2(c^2+8cd+8d^2)\tan(e+fx)(a^3-a^3\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}+\frac{2ad(2c+5d)\tan(e+fx)(a-a\sec(e+fx))^2}{5f\sqrt{a\sec(e+fx)+a}}-\frac{2d^2\tan(e+fx)(a-a\sec(e+fx))^3}{7f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*(c + 2*d)*(3*c + 2*d)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (
2*a^(7/2)*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a*d*(2*c + 5*d)*(a -
a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(a
- a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) - (2*(c^2
+ 8*c*d + 8*d^2)*(a^3 - a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec
[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^2(c+dx)^2}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \left(\frac{a^2(c+2d)(3c+2d)}{\sqrt{a-ax}}+\frac{a^2c^2}{x\sqrt{a-ax}}\right.\right.}{f\sqrt{a}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\,\mathrm{se}}{5f\sqrt{a+a}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\,\mathrm{se}}{5f\sqrt{a+a}}$$

$$= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{7/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{f\sqrt{a-a\sec(e+}}$$

Mathematica [A]
time = 2.78, size = 191, normalized size = 0.74

$$\frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}\,c^2\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos^{\frac{7}{2}}(e+fx)+4(35c^2+196cd+145d^2+(420c^2+987cd+465d^2)\cos(e+fx)+(35c^2+196cd+115d^2)\cos(2(e+fx))+140c^2\cos(3(e+fx))+301cd\cos(3(e+fx))+115d^2\cos(3(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{420f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2
]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 4*(35*c^2 + 196
*c*d + 145*d^2 + (420*c^2 + 987*c*d + 465*d^2)*Cos[e + f*x] + (35*c^2 + 196
*c*d + 115*d^2)*Cos[2*(e + f*x)] + 140*c^2*Cos[3*(e + f*x)] + 301*c*d*Cos[3
*(e + f*x)] + 115*d^2*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(234) = 468$.
time = 1.48, size = 504, normalized size = 1.95

method	result
default	$\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(105\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sin(fx+e)\left(\cos^3(fx+e)\right)c^2+3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/840/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(105*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(7/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)/cos(f*x+e)*2^(1/2))*sin(f*x+e)*cos(f*x+e)^3*c^2+315*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(7/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)/cos(f*x+e)*2^(1/2))*sin(f*x+e)*cos(f*x+e)^2*c^2+315*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(7/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*sin(f*x+e)*cos(f*x+e)*c^2+105*2^(1/2)
*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(
1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)*c^2*sin(f*x+e)-4480*cos(f*x+e)^4
*c^2-9632*cos(f*x+e)^4*c*d-3680*cos(f*x+e)^4*d^2+3920*cos(f*x+e)^3*c^2+6496
*cos(f*x+e)^3*c*d+1840*cos(f*x+e)^3*d^2+560*cos(f*x+e)^2*c^2+2464*cos(f*x+e
)^2*c*d+880*cos(f*x+e)^2*d^2+672*cos(f*x+e)*c*d+720*cos(f*x+e)*d^2+240*d^2)
/cos(f*x+e)^3/sin(f*x+e)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a
^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
```

```
cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*a^2
*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x + 2*e)^2 + 2
*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + s
in(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + si
n(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(
9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*c
os(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*co
s(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*
f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin
(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*
x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^
2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*co
s(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*co
s(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*
```

```
x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e
) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f
*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)), x) - 8*((3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f*cos
(2*f*x + 2*e)^2 + (3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f*sin(2*f*x + 2*e)^2 +
2*(3*a^2*c^2 + 5*a^2*c*d + a^2*d^2)*f*cos(2*f*x + 2*e) + (3*a^2*c^2 + 5*a^2
*c*d + a^2*d^2)*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4
*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*
f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*
f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f
*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e)...
```

Fricas [A]
time = 3.92, size = 531, normalized size = 2.06

$$\left[\frac{105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\left(15a^2d^2+2\left(140a^2c^2+301a^2cd+115a^2d^2\right)\cos(fx+e)^3+\left(35a^2c^2+196a^2cd+115a^2d^2\right)\cos(fx+e)^2+6\left(7a^2cd+10a^2d^2\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{105\left(f\cos(fx+e)^4+f\cos(fx+e)^3\right)}, -\frac{2\left(105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15a^2d^2+2\left(140a^2c^2+301a^2cd+115a^2d^2\right)\cos(fx+e)^3+\left(35a^2c^2+196a^2cd+115a^2d^2\right)\cos(fx+e)^2+6\left(7a^2cd+10a^2d^2\right)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{105\left(f\cos(fx+e)^4+f\cos(fx+e)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a
^2*d^2 + 2*(140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a
^2*c^2 + 196*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*
d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f
*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 +
a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a^2*d^2 + 2*(140*a^2*c^2 + 3
01*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a^2*c^2 + 196*a^2*c*d + 115*
a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*d^2)*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x
 + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\sec(e+fx)+1)\right)^{\frac{5}{2}}\left(c+d\sec(e+fx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2, x)
```

Giac [A]
time = 3.45, size = 441, normalized size = 1.71

$$-\frac{\frac{105\,\sqrt{-a}\,a^{3}c^{2}\log\left(\frac{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\,\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{|a|}+\frac{2\,\left(315\,\sqrt{2}\,a^{6}c^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+840\,\sqrt{2}\,a^{6}cd\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+420\,\sqrt{2}\,a^{6}d^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(875\,\sqrt{2}\,a^{6}c^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+1960\,\sqrt{2}\,a^{6}cd\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+700\,\sqrt{2}\,a^{6}d^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(805\,\sqrt{2}\,a^{6}c^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+1568\,\sqrt{2}\,a^{6}cd\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+560\,\sqrt{2}\,a^{6}d^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\left(245\,\sqrt{2}\,a^{6}c^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+448\,\sqrt{2}\,a^{6}cd\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+160\,\sqrt{2}\,a^{6}d^{2}\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(-a)*a^3*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqr
t(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(315*sqrt(2)*a^6*c^2*sgn(cos(
f*x + e)) + 840*sqrt(2)*a^6*c*d*sgn(cos(f*x + e)) + 420*sqrt(2)*a^6*d^2*sgn
(cos(f*x + e)) - (875*sqrt(2)*a^6*c^2*sgn(cos(f*x + e)) + 1960*sqrt(2)*a^6*
c*d*sgn(cos(f*x + e)) + 700*sqrt(2)*a^6*d^2*sgn(cos(f*x + e)) - (805*sqrt(2
)*a^6*c^2*sgn(cos(f*x + e)) + 1568*sqrt(2)*a^6*c*d*sgn(cos(f*x + e)) + 560*
sqrt(2)*a^6*d^2*sgn(cos(f*x + e)) - (245*sqrt(2)*a^6*c^2*sgn(cos(f*x + e))
+ 448*sqrt(2)*a^6*c*d*sgn(cos(f*x + e)) + 160*sqrt(2)*a^6*d^2*sgn(cos(f*x +
 e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^
2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f
*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2, x)
```

3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=142

$$\frac{2a^{5/2}c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\,\tan}{15f}$$

```
[Out] 2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/5*a*d*(a+
a*sec(f*x+e))^(3/2)*tan(f*x+e)/f+2/15*a^3*(35*c+32*d)*tan(f*x+e)/f/(a+a*sec
(f*x+e))^(1/2)+2/15*a^2*(5*c+8*d)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4002, 4000, 3859, 209, 3877}

$$\frac{2a^{5/2}c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{15f}+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^(5/2)*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (
2*a^3*(35*c + 32*d)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*
(5*c + 8*d)*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(15*f) + (2*a*d*(a + a*S
ec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{2}{5}\int (a+a\sec(e+$$

$$= \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{15f} + \frac{2ad(a+a$$

$$= \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{15f} + \frac{2ad(a+a$$

$$= \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+$$

$$= \frac{2a^{5/2}c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^3(35c+32d)\tan$$

Mathematica [A]
time = 1.00, size = 128, normalized size = 0.90

$$\frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}\,c\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\cos^{\frac{5}{2}}(e+fx)+2(40c+49d+2(5c+14d)\cos(e+fx)+(40c+43d)\cos(2(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]
*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(40*c + 49*d + 2
```

```
*(5*c + 14*d)*Cos[e + f*x] + (40*c + 43*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/
2]))/(30*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(124) = 248$.
time = 1.32, size = 341, normalized size = 2.40

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(15\sin(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)(\cos^2(fx+e))\sqrt{2}\,c+\text{…}\right.}{\text{…}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(15*sin(f*x+e)*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(5/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*2^(1/2)*c+30*sin(f*x+e)*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(5/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*s
in(f*x+e)/cos(f*x+e)*2^(1/2))*cos(f*x+e)*2^(1/2)*c+15*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(5/2)*c*sin(f*x+e)+320*cos(f*x+e)^3*c+344*cos(f*x+e)
^3*d-280*cos(f*x+e)^2*c-232*cos(f*x+e)^2*d-40*c*cos(f*x+e)-88*d*cos(f*x+e)-
24*d)/cos(f*x+e)^2/sin(f*x+e)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(132) = 264$.
time = 0.78, size = 1501, normalized size = 10.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3*a^2*cos
(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(a) + 3*((a
```

```
^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x + 2*e) + a
^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(
2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
 + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e
)^2 + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + 1) + (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*
f*x + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*sqrt(a)
)*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.38, size = 419, normalized size = 2.95

$$\left[\frac{15\,\left(a^{2}c\cos\left(fx+e\right)^{3}+a^{2}c\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(fx+e)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\,\left(3\,a^{2}d+\left(40\,a^{2}c+43\,a^{2}d\right)\cos\left(fx+e\right)^{2}+\left(5\,a^{2}c+14\,a^{2}d\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{15\,\left(f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)^{2}\right)},\,-\frac{2\,\left(15\,\left(a^{2}c\cos\left(fx+e\right)^{3}+a^{2}c\cos\left(fx+e\right)^{2}\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\cos(fx+e)}{\sqrt{a}\,\sin(fx+e)}\right)-\left(3\,a^{2}d+\left(40\,a^{2}c+43\,a^{2}d\right)\cos\left(fx+e\right)^{2}+\left(5\,a^{2}c+14\,a^{2}d\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\right)}{15\,\left(f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*c
os(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a^2*d +
(40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c + 14*a^2*d)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sq
```

```
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))) - (3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c
 + 14*a^2*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(124) = 248.
time = 1.62, size = 309, normalized size = 2.18

$$-\frac{\frac{15\sqrt{-a}\,a^3c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(45\sqrt{2}a^5c\operatorname{sgn}(\cos(fx+e))+60\sqrt{2}a^5d\operatorname{sgn}(\cos(fx+e))-\left(80\sqrt{2}a^5c\operatorname{sgn}(\cos(fx+e))+80\sqrt{2}a^5d\operatorname{sgn}(\cos(fx+e))-\left(35\sqrt{2}a^5c\operatorname{sgn}(\cos(fx+e))+32\sqrt{2}a^5d\operatorname{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/15*(15*sqrt(-a)*a^3*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(45*sqrt(2)*a^5*c*sgn(cos(f*x + e
)) + 60*sqrt(2)*a^5*d*sgn(cos(f*x + e)) - (80*sqrt(2)*a^5*c*sgn(cos(f*x + e
)) + 80*sqrt(2)*a^5*d*sgn(cos(f*x + e)) - (35*sqrt(2)*a^5*c*sgn(cos(f*x + e
)) + 32*sqrt(2)*a^5*d*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*
x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=203

$$\frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}}+\frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{2a^{7/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}\sqrt{c+d}\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^3*tan(f*x+e)/d/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x+
e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-2*a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)
^(1/2))*tan(f*x+e)/c/d^(3/2)/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 186, 65, 212, 214}

$$-\frac{2a^{7/2}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^3\tan(e+fx)}{df\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*a^3*Tan[e + f*x])/(d*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(7/2)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c*d^(3/2)*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}\,(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{a^2}{d\sqrt{a-ax}}+\frac{a^2}{cx\sqrt{a-ax}}-\frac{a^2(c-d)^2}{cd\sqrt{a-ax}\,(c+dx)}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e\right.}{cf\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{\left(2a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec}\right.}{cf\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+f}}$$

$$= \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx}{cf\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.61, size = 343, normalized size = 1.69

$$\frac{\cos^{\frac{3}{2}}(e+fx)(d+c\cos(e+fx))\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}\left(\frac{10(c-d)^2(c+3d+2c\cos(e+fx))\csc\left(\frac{1}{2}(e+fx)\right)\left(-\tanh^{-1}\left(\sqrt{-\frac{d(-1+\sec(e+fx))}{c+d}}\right)+\sqrt{-\frac{d(-1+\sec(e+fx))}{c+d}}\right)}{d(c+d)\sqrt{\cos(e+fx)}\sqrt{-\frac{d(-1+\sec(e+fx))}{c+d}}}+\frac{20(3c-d)\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}-\frac{16(c-d)^2d(d+c\cos(e+fx))\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{2d\sec(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)}{c+d}\right)\sin^3\left(\frac{1}{2}(e+fx)\right)}{(c+d)^3\cos^{\frac{5}{2}}(e+fx)}+10c\left(\sqrt{2}\,\mathrm{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)-\frac{2\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)\right)}{40c^2f(c+d\sec(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^(3/2)*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e +
 f*x]))^(5/2)*((10*(c - d)^2*(c + 3*d + 2*c*Cos[e + f*x])*Csc[(e + f*x)/2]*
(-ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]] + Sqrt[-((d*(-1 + Sec[e
 + f*x]))/(c + d))]))/(d*(c + d)*Sqrt[Cos[e + f*x]]*Sqrt[-((d*(-1 + Sec[e +
 f*x]))/(c + d))]) + (20*(3*c - d)*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] - (
16*(c - d)^2*d*(d + c*Cos[e + f*x])*Hypergeometric2F1[2, 5/2, 7/2, (-2*d*Se
c[e + f*x]*Sin[(e + f*x)/2]^2)/(c + d)]*Sin[(e + f*x)/2]^3)/((c + d)^3*Cos[
e + f*x]^(5/2)) + 10*c*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(
e + f*x)/2])/Sqrt[Cos[e + f*x]])))/(40*c^2*f*(c + d*Sec[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(173) = 346$.
time = 4.52, size = 1490, normalized size = 7.34

method	result	size
default	Expression too large to display	1490

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*((c+d)*(c-d))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-ln(2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d
))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*
cos(f*x+e)+d*cos(f*x+e)+c-d))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
c^2*sin(f*x+e)+2*ln(2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+
e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(
f*x+e)+c-d))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*d*sin(f*x+e)-ln
(2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+
e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e
)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1
/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))*2^(1/2
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d^2*sin(f*x+e)+ln(2*(-2^(1/2)*(d/(c-
```

```
d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d
))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1
/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d)
)^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*2^(1/2)*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*c^2*sin(f*x+e)-2*ln(2*(-2^(1/2)*(d/(c-d))^(1/2)*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))^(1/2)*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*c*d*sin(f*x+e)+ln(2*(-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(
f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*
cos(f*x+e)-c+d))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d^2*sin(f*x+e
)+4*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*cos(f*x+e)-4*((c+d)*(c-d))^(1/2)*
(d/(c-d))^(1/2)*c)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)*a^2/c/((c
+d)*(c-d))^(1/2)/d/(d/(c-d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c), x)
```

Fricas [A]
time = 7.95, size = 1210, normalized size = 5.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [(2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2 -
 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-
a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^
2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e
) + d)) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2
*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
 + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c*d*f), (
2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(a^2*d*cos
(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
```

```
os(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2
*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d
+ d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f
*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*d*f*cos(f*x + e
) + c*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) +
 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x
 + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + (a^2*d*cos(f*x
+ e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(c
os(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c*d*f), 2*(a^2*c*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2
*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d
)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/
(a*sin(f*x + e))) - (a^2*d*cos(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*d*f*c
os(f*x + e) + c*d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}{c+d\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(173) = 346.
time = 2.03, size = 412, normalized size = 2.03

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,a^3\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)d}+\frac{\sqrt{-a}\,a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)\operatorname{sgn}(\cos(fx+e))}{c}-\frac{\sqrt{-a}\,a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)\operatorname{sgn}(\cos(fx+e))}{c}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-a}\,a^3c^2\operatorname{sgn}(\cos(fx+e))-2\sqrt{2}\sqrt{-a}\,a^3cd\operatorname{sgn}(\cos(fx+e))+\sqrt{2}\sqrt{-a}\,a^3d^2\operatorname{sgn}(\cos(fx+e))\right)\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac+3ad\right)}{4\sqrt{-cd-d^2}\,a}\right)}{\sqrt{-cd-d^2}\,acd}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a^3*sgn(cos(f*x + e))*tan(1
/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*d) + sqrt(-a)*a^2*log(abs((
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a*
(2*sqrt(2) + 3)))*sgn(cos(f*x + e))/c - sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(
1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + a*(2*sqrt(2) -
3)))*sgn(cos(f*x + e))/c + sqrt(2)*(sqrt(2)*sqrt(-a)*a^3*c^2*sgn(cos(f*x +
```

```
e)) - 2*sqrt(2)*sqrt(-a)*a^3*c*d*sgn(cos(f*x + e)) + sqrt(2)*sqrt(-a)*a^3*d
^2*sgn(cos(f*x + e)))*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) -
sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) -
 sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*
a))/(sqrt(-c*d - d^2)*a*c*d))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)), x)
```

3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=329

$$\frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{a^{7/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+f}$$

```
[Out] -a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)
+2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-a
*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d)^(3/2)/f/
(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*(c-d)*arctanh(d^(1/
2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*(c+d)^(1/2)*tan(f*x+e)/c^2/d
^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\frac{2a^{7/2}(c-d)\sqrt{c+d}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^{7/2}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^3(c-d)^2\tan(e+fx)}{cdf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^(7/2)*(c - d)^2*ArcT
anh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])
/(c*d^(3/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]) + (2*a^(7/2)*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*
x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Tan[e + f*x])/(c*d*(c + d
)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \left(\frac{a^2}{c^2x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}\,(c+dx)^2} + \frac{a^2(c^2-c}{c^2d\sqrt{a-ax}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^4\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(a^4(c-d)^2\tan}{cd}$$

$$= -\frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} + \frac{(2a^3\tan(e+fx))\,\mathrm{Sub}}{c^2f\sqrt{a-a\,\mathrm{se}}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\,\tan}{c^2d^{3/2}f\sqrt{a-}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{a^{7/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.38, size = 3026, normalized size = 9.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2)*(-1
/4*((-c + d)^2*Sin[(e + f*x)/2])/(c^2*d*(c + d)) + (c^2*Sin[(e + f*x)/2] -
2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(4*c^2*(c + d)*(d + c*Cos[e
+ f*x]))))/(f*(c + d*Sec[e + f*x])^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/
4]^2*(d + c*Cos[e + f*x])^2*(c*(c^2 + 6*c*d + d^2)*EllipticF[ArcSin[Tan[(e
+ f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*d*(c + d)^2*EllipticPi
[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt
[2]] - (c^3 + 4*c^2*d - 3*c*d^2 - 2*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c
 + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt
[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/
(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*
```

```
Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]^5*((5*Cos[(e + f*x)/2]*Sqrt[
Sec[e + f*x]])/(8*(c + d)*(d + c*Cos[e + f*x])) + (c*Cos[(e + f*x)/2]*Sqrt[
Sec[e + f*x]])/(8*d*(c + d)*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*S
qrt[Sec[e + f*x]])/(8*(c + d)*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/
2]*Sqrt[Sec[e + f*x]])/(8*c*(c + d)*(d + c*Cos[e + f*x])))*(a*(1 + Sec[e +
f*x]))^(5/2)*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*
Sqrt[2])*Tan[(e + f*x)/4]^2])/(2*c^2*d*(c + d)^2*f*(c + d*Sec[e + f*x])^2*(
(Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])*(c*(c^2 + 6*c*d + d^2)*EllipticF[ArcSi
n[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*d*(c + d)^2*E
llipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17
- 12*Sqrt[2]] - (c^3 + 4*c^2*d - 3*c*d^2 - 2*d^3)*(EllipticPi[-(((-3 + 2*Sq
rt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x
)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*
(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sq
rt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*
Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*c^2*d*(c + d)^2*Sqrt[1 -
(3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-3 + 2*Sqrt[2]
)*(c*(c^2 + 6*c*d + d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[
2]]], 17 - 12*Sqrt[2]] - 4*d*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Ta
n[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^3 + 4*c^2*d - 3*
c*d^2 - 2*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sq
rt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(
c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]
]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f
*x)/4]^2])/(8*c^2*d*(c + d)^2*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]
) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]*(c*(c^2 + 6*c*d + d^2)*EllipticF[
ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*d*(c + d
)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]] - (c^3 + 4*c^2*d - 3*c*d^2 - 2*d^3)*(EllipticPi[-(((-3 +
 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e
+ f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt
[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/
4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x
)/4]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])
*Tan[(e + f*x)/4]^2])/(4*c^2*d*(c + d)^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f
*x)/4]^2*(c*(c^2 + 6*c*d + d^2)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*d*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], Ar
cSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^3 + 4*c^2
*d - 3*c*d^2 - 2*d^3)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqr
t[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 1
7 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*S
qrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e
 + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(4*c^2*d*(c + d
```

```
)^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqrt[Sec[e + f*x]]*Sqrt[1 +
(-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)
/4]^2]*((c*(c^2 + 6*c*d + d^2)*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*S
qrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan
[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - (d*(c + d)^2*Sec[(e + f*x)/4]^2)/(Sqrt
[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])]*Sqrt[1 - ((17
- 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]*(1 - ((-3 + 2*Sqrt[2])*T
an[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) - (c^3 + 4*c^2*d - 3*c*d^2 - 2*d^3)*(S
ec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2
*Sqrt[2])]*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]
*(1 + ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46081 vs. $2(281) = 562$.
time = 7.29, size = 46082, normalized size = 140.07

method	result	size
default	Expression too large to display	46082

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 47.67, size = 2117, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*
d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos
(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^
```

```
4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
 + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^
2 + (c + d)*cos(f*x + e) + d)) - 2*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*
c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*s
qrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1
)))/((c^4*d + c^3*d^2)*f*cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*c
os(f*x + e) + (c^3*d^2 + c^2*d^3)*f), -1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*
c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
4*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*
d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^3*d + 4*
a^2*c^2*d^2 - 3*a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*
d^2 - 2*a^2*c*d^3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 -
5*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d
^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x
+ e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4*d + c^3*d^2)*f*
cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 +
c^2*d^3)*f), -((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*
a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^
3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*
a^2*d^4)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) -
 (a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d
 + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 -
2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4*d + c^3*d^2)*f*cos(f*x +
 e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2*d^3)*
f), -((a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*d^3
 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*
x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^4)*
cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + 2*(a^2*c
*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^
2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/((c^4*d + c^3*d^2)*f*cos
(f*x + e)^2 + (c^4*d + 2*c^3*d^2 + c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2
*d^3)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sec\left(e+fx\right)+1)\right)^{\frac{5}{2}}}{\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(281) = 562.
time = 2.02, size = 661, normalized size = 2.01

$$\frac{\sqrt{2}\sqrt{-a}\,a^{5}\left(\frac{\sqrt{2}\left(c^{3}+4c^{2}d-3cd^{2}-2d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}d+ac+3ad\right)}{4\sqrt{-cd-d^{2}}\,a}\right)}{\left(a^{2}c^{3}d+a^{2}c^{2}d^{2}\right)\sqrt{-cd-d^{2}}\,a}+\frac{4\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}c^{2}+2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}cd-3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}d^{2}+ac^{2}-2acd+ad^{2}\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{4}d+2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}ac+6\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}ad+a^{2}c-a^{2}d\right)\left(a^{2}c^{2}d+a^{2}cd^{2}\right)}-\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}-4\sqrt{2}\left|a\right|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+4\sqrt{2}\left|a\right|-6a\right|}\right)}{a^{2}c^{2}\left|a\right|}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a)*a^5*(sqrt(2)*(c^3 + 4*c^2*d - 3*c*d^2 - 2*d^3)*arctan(
1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)
^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((a^2*c^3*d + a^2*c^2*d^2
)*sqrt(-c*d - d^2)*a) + 4*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2
*f*x + 1/2*e)^2 + a))^2*c^2 + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*c*d - 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d^2 + a*c^2 - 2*a*c*d + a*d^2)/(((sqrt(-a
)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*c - (sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*d + 2*(sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*c + 6
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*
d + a^2*c - a^2*d)*(a^2*c^2*d + a^2*c*d^2)) - sqrt(2)*log(abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*ab
s(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/
2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(a^2*c^2*abs(a)))*sgn(cos(f*x + e
))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}}{{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2, x)
```

3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=536

$$\frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{3a^{7/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}$$

```
[Out] -1/2*a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))
^(1/2)+a^3*(c-d)*tan(f*x+e)/c^2/d/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)
-3/4*a^3*(c-d)^2*tan(f*x+e)/c/d/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))
^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(
a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*a^(7/2)*(c-d)^2*arctanh(d^
(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d
)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(7/2)*(c-d)*arcta
nh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/d^(3/
2)/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*a^(7/2)*ar
ctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*d^(1/2)*tan(f*x+e
)/c^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$-\frac{2a^{7/2}\sqrt{d}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^3f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^{7/2}(c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3a^{7/2}(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{4cd^{3/2}f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}-\frac{3a^3(c-d)^2\tan(e+fx)}{4cdf(c+d)^2\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}-\frac{a^3(c-d)^2\tan(e+fx)}{2cdf(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^(7/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*a^(7/2)*(c - d)^2*Ar
cTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x
])/(4*c*d^(3/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]]) + (a^(7/2)*(c - d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt
[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*d^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^(7/2)*Sqrt[d]*ArcTanh[(Sqrt[d]*Sq
rt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^3*Sqrt[c +
d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Ta
n[e + f*x])/(2*c*d*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^
2) + (a^3*(c - d)*Tan[e + f*x])/(c^2*d*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Se
c[e + f*x])) - (3*a^3*(c - d)^2*Tan[e + f*x])/(4*c*d*(c + d)^2*f*Sqrt[a + a
*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{a^2}{c^3x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}\,(c+dx)^3} + \frac{a^2(c^2-}{c^2d\sqrt{a-ax}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(}$$

$$= -\frac{(a^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(a^4(c-d)^2\tan}{cd}$$

$$= -\frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} + \frac{a^3(c-d}{c^2df\sqrt{a+a\sec(e+}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}}{}\right.}{c^3\sqrt{c+d}\,f\sqrt{a-a\,\text{se}}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a^{7/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{c^2d^{3/2}\sqrt{c+d}\,f\sqrt{a-}}$$

$$= \frac{2a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{3a^{7/2}(c-d)^2\tanh^{-1}\left(\right.}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 25.43, size = 3344, normalized size = 6.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^5*Sec[e + f*x]*(a*(1 + Sec[e + f*x
]))^(5/2)*(-1/16*((c^3 - 12*c^2*d + 5*c*d^2 + 6*d^3)*Sin[(e + f*x)/2])/(c^3
*d*(c + d)^2) + (-(c^2*d*Sin[(e + f*x)/2]) + 2*c*d^2*Sin[(e + f*x)/2] - d^3
*Sin[(e + f*x)/2])/(8*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (3*c^3*Sin[(e +
 f*x)/2] - 14*c^2*d*Sin[(e + f*x)/2] + 3*c*d^2*Sin[(e + f*x)/2] + 8*d^3*Sin
[(e + f*x)/2])/(16*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e +
f*x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e + f*x])^3*(
```

c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt
[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*d*(c + d)^3*EllipticPi[-3 + 2*Sqrt[
2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^4 +
 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt[2])*
(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sq
rt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d)
)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 -
2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]^5*((7*Cos[(e + f*x)/2]*Sqr
t[Sec[e + f*x]])/(16*(c + d)^2*(d + c*Cos[e + f*x])) + (c*Cos[(e + f*x)/2]*
Sqrt[Sec[e + f*x]])/(32*d*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*x
)/2]*Sqrt[Sec[e + f*x]])/(32*c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(e
 + f*x))/2]*Sqrt[Sec[e + f*x]])/(8*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos
[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c*(c + d)^2*(d + c*Cos[e + f*x]))
+ (d^2*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(8*c^2*(c + d)^2*(d + c*Cos
[e + f*x])))*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[1 + (-3 + 2*Sqr
t[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(8*
c^3*d*(c + d)^3*f*(c + d*Sec[e + f*x])^3*((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[
2])*(c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]
/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*d*(c + d)^3*EllipticPi[-3 + 2*
Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (
c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt
[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/
4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c
 + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt
[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sq
rt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(32*c^3*d*(c + d)^3*Sqrt[1 - (
3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-3 + 2*Sqrt[2])
(c(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sq
rt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*d*(c + d)^3*EllipticPi[-3 + 2*Sqr
t[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^4
 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt[2]
)*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/
Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c +
d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3
- 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]*Sqrt[
1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2])/(32*c^3*d*(c + d)^3*Sqrt[1 + (-3 +
 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]*(c
*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[
3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*d*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2
], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^4 +
10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(
c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqr
t[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))
/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2
*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[1 +

```
(-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x
)/4]^2])/(16*c^3*d*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(c*
(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3
 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*d*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2]
, ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^4 + 1
0*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c
 + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt
[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/
(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*
Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[1 + (-3
+ 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^
2])/(16*c^3*d*(c + d)^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*Sqrt[Sec
[e + f*x]]*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]*Sqrt[1 - (3 + 2*Sq
rt[2])*Tan[(e + f*x)/4]^2]*((c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*Sec[(e +
f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 209488 vs. $2(460) = 920$.
time = 13.28, size = 209489, normalized size = 390.84

method	result	size
default	Expression too large to display	209489

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 83.95, size = 3453, normalized size = 6.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^
2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c
^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c
^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*
a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*c
os(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2
*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 +
(c + d)*cos(f*x + e) + d)) - 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*
c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^
3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^
2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*((a^2*c^5 - 12*a^2
*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2*c^4*d + 10*a^
2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x
+ e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*
c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (c^5*d^3 + 2*c^
4*d^4 + c^3*d^5)*f), -1/8*(16*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c
^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3
*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2
*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^4
*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^
6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x
 + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2
*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 2
0*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sq
rt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x +
 e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
 + e) + d)) + 2*((a^2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*c
os(f*x + e)^2 - (a^2*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d
 + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 +
 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5
)*f*cos(f*x + e) + (c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 +
 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10
*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^
3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d
^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*
c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(
c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 4*(a^2*c^2*d^3 + 2*a^2*c*d^4 +
```

```
a^2*d^5 + (a^2*c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*
c^4*d + 4*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^
2*c^3*d^2 + 5*a^2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*l
og((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + ((a^
2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2
*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d
^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*
x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (
c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15
*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*
c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^
2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*c
os(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2
*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((
c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)/(a*sin(f*x + e))) + 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c^4*
d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3*d^
2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2*c^
2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^{\frac{5}{2}}}{(c+d\sec(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1606 vs. 2(460) = 920.
time = 2.36, size = 1606, normalized size = 3.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
```

```
/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) - sqrt(2)*(sqrt(-a)*a^3*c^4*sgn(cos
(f*x + e)) + 10*sqrt(-a)*a^3*c^3*d*sgn(cos(f*x + e)) - 15*sqrt(-a)*a^3*c^2*
d^2*sgn(cos(f*x + e)) - 20*sqrt(-a)*a^3*c*d^3*sgn(cos(f*x + e)) - 8*sqrt(-a
)*a^3*d^4*sgn(cos(f*x + e)))*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d
- d^2)*a))/((sqrt(2)*c^5*d + 2*sqrt(2)*c^4*d^2 + sqrt(2)*c^3*d^3)*sqrt(-c*d
 - d^2)*a) - 4*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*
e)^2 + a))^6*sqrt(-a)*a^3*c^4*sgn(cos(f*x + e)) + (sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^3*c^3*d*sgn(cos(f
*x + e)) - 25*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)
^2 + a))^6*sqrt(-a)*a^3*c^2*d^2*sgn(cos(f*x + e)) + 11*(sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^3*c*d^3*sgn(
cos(f*x + e)) + 12*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^6*sqrt(-a)*a^3*d^4*sgn(cos(f*x + e)) + 3*(sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^4*c^4*sgn(co
s(f*x + e)) - 17*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^4*sqrt(-a)*a^4*c^3*d*sgn(cos(f*x + e)) - 83*(sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^4*c^2*d^2*s
gn(cos(f*x + e)) - 211*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
 + 1/2*e)^2 + a))^4*sqrt(-a)*a^4*c*d^3*sgn(cos(f*x + e)) - 76*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^4*d^4
*sgn(cos(f*x + e)) + 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
 + 1/2*e)^2 + a))^2*sqrt(-a)*a^5*c^4*sgn(cos(f*x + e)) - 29*(sqrt(-a)*tan(1
/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^5*c^3*d
*sgn(cos(f*x + e)) - 75*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*
x + 1/2*e)^2 + a))^2*sqrt(-a)*a^5*c^2*d^2*sgn(cos(f*x + e)) + 65*(sqrt(-a)*
tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^5*
c*d^3*sgn(cos(f*x + e)) + 36*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^5*d^4*sgn(cos(f*x + e)) + sqrt(-a)*a^6
*c^4*sgn(cos(f*x + e)) - 11*sqrt(-a)*a^6*c^3*d*sgn(cos(f*x + e)) + 15*sqrt(
-a)*a^6*c^2*d^2*sgn(cos(f*x + e)) - sqrt(-a)*a^6*c*d^3*sgn(cos(f*x + e)) -
4*sqrt(-a)*a^6*d^4*sgn(cos(f*x + e)))/((sqrt(2)*c^4*d + 2*sqrt(2)*c^3*d^2 +
 sqrt(2)*c^2*d^3)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^4*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x +
1/2*e)^2 + a))^4*d + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
 + 1/2*e)^2 + a))^2*a*c + 6*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/
2*f*x + 1/2*e)^2 + a))^2*a*d + a^2*c - a^2*d)^2))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c+\frac{d}{\cos(e+fx)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3,x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3, x)
```

3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=258

$$\frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2d^3(1-\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{2\sqrt{a}\,c^3\tanh^{-1}\left(\sqrt{a\right.}}{f\sqrt{a-a\sec(e}$$

```
[Out] 2*(3*c-d)*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*d^3*tan(f*x+e)/f/(a+a*s
ec(f*x+e))^(1/2)-2/3*d^3*(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 186, 65, 212, 45}

$$\frac{2\sqrt{a}\,c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^2(3c-d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\sqrt{a}(c-d)^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2d^3\tan(e+fx)(1-\sec(e+fx))}{3f\sqrt{a\sec(e+fx)+a}}+\frac{2d^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*(3*c - d)*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*Tan[e
+ f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^3*(1 - Sec[e + f*x])*Tan[e + f*
x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^3*ArcTanh[Sqrt[a - a*Sec[
e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(
Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}\,(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{(3c-d)d^2}{a\sqrt{a-ax}} + \frac{c^3}{ax\sqrt{a-ax}} + \frac{d^3x}{a\sqrt{a-ax}} - \frac{\quad}{a(1+}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(ac^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{(2c^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+f}}$$

$$= \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^3(1-\sec(e+fx))\tan}{3f\sqrt{a+a\sec(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.26, size = 787, normalized size = 3.05

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^
(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((2*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2])/
(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) - (4*c^2*(c + 3*d)*Sin[(e + f*x)/2]^3)
/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) + (4*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2]
)/(3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) + (c^3*Csc[(e + f*x)/2]*Sqrt[1 - 2*Sin
[(e + f*x)/2]^2]*((4*Sin[(e + f*x)/2]^4)/(1 - 2*Sin[(e + f*x)/2]^2)^2 - (6*
Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + (3*Sqrt[2]*ArcSin[Sqrt[2]*
Sin[(e + f*x)/2]]*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/3 - ((
c - d)^3*Csc[(e + f*x)/2]^5*(-12*Cos[(e + f*x)/2]^4*HypergeometricPFQ[{2, 2
, 7/2}, {1, 9/2}, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e
+ f*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[(e + f*x)/2]^2/(1 - 2
*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^8*(4 - 7*Sin[(e + f*x)/2]^2 + 3*Sin
[(e + f*x)/2]^4) + 7*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]
*(1 - 2*Sin[(e + f*x)/2]^2)^3*(15 - 20*Sin[(e + f*x)/2]^2 + 8*Sin[(e + f*x)
```

```
/2]^4)*((3 - 7*Sin[(e + f*x)/2]^2)*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e
+ f*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2
]^2))]]*(1 - 2*Sin[(e + f*x)/2]^2))))/(63*(1 - 2*Sin[(e + f*x)/2]^2)^(7/2))
))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(5/2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(227) = 454$.
time = 1.53, size = 907, normalized size = 3.52

method	result	size
default	Expression too large to display	907

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(3*arctanh(1/2*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(3/2)*cos(f*x+e)*2^(1/2)*sin(f*x+e)*c^3+3*arctanh(1/2*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(3/2)*2^(1/2)*c^3*sin(f*x+e)+3*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
3/2)*cos(f*x+e)*sin(f*x+e)*c^3-9*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*co
s(f*x+e)*sin(f*x+e)*c^2*d+9*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*cos(f*x
+e)*sin(f*x+e)*c*d^2-3*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*cos(f*x+e)*s
in(f*x+e)*d^3+3*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x
+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*c^3*sin(f*x+e)-9*ln
((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*c^2*d*sin(f*x+e)+9*ln((sin(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(3/2)*c*d^2*sin(f*x+e)-3*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(3/2)*d^3*sin(f*x+e)-36*cos(f*x+e)^2*c*d^2+4*cos(f*x+e)^2*d^3+36*cos(f*x+e)
*c*d^2-8*cos(f*x+e)*d^3+4*d^3)/sin(f*x+e)/cos(f*x+e)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 23.42, size = 662, normalized size = 2.57

$$\left[-\frac{3\sqrt{2}\left(\left(ac^3-3ac^2d+3acd^2-ad^3\right)\cos\left(fx+e\right)^2+\left(ac^3-3ac^2d+3acd^2-ad^3\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)+1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+6\left(c^3\cos\left(fx+e\right)^2+c^3\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-4\left(d^3+\left(9cd^2-d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{6\left(af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)\right)}, -\frac{6\left(c^3\cos\left(fx+e\right)^2+c^3\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(d^3+\left(9cd^2-d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-\frac{3\sqrt{2}\left((ac^3-3ac^2d+3acd^2-ad^3)\cos(fx+e)^2+(ac^3-3ac^2d+3acd^2-ad^3)\cos(fx+e)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}}{3\left(af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 +
(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*sqrt(-1/a)*log(-(2*sq
rt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f
*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*
x + e) + 1)) + 6*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log((2*a*
cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(d^3 + (9*
c*d^2 - d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -1/3*(6*(c^3*cos(f*x + e)^2
+ c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(d^3 + (9*c*d^2 - d^3)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 3*sqrt(2)*((a*c^3
 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + (a*c^3 - 3*a*c^2*d + 3*a
*c*d^2 - a*d^3)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2
 + a*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c+d\sec\left(e+fx\right)\right)^3}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2\sqrt{a}\,c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}\,\sqrt{a}\,(c-d)^2\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\ldots}\right.}{f\sqrt{a-a\sec(e+f\ldots}}$$

```
[Out] 2*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((a-a*sec(f*x+e))^(1
/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^
(1/2)*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4025, 186, 65, 212}

$$\frac{2\sqrt{a}\,c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\,\sqrt{a}\,(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\,\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2d^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*d^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*c^2*ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*(c - d)^2*ArcTanh[Sqrt[a - a*
Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}\,(a+ax)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{d^2}{a\sqrt{a-ax}}+\frac{c^2}{ax\sqrt{a-ax}}-\frac{(c-d)^2}{a(1+x)\sqrt{a-ax}}\right) d\ldots\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{(ac^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+\ldots\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{(2c^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a-a\sec(e\ldots}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}\,c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.56, size = 295, normalized size = 1.61

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos^{\frac{3}{2}}(e+fx)(c+d\sec(e+fx))^2\left(-\frac{(c-d)^2\sqrt{-1+\cos(e+fx)}\,(2+\cos(e+fx))\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-2\tanh^{-1}\left(\sqrt{-\sec(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)}\right)+\sqrt{2-2\sec(e+fx)}\right)}{2\sqrt{2}}+\frac{4cd\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}+c^2\left(\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)-\frac{2\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)-\frac{(c-d)^2\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)}{10\cos^{\frac{5}{2}}(e+fx)}\right)}{f(d+c\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]^(3/2)*(c + d*Sec[e + f*x])^2*(-1/2*((c - d
)^2*Sqrt[-1 + Cos[e + f*x]]*(2 + Cos[e + f*x])*Csc[(e + f*x)/2]^3*(-2*ArcTa
nh[Sqrt[-(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]] + Sqrt[2 - 2*Sec[e + f*x]]))/S
qrt[2] + (4*c*d*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] + c^2*(Sqrt[2]*ArcSin[
Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]]) - ((c
- d)^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]*S
in[(e + f*x)/2]*Sin[e + f*x]^2)/(10*Cos[e + f*x]^(5/2))))/(f*(d + c*Cos[e +
 f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(158) = 316$.
time = 1.51, size = 358, normalized size = 1.96

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sqrt{2}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)c^2\sin(fx+e)+\sqrt{-\frac{2\cos(fx}{\cos(fx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos
(f*x+e)*2^(1/2))*c^2*sin(f*x+e)+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((si
n(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*c^2
*sin(f*x+e)-2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*c*d*sin(f*x+e)+(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*d^2*sin(f*x+e)+2*cos(f*x+e)*d^2-2*d^2)/si
n(f*x+e)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 7.48, size = 516, normalized size = 2.82

$$\left[\frac{4 d^{2} \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \sin\left(f x+e\right)+\sqrt{2}\left(a c^{2}-2 a c d+a d^{2}+\left(a c^{2}-2 a c d+a d^{2}\right) \cos\left(f x+e\right)\right) \sqrt{-\frac{1}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \sqrt{-\frac{1}{a}} \cos\left(f x+e\right) \sin\left(f x+e\right)+3 \cos\left(f x+e\right)^{2}+2 \cos\left(f x+e\right)-1}{\cos\left(f x+e\right)^{2}+2 \cos\left(f x+e\right)+1}\right)-2\left(c^{2} \cos\left(f x+e\right)+c^{2}\right) \sqrt{-a} \log\left(\frac{2 a \cos\left(f x+e\right)^{2}+2 \sqrt{-a} \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \cos\left(f x+e\right) \sin\left(f x+e\right)+a \cos\left(f x+e\right)-a}{\cos\left(f x+e\right)+1}\right)}{2\left(a f \cos\left(f x+e\right)+a f\right)}, \frac{2 d^{2} \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \sin\left(f x+e\right)-2\left(c^{2} \cos\left(f x+e\right)+c^{2}\right) \sqrt{a} \arctan\left(\frac{\sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \cos\left(f x+e\right)}{\sqrt{a} \sin\left(f x+e\right)}\right)+\frac{\sqrt{2}\left(a c^{2}-2 a c d+a d^{2}+\left(a c^{2}-2 a c d+a d^{2}\right) \cos\left(f x+e\right)\right) \arctan\left(\frac{\sqrt{2} \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \cos\left(f x+e\right)}{\sqrt{a} \sin\left(f x+e\right)}\right)}{\sqrt{a}}}{a f \cos\left(f x+e\right)+a f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(2)*
(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-1/
a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*
x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^
2 + 2*cos(f*x + e) + 1)) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f*cos(f*x + e
) + a*f), (2*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(
c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2
 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*c
os(f*x + e) + a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sec\left(e+f x\right)\right)^{2}}{\sqrt{a\left(\sec\left(e+f x\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

<u>**Mupad**</u> **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f/a^(1/2)-(c-d)*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4005, 3859, 209, 3880}

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) -
 (Sqrt[2]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e +
 f*x]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c\int\sqrt{a+a\sec(e+fx)}\,dx}{a}-(c-d)\int\frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= -\frac{(2c)\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{(2(c-d))\text{Subst}\left(\int\frac{1}{2a+\dots}\right.}{\dots}$$
$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+\dots}}\right.}{\sqrt{a}\,f}$$

Mathematica [A]
time = 0.31, size = 92, normalized size = 1.01

$$\frac{2\left(\sqrt{2}\,c\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(-c+d)\text{ArcTan}\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (-c + d)*ArcTan[Sin[(e + f
*x)/2]/Sqrt[Cos[e + f*x]]])*Cos[(e + f*x)/2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[a*
(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(76) = 152$.
time = 1.31, size = 194, normalized size = 2.13

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(c\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)+c\ln\left(-\frac{-\sin(fx+e)\sqrt{-\dots}}{\dots}\right.\right.}{fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(c*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/c
os(f*x+e)*2^(1/2))+c*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+
cos(f*x+e)-1)/sin(f*x+e))-d*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+cos(f*x+e)-1)/sin(f*x+e)))/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 4.45, size = 337, normalized size = 3.70

$$\left[-\frac{\sqrt{2}\,(ac-ad)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)+1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-a}\,c\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{2af},\ -\frac{2\sqrt{a}\,c\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\frac{\sqrt{2}\,(ac-ad)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c - a*d)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2
- 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*c
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*
f), -(2*sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c - a*d)*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/
(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d\sec{(e + fx)}}{\sqrt{a\left(\sec{(e + fx)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=166

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,cf}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}+\frac{2d^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\,\dots}{\sqrt{c+d}\sqrt{a+\dots}}\right)}{\sqrt{a}\,c(c-d)\sqrt{c+\dots}}$$

```
[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-arctan(1/2*
a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)+
2*d^(3/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1
/2))/c/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {4014, 4005, 3859, 209, 3880, 4052, 211}

$$\frac{2d^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,cf(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}+\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,cf}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(Sqrt[a]*(c - d)*f) + (2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(S
qrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*c*(c - d)*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4014

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*C
sc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), I
nt[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] ||
EqQ[c^2 - d^2, 0])
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\int \frac{ac-ad-ad\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{ac(c-d)} + \frac{d^2\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+f}}{c+d\sec(e+fx)}}{ac(c-d)}$$

$$= \frac{\int \sqrt{a+a\sec(e+fx)}\,dx}{ac} - \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} - \ldots$$

$$= \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,c(c-d)\sqrt{c+d}\,f} - \frac{2\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,cf} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{2}\sqrt{a+\ldots}}\right.}{\sqrt{a}\,(c-\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 39.28, size = 431980, normalized size = 2602.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(137) = 274.
time = 4.65, size = 663, normalized size = 3.99

method	result
default	$-\frac{\left(2\sqrt{(c+d)(c-d)}\,\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)c-2\sqrt{(c+d)(c-d)}\,\sqrt{2}\,\sqrt{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*((c+d)*(c-d))^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c-2*((c+d)*(c-d)
```

```
)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*d+2*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2
)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x
+e))*c-2^(1/2)*ln(2*(-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f
*x+e)-c+d))*d^2+2^(1/2)*ln(2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*c
os(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)
+d*cos(f*x+e)+c-d))*d^2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a*(cos(f*x+e
)+1)/cos(f*x+e))^(1/2)/(d/(c-d))^(1/2)/(c-d)/c/((c+d)*(c-d))^(1/2)/a
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

<u>Fricas</u> [A]
time = 26.87, size = 1120, normalized size = 6.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] [-1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(
f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*a*d*sqrt(-d/(a*c +
 a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(
f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 2*sqrt(-a)*(
c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
))/((a*c^2 - a*c*d)*f), -1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*
cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + 4*a*d*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) + 2*sqrt(-a)*(c
 - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
```

```
x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
)/((a*c^2 - a*c*d)*f), -(a*d*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)) - sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*sqrt(
a)*(c - d)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))))/((a*c^2 - a*c*d)*f), -(2*a*d*sqrt(d/(a*c + a*d))*arcta
n((c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(d*sin(f*x + e))) - sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*sqrt(a)*
(c - d)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))))/((a*c^2 - a*c*d)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=416

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2 f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\,\sqrt{a}}\right)\tan(e+fx)}{(c-d)^2 f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-$$

```
[Out] d^2*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arct
anh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/c^2/f/(a-a*sec(f*x+e
))^(1/2)/(a+a*sec(f*x+e))^(1/2)+d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1
/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c/(c-d)/(c+d)^(3/2)/f/(a-a*sec(
f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+e)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)+2*(2*c-d)*d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1
/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c^2/(c-d)^2/f/(c+d)^(1/2)/(a-a*
sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4025, 186, 65, 212, 44, 214}

$$\frac{2\sqrt{a}\,d^{3/2}(2c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)}{c^2 f(c-d)^2\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{d^2\tan(e+fx)}{cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}+\frac{2\sqrt{a}\,\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^2 f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{\sqrt{a}\,d^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)}{cf(c-d)(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\,\sqrt{a}\,\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\,\sqrt{a}}\right)}{f(c-d)^2\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^2*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (Sqrt[a]*d^(3/2)*ArcTanh
[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c
*(c - d)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + (2*Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/
(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*Sqrt[c + d]*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(c*(c^2 - d^
2)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)(c+dx)^2}\,dx, x,\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{ac^2x\sqrt{a-ax}} - \frac{1}{a(c-d)^2(1+x)}\right.\right.}{f\sqrt{a-}}$$

$$= -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{c\,(c^2-d^2)\,f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))} + \frac{(2}{}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}}{(c}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}}{(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 36.27, size = 473385, normalized size = 1137.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117714 vs. $2(355) = 710$.
time = 9.06, size = 117715, normalized size = 282.97

method	result	size
default	Expression too large to display	117715

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2), x)
```

3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=653

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\,\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}+$$

```
[Out] 1/2*d^2*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)+
3/4*d^2*tan(f*x+e)/c/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2
)+(2*c-d)*d^2*tan(f*x+e)/c^2/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+
e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*a^(1/2)*tan(f*x+e)/c^3/
f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(3/2)*arctanh(d^(1/2)
*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e)/c/(c-d)/(c+
d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+(2*c-d)*d^(3/2)*ar
ctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(1/2)*tan(f*x+e
)/c^2/(c-d)^2/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a
rctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)*tan(f*x+
e)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*d^(3/2)*(3*c^2
-3*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*a^(
1/2)*tan(f*x+e)/c^3/(c-d)^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\frac{2\sqrt{a}\,\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{\sqrt{a}\,d^{3/2}(2c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)}{c^2f(c-d)^2(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{d^2(2c-d)\tan(e+fx)}{c^2f(c-d)^2(c+d)\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}+\frac{d^2\tan(e+fx)}{2cf\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))^2}+\frac{2\sqrt{a}\,d^{3/2}\left(3c^2-3cd+d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)}{c^3f(c-d)^3\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{3\sqrt{a}\,d^{3/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\,\sqrt{c+d}}\right)}{4cf(c-d)(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{3d^2\tan(e+fx)}{4cf(c-d)(c+d)^2\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}-\frac{\sqrt{2}\,\sqrt{a}\,\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\,\sqrt{a}}\right)}{f(c-d)^3\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*Sqrt[a]*ArcTan
h[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/((c - d)^3*f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*Sqrt[a]*d^(3/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/
(4*c*(c - d)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + (Sqrt[a]*(2*c - d)*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]]
)/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*Sqrt[a]*d^(3/2)*(3*c^2 -
 3*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c +
d])]*Tan[e + f*x])/(c^3*(c - d)^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sq
```

```
rt[a + a*Sec[e + f*x]]) + (d^2*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*Sqrt[a + a*
Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^2*Tan[e + f*x])/(4*c*(c - d)*(
c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + ((2*c - d)*d^2*
Tan[e + f*x])/(c^2*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[
e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)(c+dx)^3}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{ac^3x\sqrt{a-ax}} - \frac{1}{a(c-d)^3(1+x)\sqrt{}}\right.\right.}{}$$

$$= -\frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{2c\left(c^2-d^2\right)f\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))^2} + \frac{}{c}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}}{(c-}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}}{(c-}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}}{(c-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 38.87, size = 654358, normalized size = 1002.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402965 vs. 2(564) = 1128.
time = 15.82, size = 402966, normalized size = 617.10

method	result	size
default	Expression too large to display	402966

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^3), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(c+d\sec\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3), x)
```

3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=324

$$\frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}-\frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2c^3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\mathrm{t}}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\,\mathrm{se}}}$$

```
[Out] 2*d^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^3*tan(f*x+e)/a/f/(1+s
ec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1
/2))*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4
*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/f*2
^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-d)^2*(c+2*d
)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/f/
a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 186, 65, 212, 44}

$$\frac{2c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}(c-d)^2(c+2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2d^3\tan(e+fx)}{af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*d^3*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*Tan[e + f*
x])/(2*a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a - a*Sec[e +
 f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*
x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[a - a*Sec[e
+ f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - d)^2*(c + 2*d)*ArcTanh[Sq
rt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}\,(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{d^3}{a^2\sqrt{a-ax}} + \frac{c^3}{a^2x\sqrt{a-ax}} - \frac{(c-d)^3}{a^2(1+x)^2\sqrt{a-a}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2c}{}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3}{\sqrt{a}}$$

$$= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^3}{\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 29.47, size = 21121, normalized size = 65.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(280) = 560$.
time = 1.72, size = 957, normalized size = 2.95

method	result	size
default	Expression too large to display	957

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(4*arctanh(1/2*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*2^(1/2)*sin(f*x+e)*c^3+5*ln((sin(f*x
+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*c^3-3*ln((sin(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*c^2*d-9*ln((sin(f*x+e)*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*c*d^2+7*ln((sin(f*x+e)*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*d^3+4*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*c^3*sin(f*x+e)+5*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^3
*sin(f*x+e)-3*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e
)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^2*d*sin(f*x+e)-9*ln
((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*d^2*sin(f*x+e)+7*ln((sin(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*d^3*sin(f*x+e)-2*cos(f*x+e)^2*c^3+6*cos(f*x+e)^2*c^2*
d-6*cos(f*x+e)^2*c*d^2+10*cos(f*x+e)^2*d^3+2*c^3*cos(f*x+e)-6*cos(f*x+e)*c^
2*d+6*cos(f*x+e)*c*d^2-2*cos(f*x+e)*d^3-8*d^3)/sin(f*x+e)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A]
time = 34.81, size = 744, normalized size = 2.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*
d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x
 + e))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
```

```
+ e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a
)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^2 + 2*c^3*co
s(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)) - 4*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), 1/4*(sqrt(2)*(5*c^3 - 3*c^2*d - 9
*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*
c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8
*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*(4*d^
3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e)
 + a^2*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec(e + fx))^3}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+f\,x)}\right)^3}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=290

$$-\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{(c-d)^2\,\mathrm{ta}}{2\sqrt{2}\sqrt{a}\ }$$

```
[Out] -1/2*(c-d)^2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*arc
tanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*tan(f*x+e)/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)-(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*2^(1/2)*tan(f*x+e)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 186, 65, 212, 44}

$$-\frac{\sqrt{2}\left(c^2-d^2\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((c - d)^2*Tan[e + f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*
x]]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt
[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)^2*ArcTa
nh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqr
t[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 -
 d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sq
rt[a]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}\,(a+ax)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \left(\frac{c^2}{a^2x\sqrt{a-ax}} - \frac{(c-d)^2}{a^2(1+x)^2\sqrt{a-ax}} + \frac{-c^2+c}{a^2(1+x)\sqrt{a}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{((c-d)^2\tan(}{f}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2c^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{}{1}\right.}{af\sqrt{a-a\sec(e+f}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+}}{\sqrt{a}}\right.}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+}}{\sqrt{a}}\right.}{\sqrt{a}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 27.99, size = 16153, normalized size = 55.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(248) = 496.
time = 1.57, size = 756, normalized size = 2.61

method	result

default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-4\sqrt{2}\,\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,c^2\cos(fx+e)-5\,\mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-4*2^(1/2)*sin(f*x+e)*arctanh(1/
2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^2*cos(f*x+e)-5*sin(f*x+e)*ln(-(-sin(f*x+e
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*c^2*cos(f*x+e)+2*sin(f*x+e)*ln(-(-sin(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*c*d*cos(f*x+e)+3*sin(f*x+e)*ln(-(-sin(f*x+e)*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*d^2*cos(f*x+e)-4*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*
2^(1/2))*c^2*sin(f*x+e)-5*sin(f*x+e)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*c^2+2*sin(f*x+e)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+
cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*d+3*sin(f*
x+e)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/si
n(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d^2+2*cos(f*x+e)^2*c^2-4*cos
(f*x+e)^2*c*d+2*cos(f*x+e)^2*d^2-2*cos(f*x+e)*c^2+4*cos(f*x+e)*c*d-2*cos(f*
x+e)*d^2)/(cos(f*x+e)+1)/sin(f*x+e)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A]
time = 26.04, size = 663, normalized size = 2.29

$$\left[-\frac{4(c^2-2cd+d^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\left((5c^2-2cd-3d^2)\cos(fx+e)^2+5c^2-2cd-3d^2+2(5c^2-2cd-3d^2)\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3a\cos(fx+e)^2+2a\cos(fx+e)-a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+8(c^2\cos(fx+e)^2+2c^2\cos(fx+e)+c^2)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{8(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f)},\ -\frac{2(c^2-2cd+d^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\left((5c^2-2cd-3d^2)\cos(fx+e)^2+5c^2-2cd-3d^2+2(5c^2-2cd-3d^2)\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+8(c^2\cos(fx+e)^2+2c^2\cos(fx+e)+c^2)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{4(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c
^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(-a)*log((
2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))
/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c^2 - 2*c*
d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
- sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*d - 3*d^2 +
 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 8*(c^2*
cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x
 + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec(e + fx))^2}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+f\,x)}\right)^2}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f}-\frac{(5c-d)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-1/4*(5*c-d)
*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*2^
(1/2)-1/2*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4007, 4005, 3859, 209, 3880}

$$-\frac{(5c-d)\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}f}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(3/2)*f) -
 ((5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]
)])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x]
)^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}} - \frac{\int \frac{-2ac+\frac{1}{2}a(c-d)\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}} + \frac{c\int \sqrt{a+a\sec(e+fx)}\,dx}{a^2} - \frac{(5c-d)\int \frac{\quad}{\sqrt{a+}}}{4}$$

$$= -\frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right.}{af}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{(5c-d)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right.}{2\sqrt{2}a^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 26.89, size = 11183, normalized size = 88.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(106) = 212$.

time = 1.51, size = 552, normalized size = 4.35

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-4\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sin(fx+e)c\cos(fx+e)-5\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-4*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*c*cos(f*x+e)-5*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*c*cos(f*x+e)+ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*d*cos(f*x+e)-4*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c*sin(f*x+e)-5*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*c+ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*d+2*cos(f*x+e)^2*c-2*cos(f*x+e)^2*d-2*c*cos(f*x+e)+2*d*cos(f*x+e))/(cos(f*x+e)+1)/sin(f*x+e)/a^2

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(112) = 224$.

time = 8.81, size = 591, normalized size = 4.65

$$\left[-\frac{4(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\left((5c-d)\cos(fx+e)^2+2(5c-d)\cos(fx+e)+5c-d\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3a\cos(fx+e)^2+2a\cos(fx+e)-a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+8\left(c\cos(fx+e)^2+2c\cos(fx+e)+c\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)}{8\left(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f\right)},\ -\frac{2(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\left((5c-d)\cos(fx+e)^2+2(5c-d)\cos(fx+e)+5c-d\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+8\left(c\cos(fx+e)^2+2c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{4\left(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*
c - d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)
/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x
 + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(
f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4
*(2*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*c - d
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e))) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sq
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d\sec(e + fx)}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + \frac{d}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=394

$$-\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}\,cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}\,(}{\sqrt{a}}$$

```
[Out] -1/2*tan(f*x+e)/a/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((
a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*tan(f*x+e)/(c-d)/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*
x+e))^(1/2)-(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(
1/2)*tan(f*x+e)/(c-d)^2/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-2*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*
tan(f*x+e)/c/(c-d)^2/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$-\frac{2d^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}\,cf(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)}{2af(c-d)(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}\,f(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}\,(c-2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}\,f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] -1/2*Tan[e + f*x]/(a*(c - d)*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]])
 + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c*f*
Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 2*d)*Arc
Tanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(c
- d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sqrt
[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*(c
 - d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2)*Arc
Tanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x]
)/(Sqrt[a]*c*(c - d)^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)^2(c+dx)}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^2cx\sqrt{a-ax}} - \frac{1}{a^2(c-d)(1+x)^2}\right.\right.}{f\sqrt{a-a}}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{cf\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \cdots$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{(2\,\text{ta}}{}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{t}}{\sqrt{a}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{t}}{\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 35.32, size = 378865, normalized size = 961.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2079 vs. 2(334) = 668.
time = 5.36, size = 2080, normalized size = 5.28

method	result	size
default	Expression too large to display	2080

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -1/4/f*(-1+cos(f*x+e))*(-4*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(
f*x+e)*2^(1/2))*((c+d)*(c-d))^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c^2+8*cos(f*x+e
)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*((c+d)*(c-d))^(1/2)*
2^(1/2)*(d/(c-d))^(1/2)*c*d-4*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/c
os(f*x+e)*2^(1/2))*((c+d)*(c-d))^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*d^2+2*cos(f*
x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-2*(2^(1/2)*(d/(c-d
))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d)
)^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*si
n(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-
d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))*2^(1/2)*d^3-5*cos(f*x+e)*
sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*((c+d)*(c-d))^(1/2)*(d
/(c-d))^(1/2)*c^2+9*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin
(f*x+e))*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*d-2*cos(f*x+e)*sin(f*x+e)*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(2*(-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*si
n(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+
c*cos(f*x+e)-d*cos(f*x+e)-c+d))*2^(1/2)*d^3-4*sin(f*x+e)*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)/cos(f*x+e)*2^(1/2))*((c+d)*(c-d))^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c^2+8*s
in(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*((c+d)*(c-d))^(1/2)*2^(
1/2)*(d/(c-d))^(1/2)*c*d-4*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1
/2))*((c+d)*(c-d))^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*d^2+2*sin(f*x+e)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1
/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-((c+d)*(c-d)
)^(1/2)*sin(f*x+e)-c+d))*2^(1/2)*d^3-5*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e
)-1)/sin(f*x+e))*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2+9*sin(f*x+e)*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*
d-2*sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(2*(-2^(1/2)*(d/(c-d)
)^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(d/(c-d))
^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2
)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^
```

```
(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*2^(1/2)*d^3+2*cos(f*x+e)^2
*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2-2*cos(f*x+e)^2*((c+d)*(c-d))^(1/2)
*(d/(c-d))^(1/2)*c*d-2*cos(f*x+e)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c^2+2
*cos(f*x+e)*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)*c*d)*(a*(cos(f*x+e)+1)/cos(
f*x+e))^(1/2)/sin(f*x+e)^3/(d/(c-d))^(1/2)/(c-d)^2/c/((c+d)*(c-d))^(1/2)/a^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c + \frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))), x)
```

3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=560

$$-\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}\,c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{\sqrt{2}\cdots}{\sqrt{\cdots}}$$

[Out] -1/2*tan(f*x+e)/a/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-d^3*tan(f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)^2/(c+d)^(3/2)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^2/f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/(c-d)^3/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*(3*c-d)*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c-d)^3/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)

Rubi [A]
time = 0.36, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$,
Rules used = {4025, 186, 65, 212, 44, 214}

$$-\frac{2d^{5/2}(3c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}\,c^2f(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d^{5/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}\,cf(c-d)^2(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d^3\tan(e+fx)}{acf(c-d)^2(c+d)\sqrt{a\sec(e+fx)+a}\,(c+d\sec(e+fx))}-\frac{\tan(e+fx)}{2af(c-d)^2(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}\,f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}(c-3d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}\,f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]

[Out] -1/2*Tan[e + f*x]/(a*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*(3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*x])/(a*c*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)^2(c+dx)^2}\,dx, x\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \left(\frac{1}{a^2c^2x\sqrt{a-ax}} - \frac{}{a^2(c-d)^2(1+}\right.\right.}{}$$

$$= -\frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} +$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{}{ac}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2}{\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2}{\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 37.91, size = 582620, normalized size = 1040.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164977 vs. $2(480) = 960$.
time = 8.36, size = 164978, normalized size = 294.60

method	result	size
default	Expression too large to display	164978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2),x)
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2), x)
```

3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=802

$$-\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}\,c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\cdots$$

```
[Out] -1/2*tan(f*x+e)/a/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*d^3*t
an(f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-(3*
c-d)*d^3*tan(f*x+e)/a/c^2/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))
^(1/2)-3/4*d^3*tan(f*x+e)/a/c/(c^2-d^2)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e
))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/a^(1/2)
/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*d^(5/2)*arctanh(d^(1/2)*
(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)^2/(c+d)^(5/2
)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(3*c-d)*d^(5/2)*a
rctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(
c-d)^3/(c+d)^(3/2)/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-
1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^3/
f*2^(1/2)/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c-4*d)*arc
tanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/(c-d)^4
/f/a^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*d^(5/2)*(6*c^2-4
*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f
*x+e)/c^3/(c-d)^4/f/a^(1/2)/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x
+e))^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] -1/2*Tan[e + f*x]/(a*(c - d)^3*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]
]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^
3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 4*d)
*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]
*(c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[
Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a
]*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*d^(5/
2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e
```

```
+ f*x])/(4*Sqrt[a]*c*(c - d)^2*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]]) - ((3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec
[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*(c
+ d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^(5/2
)*(6*c^2 - 4*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]
*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^3*(c - d)^4*Sqrt[c + d]*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*x])/(2*a*c*(c -
d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) - ((3*c - d
)*d^3*Tan[e + f*x])/(a*c^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c
+ d*Sec[e + f*x])) - (3*d^3*Tan[e + f*x])/(4*a*c*(c^2 - d^2)^2*f*Sqrt[a + a
*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)^2(c+dx)^3}\,dx,\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx}}$$

$$= -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \left(\frac{1}{a^2c^3x\sqrt{a-ax}} - \frac{}{a^2(c-d)^3(1}\right.\right.}{}$$

$$= -\frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{c^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - $$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + $$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + $$

$$= -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + $$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 40.80, size = 776222, normalized size = 967.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 480552 vs. $2(694) = 1388$.
time = 15.05, size = 480553, normalized size = 599.19

method	result	size
default	Expression too large to display	480553

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] \text{Hanged}
```

3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=480

$$-\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}-\frac{(c-d}{2a^2f(1+\sec(}$$

```
[Out] -1/4*(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16*
(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^2*
(c+2*d)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arctan
h((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)-3/32*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*se
c(f*x+e))^(1/2)-1/4*(c-d)^2*(c+2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)-(c^3-d^3)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 186, 65, 212, 44}

$$-\frac{\sqrt{2}\left(c^3-d^3\right)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3(c-d)^3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2(c+2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3(c-d)^3\tan(e+fx)}{16a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^3\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*((c - d)^3*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e
+ f*x]]) - (3*(c - d)^3*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a +
 a*Sec[e + f*x]]) - ((c - d)^2*(c + 2*d)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e
+ f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]
/Sqrt[a]]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]) - (3*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a]
)]*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*
Sec[e + f*x]]) - ((c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqr
t[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^3 - d^3)*ArcTanh[Sqrt[a - a*Sec[e +
 f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(a^2\tan(e+fx))\operatorname{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}\,(a+ax)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\operatorname{Subst}\left(\int\left(\frac{c^3}{a^3x\sqrt{a-ax}} - \frac{(c-d)^3}{a^3(1+x)^3\sqrt{a-ax}} - \frac{(c-d)^2(c+}{a^3(1+x)^2\sqrt{a}}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e}$$

$$= -\frac{(c^3\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{((c-d)^3\tan(e}{af}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)^2(c+2d)\tan(}{2a^2f(1+\sec(e+fx))\sqrt{a+}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^3\tan(e+}{16a^2f(1+\sec(e+fx))\sqrt{a}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^3\tan(e+}{16a^2f(1+\sec(e+fx))\sqrt{a}}$$

$$= -\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^3\tan(e+}{16a^2f(1+\sec(e+fx))\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 29.39, size = 21194, normalized size = 44.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. 2(415) = 830.
time = 2.09, size = 1444, normalized size = 3.01

method	result	size

default	Expression too large to display	1444

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/32/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(26*cos(f*x+e)*d^3+22*c^3*cos(f*x+e)+42*cos(f*x+e)^3*c^2*d-24*cos(f*x+e)^2*c^2*d+24*cos(f*x+e)^2*c*d^2-30*cos(f*x+e)*c*d^2+6*cos(f*x+e)^3*c*d^2-30*cos(f*x+e)^3*c^3+32*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^3*sin(f*x+e)-18*cos(f*x+e)*c^2*d-9*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^2*d-15*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c*d^2-18*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^2*d*cos(f*x+e)-30*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c*d^2*cos(f*x+e)+32*2^(1/2)*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*sin(f*x+e)*c^3+8*cos(f*x+e)^2*c^3+64*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*2^(1/2)*sin(f*x+e)*c^3+43*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^3-19*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*d^3-18*cos(f*x+e)^3*d^3-8*cos(f*x+e)^2*d^3+43*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^3-19*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*d^3+86*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^3*cos(f*x+e)-38*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*d^3*cos(f*x+e)-9*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^2*d-15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c*d^2)/(cos(f*x+e)+1)/sin(f*x+e)^3/a^3

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 83.55, size = 931, normalized size = 1.94

$$\left[\frac{\sqrt{2}\left((43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)^3+43c^3-9c^2d-15cd^2-19d^3+3(43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)^2+3(43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3a\cos(fx+e)^2+2a\cos(fx+e)-a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-64\left(c^3\cos(fx+e)^3+3c^3\cos(fx+e)^2+3c^3\cos(fx+e)+c^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-4\left(3(5c^3-7c^2d-cd^2+3d^3)\cos(fx+e)^2+(11c^3-9c^2d-15cd^2+13d^3)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{64\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)},\ \frac{\sqrt{2}\left((43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)^3+43c^3-9c^2d-15cd^2-19d^3+3(43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)^2+3(43c^3-9c^2d-15cd^2-19d^3)\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-64\left(c^3\cos(fx+e)^3+3c^3\cos(fx+e)^2+3c^3\cos(fx+e)+c^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left(3(5c^3-7c^2d-cd^2+3d^3)\cos(fx+e)^2+(11c^3-9c^2d-15cd^2+13d^3)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{32\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*
c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3
)*cos(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*s
qrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*
x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x +
e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*c
os(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*(5*c^3 - 7*c^2*d - c*d^2 + 3*d^
3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x + e))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 +
 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43
*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*c^3 - 9*c^2*d - 15*
c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^2 +
3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(
sqrt(a)*sin(f*x + e))) - 2*(3*(5*c^3 - 7*c^2*d - c*d^2 + 3*d^3)*cos(f*x + e
)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x + e))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f
*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sec(e+fx))^3}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=468

$$-\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}-\frac{(c^2}{2a^2f(1+\sec(}$$

```
[Out] -1/4*(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16*
(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c^2-d^2
)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((a-a
*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2)-3/32*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+
e))^(1/2)-1/4*(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*tan(f*x+e)/a^(3/2)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
-c^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)
/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4025, 186, 65, 212, 44}

$$-\frac{(c^2-d^2)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}c^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3(c-d)^2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c^2-d^2)\tan(e+fx)}{2a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{3(c-d)^2\tan(e+fx)}{16a^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-d)^2\tan(e+fx)}{4a^2f(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*((c - d)^2*Tan[e + f*x])/(a^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e
+ f*x]]) - (3*(c - d)^2*Tan[e + f*x])/(16*a^2*f*(1 + Sec[e + f*x])*Sqrt[a +
 a*Sec[e + f*x]]) - ((c^2 - d^2)*Tan[e + f*x])/(2*a^2*f*(1 + Sec[e + f*x])*
Sqrt[a + a*Sec[e + f*x]]) + (2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]
]*Tan[e + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
]) - (Sqrt[2]*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e
 + f*x])/(a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3
*(c - d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x]
)/(16*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
- ((c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e +
f*x])/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}\,(a+ax)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{c^2}{a^3x\sqrt{a-ax}} - \frac{(c-d)^2}{a^3(1+x)^3\sqrt{a-ax}} + \frac{-c^2+d^2}{a^3(1+x)^2\sqrt{a}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e}$$

$$= -\frac{(c^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c^2\tan(e+fx)}{af\sqrt{a}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c^2-d^2)\tan(e+}{2a^2f(1+\sec(e+fx))\sqrt{a+}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+}{16a^2f(1+\sec(e+fx))\sqrt{a}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+}{16a^2f(1+\sec(e+fx))\sqrt{a}}$$

$$= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{3(c-d)^2\tan(e+}{16a^2f(1+\sec(e+fx))\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 28.03, size = 16249, normalized size = 34.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. 2(403) = 806.
time = 1.58, size = 1133, normalized size = 2.42

method	result	size

default	Expression too large to display	1133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(32*2^(1/2)*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*arctanh(1/2*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^2+64*2^(1/2)*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^2+43*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*ln((sin(f*x+e)*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*c^2-6*(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*c*d-5*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*d^2+32*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/c
os(f*x+e)*2^(1/2))*c^2*sin(f*x+e)+86*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*s
in(f*x+e)*cos(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-co
s(f*x+e)+1)/sin(f*x+e))*c^2-12*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)*cos(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+
e)+1)/sin(f*x+e))*c*d-10*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*co
s(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/
sin(f*x+e))*d^2+43*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*c^2*sin(f*x+e)-6
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*c*d*sin(f*x+e)-5*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos
(f*x+e)+1)/sin(f*x+e))*d^2*sin(f*x+e)-30*cos(f*x+e)^3*c^2+28*cos(f*x+e)^3*c
*d+2*cos(f*x+e)^3*d^2+8*cos(f*x+e)^2*c^2-16*cos(f*x+e)^2*c*d+8*cos(f*x+e)^2
*d^2+22*cos(f*x+e)*c^2-12*cos(f*x+e)*c*d-10*cos(f*x+e)*d^2)/(cos(f*x+e)+1)^
2/sin(f*x+e)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

Fricas [A]
time = 31.18, size = 833, normalized size = 1.78

$$\left[\frac{\sqrt{2}\left((43c^2-6cd-5d^2)\cos(fx+e)^3+3(43c^2-6cd-5d^2)\cos(fx+e)^2+43c^2-6cd-5d^2+3(43c^2-6cd-5d^2)\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3a\cos(fx+e)^2+2a\cos(fx+e)-a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-64\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2+2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-4\left((15c^2-14cd-d^2)\cos(fx+e)^2+(11c^2-6cd-5d^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{64\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)},\ \frac{\sqrt{2}\left((43c^2-6cd-5d^2)\cos(fx+e)^3+3(43c^2-6cd-5d^2)\cos(fx+e)^2+43c^2-6cd-5d^2+3(43c^2-6cd-5d^2)\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-64\left(c^2\cos(fx+e)^3+3c^2\cos(fx+e)^2+3c^2\cos(fx+e)+c^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-2\left((15c^2-14cd-d^2)\cos(fx+e)^2+(11c^2-6cd-5d^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{32\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d
 - 5*d^2)*cos(f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d
^2)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*
x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^2*cos(f*x + e)^3
 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*((15*c^2 - 14*c
*d - d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c^2 -
6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^2 +
 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt(a)*
arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a
)*sin(f*x + e))) - 64*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*co
s(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c^2 - 14*c*d - d^2)*cos(f*x + e
)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*
a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sec(e+fx))^2}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{(43c-3d)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-1/32*(43*c-
3*d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/
f*2^(1/2)-1/4*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-1/16*(11*c-3*d)*tan
(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4007, 4005, 3859, 209, 3880}

$$-\frac{(43c-3d)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}f}-\frac{(11c-3d)\tan(e+fx)}{16af(a\sec(e+fx)+a)^{3/2}}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(5/2)*f) -
 ((43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Tan[e + f*x])/(4*f*(a + a*Sec[e +
f*x])^(5/2)) - ((11*c - 3*d)*Tan[e + f*x])/(16*a*f*(a + a*Sec[e + f*x])^(3/
2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{\int \frac{-4ac+\frac{3}{2}a(c-d)\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} + \frac{\int \frac{8a^2c-\frac{1}{4}a^2(11c-3c}{\sqrt{a+a\sec(}}}{8a^4}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} + \frac{c\int \sqrt{a+a\sec(}}{a^3}$$

$$= -\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}} - \frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\text{Subst}\left(\int \frac{}{a+}\right.}{}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{(43c-3d)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+}}\right.}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 27.00, size = 11243, normalized size = 68.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(139) = 278$.
time = 1.62, size = 824, normalized size = 5.02

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(-32(\cos^2(fx+e))\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{2}}{2\cos(fx+e)}\right)\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sin(fx+e)c-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-32*cos(f*x+e)^2*arctanh(1/2*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*c-43*cos(f*x+e)^2*ln(-(-sin(
f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*c+3*cos(f*x+e)^2*ln(-(-sin(f*x+e
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*d-64*arctanh(1/2*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)*c*cos(f*x+e)-86*ln(-(-sin(f*x+e)*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)*c*cos(f*x+e)+6*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)*d*cos(f*x+e)-32*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*2^(1/2
)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^
(1/2))*c*sin(f*x+e)+30*cos(f*x+e)^3*c-14*cos(f*x+e)^3*d-43*ln(-(-sin(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*c+3*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)*d-8*cos(f*x+e)^2*c+8*cos(f*x+e)^2*d-22*c*cos(f*x+e)+6*d*
cos(f*x+e))/(cos(f*x+e)+1)^2/sin(f*x+e)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(147) = 294$.
time = 8.98, size = 721, normalized size = 4.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2
 + 3*(43*c - 3*d)*cos(f*x + e) + 43*c - 3*d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*
cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
- 4*((15*c - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c - 3*d)*c
os(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2 + 3*(43*c - 3*d)*cos(f*x + e)
 + 43*c - 3*d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*
x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c - 7*d)*cos(f*x
 + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*co
s(f*x + e) + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec{\left(e + fx \right)}}{\left(a \left(\sec{\left(e + fx \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=592

$$-\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}\cdot$$

```
[Out] -1/4*tan(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(c-
2*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*ta
n(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((a-a*s
ec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c/f/(a-a*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2)-1/4*(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^2/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*t
an(f*x+e)/a^(3/2)/(c-d)/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2)-(c^2-3*c*d+3*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)
)^(1/2)+2*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2
))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$-\frac{\sqrt{2}(c^2-3cd+3d^2)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2d^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}cf(c-d)^3\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-2d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3\tan(e+fx)}{16a^2f(c-d)(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-2d)\tan(e+fx)}{2a^2f(c-d)^2(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)}{4a^2f(c-d)(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] -1/4*Tan[e + f*x]/(a^2*(c - d)*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e + f*
x]]) - ((c - 2*d)*Tan[e + f*x])/(2*a^2*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[
a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)*f*(1 + Sec[e + f*x]
)*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*
Tan[e + f*x])/(a^(3/2)*c*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
]) - ((c - 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e +
 f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[
e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Sqrt[a - a*Sec[e +
 f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec
[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a -
```

```
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c - d)^3
*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)^3(c+dx)}\,dx, x,\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^3cx\sqrt{a-ax}} - \frac{}{a^3(c-d)(1+x}\right.\right.}{}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{acf\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} +$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 37.04, size = 486155, normalized size = 821.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3862 vs. 2(509) = 1018.
time = 5.31, size = 3863, normalized size = 6.53

method	result	size

default	Expression too large to display	3863

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -1/32/f*(-1+cos(f*x+e))^2*(230*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2+43*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^3+86*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^3+32*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*c^3-32*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*d^3-126*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^2*d+115*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2-16*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*2^(1/2)*d^4-16*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*2^(1/2)*d^4+16*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))*2^(1/2)*d^4+76*cos(f*x+e)^3*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^2*d-46*cos(f*x+e)^3*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2-16*cos(f*x+e)^2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^2*d+8*cos(f*x+e)^2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2-60*cos(f*x+e)*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^2*d+38*cos(f*x+e)*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2-96*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/

```
2))*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*c^2*d+96*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*2^(1/2)*((c
+d)*(c-d))^(1/2)*c*d^2-192*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*
sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(
f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*c^2*d+192*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*
2^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2-30*cos(f*x+e)^3*(d/(c-d))^(1/2)*((c+d)*(c
-d))^(1/2)*c^3+8*cos(f*x+e)^2*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^3+22*co
s(f*x+e)*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^3+16*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)
*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-((
c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))*2^(1/2)*d^4-96*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*c^2
*d+96*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*
2^(1/2)*((c+d)*(c-d))^(1/2)*c*d^2+32*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)^2*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*c^3
-32*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*arctanh(1/
2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(d/(c
-d))^(1/2)*2^(1/2)*((c+d)*(c-d))^(1/2)*d^3-126*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))*(d/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c^2
*d+115*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((a*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))), x)
```

3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=756

$$-\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/4*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(
c-3*d)*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*
tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+d^4*tan(f*x+
e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh(
(a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c^2/f/(a-a*sec(f*x+e))^(
1/2)/(a+a*sec(f*x+e))^(1/2)+d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/
a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/(c+d)^(3/2)/f/(a-a*sec(f*
x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^3/f*2^(1/2)/(a-a*sec(f*x+e
))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^2/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)-(c^2-4*c*d+6*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*2^(1/2)*tan(f*x+e)/a^(3/2)/(c-d)^4/f/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)+2*(4*c-d)*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))
^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^2/(c-d)^4/f/(c+d)^(1/2)/(a
-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 756, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\frac{2d^{7/2}(4c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}c^2f(c-d)^4\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\sqrt{2}(c^2-4cd+6d^2)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}f(c-d)^4\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{d^{7/2}\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{a^{3/2}cf(c-d)^3(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}f(c-d)^2\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{(c-3d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{d^4\tan(e+fx)}{a^2cf(c-d)^3(c+d)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}-\frac{3\tan(e+fx)}{16a^2f(c-d)^2(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{(c-3d)\tan(e+fx)}{2a^2f(c-d)^3(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}}-\frac{\tan(e+fx)}{4a^2f(c-d)^2(\sec(e+fx)+1)^2\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] -1/4*Tan[e + f*x]/(a^2*(c - d)^2*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 3*d)*Tan[e + f*x])/(2*a^2*(c - d)^3*f*(1 + Sec[e + f*x])*Sqr
t[a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^2*f*(1 + Sec[e +
f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a]]*Tan[e + f*x])/(a^(3/2)*c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) - ((c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*T
an[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
 + a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])
]*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sq
```

```
rt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 4*c*d + 6*d^2)*ArcTanh[Sqrt[a - a
*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^4*f*Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^(7/2)*ArcTanh[(Sqrt[d]*Sq
rt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c*(c
- d)^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) +
 (2*(4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*S
qrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*Sqrt[c + d]*f*Sqrt[a - a*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(a^2*c*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
```

```
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{\frac{1}{(a+ax)^3(c+dx)^2}}{x\sqrt{a-ax}}\,dx,\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^3c^2x\sqrt{a-ax}} - \frac{}{a^3(c-d)^2(1}\right.\right.}{}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{ac^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 39.49, size = 688080, normalized size = 910.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197499 vs. $2(653) = 1306$.
time = 8.95, size = 197500, normalized size = 261.24

method	result	size
default	Expression too large to display	197500

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] \text{Hanged}
```

3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=999

$$-\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}-\frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}-$$

```
[Out] -1/4*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(
c-4*d)*tan(f*x+e)/a^2/(c-d)^4/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*
tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+1/2*d^4*tan(
f*x+e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)+3/4*
d^4*tan(f*x+e)/a^2/c/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1
/2)+(4*c-d)*d^4*tan(f*x+e)/a^2/c^2/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))/(a+a*se
c(f*x+e))^(1/2)+2*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2
)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(7/2)*arctanh(d
^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-
d)^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+(4*c-d)*d^
(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e
)/a^(3/2)/c^2/(c-d)^4/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-1/4*(c-4*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(
f*x+e)/a^(3/2)/(c-d)^4/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1
/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(
3/2)/(c-d)^3/f*2^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-(c^2-5
*c*d+10*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*ta
n(f*x+e)/a^(3/2)/(c-d)^5/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*
d^(7/2)*(10*c^2-5*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(
c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^3/(c-d)^5/f/(c+d)^(1/2)/(a-a*sec(f*x+e))^(
1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 999, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4025, 186, 65, 212, 44, 214}

$$\frac{(4c-d)\tan(e+fx)d^4}{a^2c^2(c-d)^4(c+d)f\sqrt{\sec(e+fx)a+a}\,(c+d\sec(e+fx))}+\frac{3\tan(e+fx)d^4}{4a^2c(c-d)^3(c+d)^2f\sqrt{\sec(e+fx)a+a}\,(c+d\sec(e+fx))}+\frac{\tan(e+fx)d^4}{2a^2c(c-d)^3(c+d)f\sqrt{\sec(e+fx)a+a}\,(c+d\sec(e+fx))^2}+\frac{2(10c^2-5dc+d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{7/2}}{a^{3/2}c^3(c-d)^5\sqrt{c+d}\,f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}+\frac{(4c-d)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{7/2}}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)d^{7/2}}{4a^{3/2}c(c-d)^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}-\frac{\sqrt{2}(c^2-5dc+10d^2)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}-\frac{(c-4d)\tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}-\frac{3\tan(e+fx)}{16a^2(c-d)^3f(\sec(e+fx)+1)\sqrt{\sec(e+fx)a+a}}-\frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(\sec(e+fx)+1)\sqrt{\sec(e+fx)a+a}}-\frac{\tan(e+fx)}{4a^2(c-d)^3f(\sec(e+fx)+1)^2\sqrt{\sec(e+fx)a+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] -1/4*Tan[e + f*x]/(a^2*(c - d)^3*f*(1 + Sec[e + f*x])^2*Sqrt[a + a*Sec[e +
f*x]]) - ((c - 4*d)*Tan[e + f*x])/(2*a^2*(c - d)^4*f*(1 + Sec[e + f*x])*Sqr
t[a + a*Sec[e + f*x]]) - (3*Tan[e + f*x])/(16*a^2*(c - d)^3*f*(1 + Sec[e +
f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
```

```
a]]*Tan[e + f*x])/(a^(3/2)*c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) - ((c - 4*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*T
an[e + f*x])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
 + a*Sec[e + f*x]]) - (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])
]*Tan[e + f*x])/(16*Sqrt[2]*a^(3/2)*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c^2 - 5*c*d + 10*d^2)*ArcTanh[Sqrt[a -
a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(a^(3/2)*(c - d)^5*f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (3*d^(7/2)*ArcTanh[(Sqrt[d]
*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(4*a^(3/2)*
c*(c - d)^3*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]) + ((4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a
]*Sqrt[c + d])]*Tan[e + f*x])/(a^(3/2)*c^2*(c - d)^4*(c + d)^(3/2)*f*Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*d^(7/2)*(10*c^2 - 5*c*d +
 d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan
[e + f*x])/(a^(3/2)*c^3*(c - d)^5*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) + (d^4*Tan[e + f*x])/(2*a^2*c*(c - d)^3*(c + d)*f*S
qrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (3*d^4*Tan[e + f*x])/(4*a
^2*c*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) +
 ((4*c - d)*d^4*Tan[e + f*x])/(a^2*c^2*(c - d)^4*(c + d)*f*Sqrt[a + a*Sec[e
 + f*x]]*(c + d*Sec[e + f*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
```

```
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}\,(a+ax)^3(c+dx)^3}\,dx, x\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \left(\frac{1}{a^3c^3x\sqrt{a-ax}} - \frac{}{a^3(c-d)^3(1+}\right.\right.}{}$$

$$= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{ac^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} +$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

$$= -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} -$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 42.76, size = 893714, normalized size = 894.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 19.14, size = 556423, normalized size = 556.98

method	result	size
default	Expression too large to display	556423

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] \text{Hanged}
```

3.184 $\int \sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{a}\,\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{a}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*a^(1/2)*c^(1/2)/f+2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*a^(1/2)*d^(1/2)/f
```

Rubi [A]
time = 0.21, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4017, 4019, 209, 4065, 212}

$$\frac{2\sqrt{a}\,\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{a}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e
+ f*x]]*Sqrt[c + d*Sec[e + f*x]])])/f + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]
)/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4017

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
d*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]
]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
```

```
b*c - a*d, 0]
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}\,dx = c\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)\sqrt{a+a\sec}}{\sqrt{c+d\sec(e+}}$$

$$= -\frac{(2ac)\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+}}\right.}{f}$$

$$= \frac{2\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right.}{f}$$

Mathematica [A]
time = 20.51, size = 240, normalized size = 1.95

$$-\frac{2\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c+d\sec(e+fx)}\left(-2\sqrt{c}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\,\sqrt{c-c\cos(e+fx)}}\right)\sqrt{c(1+\cos(e+fx))}\,\sin^2\left(\frac{1}{2}(e+fx)\right)+\text{ArcTan}\left(\frac{\sqrt{c(1+\cos(e+fx))}\,\sqrt{d+c\cos(e+fx)}}{\sqrt{c^2\sin^2(e+fx)}}\right)\sqrt{c-c\cos(e+fx)}\,\sqrt{c^2\sin^2(e+fx)}\right)}{f\sqrt{c(1+\cos(e+fx))}\,\sqrt{c-c\cos(e+fx)}\,\sqrt{d+c\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]]*(-2*Sq
rt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c -
c*Cos[e + f*x]])]*Sqrt[c*(1 + Cos[e + f*x])]*Sin[(e + f*x)/2]^2 + ArcTan[(S
qrt[c*(1 + Cos[e + f*x])]*Sqrt[d + c*Cos[e + f*x]])/Sqrt[c^2*Sin[e + f*x]^2
```

```
]]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[c^2*Sin[e + f*x]^2]))/(f*Sqrt[c*(1 + Cos[e
 + f*x])]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1550 vs. $2(99) = 198$.
time = 2.07, size = 1551, normalized size = 12.61

method	result	size
default	Expression too large to display	1551

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2
)*cos(f*x+e)*(-1+cos(f*x+e))*(-ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(
1/2))*2^(1/2)*(-d)^(1/2)*c^3+3*ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(
1/2))*2^(1/2)*(-d)^(1/2)*c^2*d-3*ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)
^(1/2))*2^(1/2)*(-d)^(1/2)*c*d^2+ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)
^(1/2))*2^(1/2)*(-d)^(1/2)*d^3+ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^
(1/2)*c^3-3*ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)
^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^(1/2)*c^2*d+3*ln(((
-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)
+(c-d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^(1/2)*c*d^2-ln(((-2*(d+c*cos(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f
*x+e))*2^(1/2)*(-d)^(1/2)*d^3+ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-d
*sin(f*x+e)-c+d)/(cos(f*x+e)-1-sin(f*x+e)))*(c-d)^(1/2)*c^2*d-2*ln(-2*(2^(1
/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(
f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(cos(f*x+e)-1-sin(f*x+e)
))*(c-d)^(1/2)*c*d^2+ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+
e)-c+d)/(cos(f*x+e)-1-sin(f*x+e)))*(c-d)^(1/2)*d^3-ln(-2*(2^(1/2)*(-d)^(1/2
)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(
f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2
)*c^2*d+2*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c-d)/(co
s(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2)*c*d^2-ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+
c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*
sin(f*x+e)-d*sin(f*x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2)*d^3+2*(
-(c-d)^4*c)^(1/2)*arctan((-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1
```

```
))^(1/2)/sin(f*x+e)*(c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2))*(c-d)^(1/2)*(-d)^
(1/2))/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)^2/(c-d)^(1/2)*
2^(1/2)/(-d)^(1/2)/(c^2-2*c*d+d^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Fricas [A]
time = 12.18, size = 868, normalized size = 7.06

$$\left[\frac{\sqrt{ad}\log\left(\frac{2\sqrt{ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac-ad)\cos(fx+e)^2+2ad+(ac+ad)\cos(fx+e)}{\cos(fx+e)^2+\cos(fx+e)}\right)+\sqrt{-ac}\log\left(\frac{2ac\cos(fx+e)^2-2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-ac+ad+(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f}, -\frac{2\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ac\sin(fx+e)}\right)-\sqrt{ad}\log\left(\frac{2\sqrt{ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+(ac-ad)\cos(fx+e)^2+2ad+(ac+ad)\cos(fx+e)}{\cos(fx+e)^2+\cos(fx+e)}\right)}{f}, -\frac{2\sqrt{-ad}\arctan\left(\frac{\sqrt{-ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)-\sqrt{-ac}\log\left(\frac{2ac\cos(fx+e)^2-2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-ac+ad+(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f}, -\frac{2\left(\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ac\sin(fx+e)}\right)+\sqrt{-ad}\arctan\left(\frac{\sqrt{-ad}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{ad\sin(fx+e)}\right)\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [(sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*c
os(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*x
 + e))) + sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1
)))/f, -(2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e)))
 - sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*
cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*
x + e))))/f, -(2*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*
x + e))) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) +
1)))/f, -2*(sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))
) + sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/f]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,(\sec{(e+fx)}+1)}\,\sqrt{c+d\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}\,f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*a^(1/2)/f/c^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4019, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[c]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{(2a)\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}\,f}$$

Mathematica [A]
time = 0.22, size = 102, normalized size = 1.67

$$\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c}\,f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]
]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(Sq
rt[c]*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(49) = 98.
time = 2.13, size = 189, normalized size = 3.10

method	result
default	$-\frac{2\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\cos(fx+e)(-1+\cos(fx+e))\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\arctan\left(\frac{(-1+\cos(fx+e))(c-d)^2c\sqrt{2}}{\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\sin(fx+e)\sqrt{-(c-}}\right.}{f\sin(fx+e)^2\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}(c^2-2cd+d^2)c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*(a*(cos
(f*x+e)+1)/cos(f*x+e))^(1/2)*arctan((-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)/sin(f*x+e)*(c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2))/sin(f*
x+e)^2/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*2^(1/2)*(-(c-d)^4*c)^(1/2
)/(c^2-2*c*d+d^2)/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 5.62, size = 221, normalized size = 3.62

$$\left[\frac{\sqrt{-\frac{a}{c}}\log\left(-\frac{2c\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-2ac\cos(fx+e)^2+ac-ad-(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f}, -\frac{2\sqrt{\frac{a}{c}}\arctan\left(\frac{\sqrt{\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos
(f*x + e)^2 + a*c - a*d - (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1))/f,
-2*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+f}}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*a^(1/2)/c^(3/2)/f-2*a*d*tan(f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/2)
/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4024, 4019, 209, 4072, 37}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(c^(3/2)*f) - (2*a*d*Tan[e + f*x])/(c*(c + d)*f
*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
```

```
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4024

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[
c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e
+ f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{cf} +$$

$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{}{c(c+d)f\sqrt{a+}}$$

Mathematica [A]
time = 0.98, size = 135, normalized size = 1.22

$$-\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(-\sqrt{2}(c+d)^{3/2}\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}+2\sqrt{c}\,d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{c^{3/2}(c+d)f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(-(Sqrt[2]*(c + d)^(3/2)*Arc
Sin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x
])/(c + d)]) + 2*Sqrt[c]*d*Sin[(e + f*x)/2]))/(c^(3/2)*(c + d)*f*Sqrt[c + d
*Sec[e + f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(95) = 190$.
time = 1.98, size = 377, normalized size = 3.40

method	result
default	$-\frac{\left(\sqrt{2}\,\sqrt{-(c-d)^{4} c}\,\arctan\left(\frac{(-1+\cos(fx+e))(c-d)^{2} c\sqrt{2}}{\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sin(fx+e)\sqrt{-(c-d)^{4} c}}\right)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\, c\sin(fx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e)
)/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)*(c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2))*(-
2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^(1/2)*(-(c-d)^4*c)^
(1/2)*arctan((-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/sin
(f*x+e)*(c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2))*(-2*(d+c*cos(f*x+e))/(cos(f*x
+e)+1))^(1/2)*d*sin(f*x+e)-2*c^3*d*cos(f*x+e)+4*c^2*d^2*cos(f*x+e)-2*cos(f*
x+e)*c*d^3+2*c^3*d-4*c^2*d^2+2*c*d^3)*cos(f*x+e)*(a*(cos(f*x+e)+1)/cos(f*x+
e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/(d+c*cos(f*x+e))/sin(f*x+e)/(
c+d)/c^2/(c^2-2*c*d+d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(101) = 202$.

time = 3.03, size = 552, normalized size = 4.97

$$\left[-\frac{2\,d\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(c^{2}+cd\right)\cos\left(fx+e\right)^{2}+cd+d^{2}+\left(c^{2}+2\,cd+d^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{a}{c}}\log\left(-\frac{2\,c\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\cos(fx+e)\sin(fx+e)-2\,ac\cos(fx+e)^{2}+ac-ad-(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{\left(c^{3}+c^{2}d\right)f\cos\left(fx+e\right)^{2}+\left(c^{3}+2\,c^{2}d+cd^{2}\right)f\cos\left(fx+e\right)+\left(c^{2}d+cd^{2}\right)f},\ -\frac{2\left(d\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(c^{2}+cd\right)\cos\left(fx+e\right)^{2}+cd+d^{2}+\left(c^{2}+2\,cd+d^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a}{c}}\arctan\left(\frac{\sqrt{\frac{a}{c}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\cos(fx+e)}{a\sin(fx+e)}\right)\right)}{\left(c^{3}+c^{2}d\right)f\cos\left(fx+e\right)^{2}+\left(c^{3}+2\,c^{2}d+cd^{2}\right)f\cos\left(fx+e\right)+\left(c^{2}d+cd^{2}\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-(2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((c^2 + c*d)*cos(f*x + e)^2 + c*d +
 d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c + a*
d)*cos(f*x + e))/(cos(f*x + e) + 1)))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^
3 + 2*c^2*d + c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f), -2*(d*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + ((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*
d + d^2)*cos(f*x + e))*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(
f*x + e))))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^3 + 2*c^2*d + c*d^2)*f*cos
(f*x + e) + (c^2*d + c*d^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,\sqrt{c-d}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c-d}}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right.}{\sqrt{a}\,f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))*c^(1/2)/f/a^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)
/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/
2)
```

Rubi [A]
time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4020, 4019, 209, 4068}

$$\frac{2\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,\sqrt{c-d}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt
[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4020

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[a/c, Int[Sqrt[c + d*Csc[e + f*x]]/Sqrt[a
+ b*Csc[e + f*x]], x], x] + Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a +
b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} + (-c+d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e-}}$$

$$= -\frac{(2c)\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{(2}{}$$

$$= \frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\sqrt{2}\,\sqrt{c-d}\,\tan^{-1}}{}$$

Mathematica [A]
time = 14.46, size = 184, normalized size = 1.30

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{-c+d}\,\tanh^{-1}\left(\frac{\sqrt{-c+d}\,\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)+\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{c+d}\,\mathrm{ArcSin}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{c+d\sec(e+fx)}}{f\sqrt{d+c\cos(e+fx)}\,\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(Sqrt[-c + d]*ArcTanh[(Sqrt[-c + d]*Sin[(e + f*x)/2])/S
qrt[d + c*Cos[e + f*x]]] + (Sqrt[2]*Sqrt[c]*Sqrt[c + d]*ArcSin[(Sqrt[2]*Sqr
t[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)])/Sqr
t[d + c*Cos[e + f*x]])*Sqrt[c + d*Sec[e + f*x]])/(f*Sqrt[d + c*Cos[e + f*x]
]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(114) = 228$.
time = 1.96, size = 482, normalized size = 3.42

method	result
default	$-\frac{2\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\,\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\cos(fx+e)(-1+\cos(fx+e))\left(\sqrt{2}\,\sqrt{-(c-d)^4 c}\,\arctan\left(\frac{(-1+}{\sqrt{-\frac{2(d+c\cos(f}{\cos(fx+e}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2
)*cos(f*x+e)*(-1+cos(f*x+e))*(2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((-1+cos(f*x
+e))/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)*(c-d)^2*c*2^(1/2
)/(-(c-d)^4*c)^(1/2))*(c-d)^(1/2)+ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f*x+e))*c^3-3*ln((
(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e
)+(c-d)^(1/2))/sin(f*x+e))*c^2*d+3*ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f*x+e))*c*d^2-ln(
((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+
e)+(c-d)^(1/2))/sin(f*x+e))*d^3)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
/sin(f*x+e)^2/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-c>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.96, size = 945, normalized size = 6.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c
+ 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-(2*sqrt(-
c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x
+ e) + c - d)/(cos(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*
sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x
 + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e
) + 1)) - 4*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(c*sin(f*x + e))))
/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)/((c - d)*sin(f*x + e))) - sqrt(-c/a)*log(-(2*sqrt(-c/a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x + e) + c - d)/(cos(
f*x + e) + 1)))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + 2*sqrt(c/a)*arctan(sqrt(c/a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
 e))*cos(f*x + e)/(c*sin(f*x + e))))/f]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sec{(e + fx)}}}{\sqrt{a(\sec{(e + fx)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

```
[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2))/f/a^(1/2)/c^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)
/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/
2)
```

Rubi [A]
time = 0.21, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4023, 4019, 209, 4068}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c -
 d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4019

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(1 + a*c*x^2), x],
x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

Rule 4023

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqr
t[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\sec(e+f}{\sqrt{a+a\sec(e+fx)}\sqrt{c}$$

$$= -\frac{2\mathrm{Subst}\left(\int \frac{1}{1+acx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d}}\right.}{f}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}\,f} -$$

Mathematica [A]
time = 0.34, size = 171, normalized size = 1.21

$$\frac{2\left(\sqrt{2}\sqrt{c-d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)-\sqrt{c}\,\mathrm{ArcTan}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+fx)}\sec(e+fx)}{\sqrt{c}\sqrt{c-d}\,f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d +
c*Cos[e + f*x]]] - Sqrt[c]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c
*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(S
qrt[c]*Sqrt[c - d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(114) = 228.
time = 2.02, size = 415, normalized size = 2.94

method	result
default	$-\frac{2\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\,\cos(fx+e)(-1+\cos(fx+e))\left(\ln\left(\frac{\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sin(fx+e)-\sqrt{c-d}\,\cos(}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2
)*cos(f*x+e)*(-1+cos(f*x+e))*(ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f*x+e))*c^3-2*ln(((-2*
(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c
-d)^(1/2))/sin(f*x+e))*c^2*d+ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)-(c-d)^(1/2)*cos(f*x+e)+(c-d)^(1/2))/sin(f*x+e))*c*d^2+2^(1/2)*(
-(c-d)^4*c)^(1/2)*arctan((-1+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1
))^(1/2)/sin(f*x+e)*(c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2))*(c-d)^(1/2))/(-2*
(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)^2/a/(c-d)^(1/2)/(c^2-2*c*
d+d^2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Fricas [A]
time = 4.74, size = 975, normalized size = 6.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
```

```
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/(a*c*f), 1/
2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*
c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*
cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 4*sqrt(a*c
)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))))/(a*c*f), (sqrt(2)
*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(
a*c - a*d) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)
+ 1)))/(a*c*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a
*d)*sin(f*x + e)))/sqrt(a*c - a*d) - 2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(
f*x + e)/(a*c*sin(f*x + e))))/(a*c*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=67

$$\frac{ax}{c}+\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\,\sqrt{c+d}\,f}$$

```
[Out] a*x/c+2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c/f/
(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4004, 3916, 2738, 214}

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\,\sqrt{c+d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*x)/c + (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx &= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c}\\
&= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{cd}\\
&= \frac{ax}{c}+\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{cdf}\\
&= \frac{ax}{c}+\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\,\sqrt{c+d}\,f}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 68, normalized size = 1.01

$$\frac{a(e+fx)+\frac{2(-bc+ad)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
[Out] (a*(e + f*x) + (2*(-(b*c) + a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
^2 - d^2]])/Sqrt[c^2 - d^2])/(c*f)
```

Maple [A]
time = 0.17, size = 73, normalized size = 1.09

method	result
derivativedivides	$\frac{-\frac{2(ad-bc)\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{f}$

default	$\frac{-\frac{2(ad-bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}}{f}$
risch	$\frac{ax}{c}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)ad}{\sqrt{c^2-d^2}\,fc}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)b}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2}{}\right.}{\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(a*d-b*c)/c/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((
c+d)*(c-d))^(1/2))+2*a/c*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.59, size = 258, normalized size = 3.85

$$\left[\frac{2\,(ac^2-ad^2)fx-(bc-ad)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,\sqrt{c^2-d^2}\,(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)}{2\,(c^3-cd^2)f},\ \frac{(ac^2-ad^2)fx+(bc-ad)\sqrt{-c^2+d^2}\,\arctan\left(-\frac{\sqrt{-c^2+d^2}\,(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)}{(c^3-cd^2)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^2 - a*d^2)*f*x - (b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*
x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) +
 c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) +
d^2)))/((c^3 - c*d^2)*f), ((a*c^2 - a*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^
2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))
))/((c^3 - c*d^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(58) = 116.
time = 0.50, size = 274, normalized size = 4.09

$$\frac{\frac{\left(\sqrt{-c^2+d^2}\,a(c-2\,d)|-c+d|+\sqrt{-c^2+d^2}\,bc|-c+d|-\sqrt{-c^2+d^2}\,a|c||-c+d|+\sqrt{-c^2+d^2}\,b|c||-c+d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-\frac{d+\sqrt{(c+d)(c-d)+d^2}}{c-d}}}\right)\right)}{(c^2-2\,cd+d^2)c^2+(c^2d-2\,cd^2+d^3)|c|}+\frac{(ac+bc-2\,ad+a|c|-b|c|)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-\frac{d-\sqrt{(c+d)(c-d)+d^2}}{c-d}}}\right)\right)}{c^2-d|c|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((sqrt(-c^2 + d^2)*a*(c - 2*d)*abs(-c + d) + sqrt(-c^2 + d^2)*b*c*abs(-c +
d) - sqrt(-c^2 + d^2)*a*abs(c)*abs(-c + d) + sqrt(-c^2 + d^2)*b*abs(c)*abs(
-c + d))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sq
rt(-(d + sqrt((c + d)*(c - d) + d^2))/(c - d))))/((c^2 - 2*c*d + d^2)*c^2 +
 (c^2*d - 2*c*d^2 + d^3)*abs(c)) + (a*c + b*c - 2*a*d + a*abs(c) - b*abs(c)
)*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sqrt(-(d
- sqrt((c + d)*(c - d) + d^2))/(c - d))))/(c^2 - d*abs(c)))/f
```

Mupad [B]
time = 2.68, size = 573, normalized size = 8.55

$$\frac{b\,c^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}}-\frac{b\,d^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}}+\frac{2\,a\,c\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)}-\frac{b\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(c+d\right)\,\left(c-d\right)}}{f\,\left(c^2-d^2\right)}-\frac{a\,c\,d\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}}-\frac{2\,a\,d^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f\,\left(c^2-d^2\right)}+\frac{a\,d^3\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f\,{\left(c^2-d^2\right)}^{3/2}}+\frac{a\,d\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(c+d\right)\,\left(c-d\right)}}{c\,f\,\left(c^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x)),x)
```

```
[Out] (b*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (b*d^2*lo
g((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*a*c*atan(sin(e/
2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (b*log((c*cos(e/2 + (f*
x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/
2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) - (a*c*d*log((c*sin(
e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2
))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*a*d^2*atan(sin(e/2 + (f*
x)/2)/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)) + (a*d^3*log((c*sin(e/2 + (f*x
)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2
 + (f*x)/2)))/(c*f*(c^2 - d^2)^(3/2)) + (a*d*log((c*cos(e/2 + (f*x)/2) + d*
cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/
2))*((c + d)*(c - d))^(1/2))/(c*f*(c^2 - d^2))
```

3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{ax}{c^2}+\frac{2(bc^3-2ac^2d+ad^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f}-\frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}$$

```
[Out] a*x/c^2+2*(-2*a*c^2*d+a*d^3+b*c^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(
c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f-d*(-a*d+b*c)*tan(f*x+e)/c/(c^2-d^
2)/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4008, 4004, 3916, 2738, 214}

$$-\frac{d(bc-ad)\tan(e+fx)}{cf\,(c^2-d^2)\,(c+d\sec(e+fx))}+\frac{2(-2ac^2d+ad^3+bc^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}}+\frac{ax}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*x)/c^2 + (2*(b*c^3 - 2*a*c^2*d + a*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) - (d*(b*c - a*d)*T
an[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx &= -\frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} - \frac{\int \frac{-a\left(c^2-d^2\right)-c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c\left(c^2-d^2\right)} \\
&= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\left(c^2(bc-ad)-ad(c^2-d^2)\right)\int \frac{\sec(e+fx}{c+d\sec(e+\cdots}}{c^2\left(c^2-d^2\right)} \\
&= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\left(c^2(bc-ad)-ad(c^2-d^2)\right)\int \frac{1}{1+\frac{c\cos(e+f\cdots}{d}}}{c^2d\left(c^2-d^2\right)} \\
&= \frac{ax}{c^2} - \frac{d(bc-ad)\tan(e+fx)}{c\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\left(2\left(c^2(bc-ad)-ad(c^2-d^2)\right)\right)\text{Subst}\Big(\cdots}{c^2d\left(c^2-\cdots\right.} \\
&= \frac{ax}{c^2} + \frac{2\left(bc^3-2ac^2d+ad^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc-ad)\tan\cdots}{c\left(c^2-d^2\right)f(c+d\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 155, normalized size = 1.26

$$\frac{-\frac{2\left(bc^3+ad\left(-2c^2+d^2\right)\right)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{ad\left(c^2-d^2\right)(e+fx)+ac\left(c^2-d^2\right)(e+fx)\cos(e+fx)-cd(bc-ad)\sin(e+fx)}{d+c\cos(e+fx)}}{c^2(c-d)(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(b*c^3 + a*d*(-2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] + (a*d*(c^2 - d^2)*(e + f*x) + a*c*(c^2 - d^2)
*(e + f*x)*Cos[e + f*x] - c*d*(b*c - a*d)*Sin[e + f*x])/(d + c*Cos[e + f*x]
))/(c^2*(c - d)*(c + d)*f)
```

Maple [A]
time = 0.22, size = 168, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2d(ad-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan}{\sqrt{(c+d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)\,(c-}}}{c^2}}{f}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2d(ad-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan}{\sqrt{(c+d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)\,(c-}}}{c^2}}{f}$
risch	$\frac{ax}{c^2}-\frac{2id(-ad+bc)\left(d\,e^{i(fx+e)}+c\right)}{c^2(c^2-d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{2\ln\left(e^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)ad}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}-\frac{ic^2}{}\right.}{\sqrt{c^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a/c^2*arctan(tan(1/2*f*x+1/2*e))+2/c^2*(-d*(a*d-b*c)*c/(c^2-d^2)*tan
(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)-(2*a*c^
2*d-a*d^3-b*c^3)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+
1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(117) = 234$.
time = 4.42, size = 577, normalized size = 4.69

$$\left[\frac{2\,(ac^5-2\,ac^3d^2+acd^4)fx\cos(fx+e)+2\,(ac^4d-2\,ac^2d^3+ad^5)fx-(bc^3d-2\,ac^2d^2+ad^4+(bc^4-2\,ac^3d+acd^3)\cos(fx+e))\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-2\,(bc^4d-ac^3d^2-bc^2d^3+acd^4)\sin(fx+e)}{2\,((c^7-2\,c^5d^2+c^3d^4)f\cos(fx+e)+(c^6d-2\,c^4d^3+c^2d^5)f)}, \frac{(ac^5-2\,ac^3d^2+acd^4)fx\cos(fx+e)+(ac^4d-2\,ac^2d^3+ad^5)fx+(bc^3d-2\,ac^2d^2+ad^4+(bc^4-2\,ac^3d+acd^3)\cos(fx+e))\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(bc^4d-ac^3d^2-bc^2d^3+acd^4)\sin(fx+e)}{(c^7-2\,c^5d^2+c^3d^4)f\cos(fx+e)+(c^6d-2\,c^4d^3+c^2d^5)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + 2*(a*c^4*d - 2*a
*c^2*d^3 + a*d^5)*f*x - (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d
 + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^4
*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3
*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a*c^5 - 2*a*c^3*
d^2 + a*c*d^4)*f*x*cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f*x + (b*
c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*cos(f*x + e))*s
qrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*
sin(f*x + e))) - (b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/
((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)
*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(e + fx)}{(c + d \sec(e + fx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**2, x)
```

Giac [A]
time = 0.51, size = 201, normalized size = 1.63

$$\frac{\frac{2\,(bc^3-2\,ac^2d+ad^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^2}+\frac{2\,\left(bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{(c^3-cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(b*c^3 - 2*a*c^2*d + a*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
```

```
 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) + (f*x + e)*a/c^2 + 2*(b*c*d*t
an(1/2*f*x + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2
*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

Mupad [B]
time = 9.33, size = 2500, normalized size = 20.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^2,x)
```

```
[Out] (2*a*atan(((a*((a*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c
^4*d^2) - (a*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^
4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i
)/c^2 + (32*tan(e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5
 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*
d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2 - (a*((a*((32*(
a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2
 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*tan(e/2 + (
f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^
2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 - (32*tan(e/2 + (f
*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c
^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4
*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2)/((64*(a^3*d^5 + a*b^2*c^5 - a^2*b*c^5
 - a^3*c*d^4 + 2*a^3*c^4*d - 3*a^3*c^2*d^3 + 2*a^3*c^3*d^2 + a^2*b*c^2*d^3 +
 a^2*b*c^3*d^2 - 3*a^2*b*c^4*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*((a
*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*
c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a*tan(
e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2
*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 + (32*tan(e
/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d -
5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3
))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2))*1i)/c^2 + (a*((a*((32*(a*c^4*d^5 - b*
c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d +
 b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*tan(e/2 + (f*x)/2)*(2*c^9
*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(
c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 - (32*tan(e/2 + (f*x)/2)*(a^2*c^
6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2
*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2
*d^3 - c^3*d^2))*1i)/c^2)))/(c^2*f) + (atan(((((32*tan(e/2 + (f*x)/2)*(a^2*
c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a
^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c
^2*d^3 - c^3*d^2) + (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*
```

```
d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3
- c^4*d^2) - (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*
c^3 - 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 -
 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4
- 3*c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^
8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 +
b*c^3 - 2*a*c^2*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) + (((32*tan(
e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d -
 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^
3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(a*c^4*d^5 - b*c^9 - a*c^9 -
3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^
5*d + c^6 - c^3*d^3 - c^4*d^2) + (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^
3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c
^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 -
c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c
^3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d
)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c
^6*d^2))/((64*(a^3*d^5 + a*b^2*c^5 - a^2*b*c^5 - a^3*c*d^4 + 2*a^3*c^4*d -
3*a^3*c^2*d^3 + 2*a^3*c^3*d^2 + a^2*b*c^2*d^3 + a^2*b*c^3*d^2 - 3*a^2*b*c^4
*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (((32*tan(e/2 + (f*x)/2)*(a^2*c^6
+ 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c
^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d
^3 - c^3*d^2) + (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
- b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^
4*d^2) - (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3
- 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c
^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*
c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^8 -
c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^
3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) - (((32*tan(e/2 + (
f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*
c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^
4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6
*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*...
```

3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=204

$$\frac{ax}{c^3}+\frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f}-\frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+f$$

```
[Out] a*x/c^3+(b*c^3*(2*c^2+d^2)-a*d*(6*c^4-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)
*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f-1/2*d*(-a*d+
b*c)*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2-1/2*d*(-5*a*c^2*d+2*a*d^3+
3*b*c^3)*tan(f*x+e)/c^2/(c^2-d^2)^2/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.37, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4008, 4145, 4004, 3916, 2738, 214}

$$-\frac{d(bc-ad)\tan(e+fx)}{2cf\,(c^2-d^2)\,(c+d\sec(e+fx))^2}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{2c^2f\,(c^2-d^2)^2\,(c+d\sec(e+fx))}+\frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{ax}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*x)/c^3 + ((b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan
h[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(
5/2)*f) - (d*(b*c - a*d)*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*(c + d*Sec[e + f*
x])^2) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*d^3)*Tan[e + f*x])/(2*c^2*(c^2 - d^2
)^2*f*(c + d*Sec[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = -\frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{\int \frac{-2a(c^2-d^2)-2c(bc-ad)\sec(e+fx)+d(bc-ad)\sec^2}{(c+d\sec(e+fx))^2}}{2c\,(c^2-d^2)}$$

$$= -\frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx)}{2c^2\,(c^2-d^2)^2\,f(c+d\sec(e+fx))} +$$

$$= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx}{2c^2\,(c^2-d^2)^2\,f(c+d\sec(e+fx))}$$

$$= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx}{2c^2\,(c^2-d^2)^2\,f(c+d\sec(e+fx))}$$

$$= \frac{ax}{c^3} - \frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx}{2c^2\,(c^2-d^2)^2\,f(c+d\sec(e+fx))}$$

$$= \frac{ax}{c^3} + \frac{(2bc^5-6ac^4d+bc^3d^2+5ac^2d^3-2ad^5)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f}$$

Mathematica [A]
time = 1.32, size = 267, normalized size = 1.31

$$\frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))\left(2a(e+fx)(d+c\cos(e+fx))^2 - \frac{2(bc^3(2c^2+d^2)+ad(-6c^4+5c^2d^2-2d^4))\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^2}{(c^2-d^2)^{5/2}} + \frac{cd^2(bc-ad)\sin(e+fx)}{(c-d)(c+d)} - \frac{cd(4bc^3-6ac^2d-bcd^2+3ad^3)(d+c\cos(e+fx))\sin(e+fx)}{(c-d)^2(c+d)^2}\right)}{2c^3f(b+a\cos(e+fx))(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])*(2*a*(e + f*x)*(d
 + c*Cos[e + f*x])^2 - (2*(b*c^3*(2*c^2 + d^2) + a*d*(-6*c^4 + 5*c^2*d^2 -
2*d^4))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e +
 f*x])^2)/(c^2 - d^2)^(5/2) + (c*d^2*(b*c - a*d)*Sin[e + f*x])/((c - d)*(c
+ d)) - (c*d*(4*b*c^3 - 6*a*c^2*d - b*c*d^2 + 3*a*d^3)*(d + c*Cos[e + f*x])
*Sin[e + f*x])/((c - d)^2*(c + d)^2)))/(2*c^3*f*(b + a*Cos[e + f*x])*(c + d
*Sec[e + f*x])^3)
```

Maple [A]
time = 0.34, size = 287, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a*c^2*d+a*c*d^2-2*a
*d^3-4*b*c^3-b*c^2*d)*c*d/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*
```

```
c*(6*a*c^2*d-a*c*d^2-2*a*d^3-4*b*c^3+b*c^2*d)/(c+d)/(c-d)^2*tan(1/2*f*x+1/2
*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-1/2*(6*a*c^4*d-5
*a*c^2*d^3+2*a*d^5-2*b*c^5-b*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/
2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(196) = 392$.
time = 3.04, size = 1176, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2
+ 8*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 4*(a
*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x - (2*b*c^5*d^2 - 6*a*c^4*
d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*d^2
+ 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d^2 +
b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c
*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f
*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*
x + e) + d^2)) - 2*(3*b*c^6*d^2 - 5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*c^3*d^5 -
 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4 + b*c^3*d
^5 - 3*a*c^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^
4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7
)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(
a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a*c^
7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 2*(a*c^6*d^2
- 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x + (2*b*c^5*d^2 - 6*a*c^4*d^3 + b*c
^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*d^2 + 5*a*c^4
*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d^2 + b*c^4*d^3
 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^
```

```
2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (3*b*c^6*d^2 -
5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*c^3*d^5 - 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*
d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4 + b*c^3*d^5 - 3*a*c^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*
(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^
7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec(e + fx)}{(c + d\sec(e + fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(191) = 382.
time = 0.58, size = 457, normalized size = 2.24

$$\frac{\frac{(2bc^5-6ac^4d+bc^3d^2+5ac^2d^3-2ad^5)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^7-2c^5d^2+c^3d^4)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^3}+\frac{4bc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6ac^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3bc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+5ac^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-bc^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3acd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ad^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4bc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+6ac^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3bc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+5ac^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+bc^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3acd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2ad^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(c^6-2c^4d^2+c^2d^4)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*b*c^5 - 6*a*c^4*d + b*c^3*d^2 + 5*a*c^2*d^3 - 2*a*d^5)*(pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan
(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(-c^
2 + d^2)) + (f*x + e)*a/c^3 + (4*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 6*a*c^3*d
^2*tan(1/2*f*x + 1/2*e)^3 - 3*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a*c*d^4*tan
(1/2*f*x + 1/2*e)^3 - 2*a*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^4*d*tan(1/2*f*
x + 1/2*e) + 6*a*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*b*c^3*d^2*tan(1/2*f*x + 1
/2*e) + 5*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + b*c^2*d^3*tan(1/2*f*x + 1/2*e) -
 3*a*c*d^4*tan(1/2*f*x + 1/2*e) - 2*a*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c
^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c
- d)^2))/f
```

Mupad [**B**]
time = 11.35, size = 2500, normalized size = 12.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^3,x)

[Out] (2*a*atan(((a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3))/c^3 + (a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3))/c^3)/((16*(4*a^3*d^9 + 4*a*b^2*c^9 - 4*a^2*b*c^9 - 2*a^3*c*d^8 + 12*a^3*c^8*d - 18*a^3*c^2*d^7 + 13*a^3*c^3*d^6 + 36*a^3*c^4*d^5 - 26*a^3*c^5*d^4 - 34*a^3*c^6*d^3 + 24*a^3*c^7*d^2 + a*b^2*c^5*d^4 + 4*a*b^2*c^7*d^2 - 2*a^2*b*c^2*d^7 - 2*a^2*b*c^3*d^6 + 2*a^2*b*c^4*d^5 + 2*a^2*b*c^6*d^3 + 6*a^2*b*c^7*d^2 - 20*a^2*b*c^8*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3

```
- 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c
^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3)*1i)/c^3 + (a*((8*tan(e/2 + (f*x)
/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*
a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6
*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a
*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 -
 c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((
8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9
*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^
8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*
d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3
 - 3*c^11*d^2) + (a*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 +
 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^1
3*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5
 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3)*1i)/c^3)))/(c^3*f) - ((tan
(e/2 + (f*x)/2)^3*(2*a*d^4 - 6*a*c^2*d^2 + b*c^2*d^2 - a*c*d^3 + 4*b*c^3*d)
)/((c^2*d - c^3)*(c + d)^2) + (tan(e/2 + (f*x)/2)*(2*a*d^4 - 6*a*c^2*d^2 -
b*c^2*d^2 + a*c*d^3 + 4*b*c^3*d))/((c + d)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(
2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 -
2*c*d + d^2) + c^2 + d^2)) + (atan(((((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8
*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^
2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d
^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^
3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d...
```

3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{a^2x}{c^2}+\frac{2(bc-ad)\left(2ac^2-bcd-ad^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f}+\frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))}$$

```
[Out] a^2*x/c^2+2*(-a*d+b*c)*(2*a*c^2-a*d^2-b*c*d)*arctanh((c-d)^(1/2)*tan(1/2*f*
x+1/2*e)/(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f+(-a*d+b*c)^2*sin(f*x+e)
/c/(c^2-d^2)/f/(d+c*cos(f*x+e))
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4026, 2869, 2814, 2738, 214}

$$\frac{a^2x}{c^2}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)(c\cos(e+fx)+d)}+\frac{2(bc-ad)\left(2ac^2-ad^2-bcd\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[(Sqrt[c - d]
*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) + ((b*
c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^2}\,dx$$

$$= \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))} - \frac{\int \frac{-c(2abc-(a^2+b^2)d)-a^2(c^2-d^2)\cos(e+fx)}{d+c\cos(e+fx)}\,dx}{c\,(c^2-d^2)}$$

$$= \frac{a^2x}{c^2} + \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))} + \frac{(c^2(2abc-(a^2+b^2)\,d)-a^2d(c^2-d \cdots}{c^2\,(c^2-d^2)}$$

$$= \frac{a^2x}{c^2} + \frac{(bc-ad)^2\sin(e+fx)}{c\,(c^2-d^2)\,f(d+c\cos(e+fx))} + \frac{(2(c^2(2abc-(a^2+b^2)\,d)-a^2d(c^2- \cdots}{\cdots}$$

$$= \frac{a^2x}{c^2} + \frac{2(bc-ad)\,(2ac^2-bcd-ad^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc \cdots}{c\,(c^2- \cdots}$$

Mathematica [A]
time = 0.73, size = 136, normalized size = 1.02

$$\frac{a^2(e+fx) + \frac{2\left(-2abc^3+b^2c^2d+a^2\left(2c^2d-d^3\right)\right)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + \frac{c(bc-ad)^2\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(e + f*x) + (2*(-2*a*b*c^3 + b^2*c^2*d + a^2*(2*c^2*d - d^3))*ArcTanh[
((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (c*(b*c -
 a*d)^2*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x])))/(c^2*f)
```

Maple [A]
time = 0.25, size = 195, normalized size = 1.47

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(a^{2} d^{2}-2 a b d c+b^{2} c^{2}\right) c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)\left(c\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)-d\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{2\left(2 a^{2} c^{2} d-a^{2} d^{3}-2 a b\, c^{3}+b^{2} c^{2} d\right) \operatorname{arctanh}\left(\frac{(c-d) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d) \sqrt{(c+d)(c-d)}}}{c^{2}}+}{f}$
default	$\frac{\frac{-\frac{2\left(a^{2} d^{2}-2 a b d c+b^{2} c^{2}\right) c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)\left(c\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)-d\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{2\left(2 a^{2} c^{2} d-a^{2} d^{3}-2 a b\, c^{3}+b^{2} c^{2} d\right) \operatorname{arctanh}\left(\frac{(c-d) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d) \sqrt{(c+d)(c-d)}}}{c^{2}}+}{f}$
risch	$\frac{a^{2} x}{c^{2}}+\frac{2 i\left(a^{2} d^{2}-2 a b d c+b^{2} c^{2}\right)\left(d\, e^{i(f x+e)}+c\right)}{c^{2}\left(c^{2}-d^{2}\right) f\left(e^{2 i(f x+e)} c+2 d\, e^{i(f x+e)}+c\right)}+\frac{2 \ln\left(e^{i(f x+e)}-\frac{i c^{2}-i d^{2}-\sqrt{c^{2}-d^{2}}\, d}{\sqrt{c^{2}-d^{2}}\, c}\right) a^{2} d}{\sqrt{c^{2}-d^{2}}\,(c+d)(c-d) f}-\frac{\ln\left(e^{i(f x+e)}-\frac{i c^{2}}{}\right.}{\sqrt{c^{2}-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/c^2*(-(a^2*d^2-2*a*b*c*d+b^2*c^2)*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(c*
tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)-(2*a^2*c^2*d-a^2*d^3-2*a*b
*c^3+b^2*c^2*d)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1
/2*e)/((c+d)*(c-d))^(1/2)))+2*a^2/c^2*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(127) = 254.
time = 2.83, size = 687, normalized size = 5.17

$$\left[\frac{2\left(a^{2} c^{5}-2 a^{2} c^{3} d^{2}+a^{2} c d^{4}\right) f x \cos\left(f x+e\right)+2\left(a^{2} c^{4} d-2 a^{2} c^{2} d^{3}+a^{2} d^{5}\right) f x+\left(2 a b c^{3} d+a^{2} d^{4}-\left(2 a^{2}+b^{2}\right) c^{2} d^{2}+\left(2 a b c^{4}+a^{2} c d^{3}-\left(2 a^{2}+b^{2}\right) c^{3} d\right) \cos\left(f x+e\right)\right) \sqrt{c^{2}-d^{2}} \log\left(\frac{2 c d \cos\left(f x+e\right)-\left(c^{2}-2 d^{2}\right) \cos\left(f x+e\right)^{2}+2 \sqrt{c^{2}-d^{2}}\left(d \cos\left(f x+e\right)+c\right) \sin\left(f x+e\right)+2 c^{2}-d^{2}}{c^{2} \cos\left(f x+e\right)^{2}+2 c d \cos\left(f x+e\right)+d^{2}}\right)+2\left(b^{2} c^{5}-2 a b c^{4} d+2 a b c^{2} d^{3}-a^{2} c d^{4}+\left(a^{2}-b^{2}\right) c^{3} d^{2}\right) \sin\left(f x+e\right)}{2\left(\left(c^{7}-2 c^{5} d^{2}+c^{3} d^{4}\right) f \cos\left(f x+e\right)+\left(c^{6} d-2 c^{4} d^{3}+c^{2} d^{5}\right) f\right)}, \frac{\left(a^{2} c^{5}-2 a^{2} c^{3} d^{2}+a^{2} c d^{4}\right) f x \cos\left(f x+e\right)+\left(a^{2} c^{4} d-2 a^{2} c^{2} d^{3}+a^{2} d^{5}\right) f x+\left(2 a b c^{3} d+a^{2} d^{4}-\left(2 a^{2}+b^{2}\right) c^{2} d^{2}+\left(2 a b c^{4}+a^{2} c d^{3}-\left(2 a^{2}+b^{2}\right) c^{3} d\right) \cos\left(f x+e\right)\right) \sqrt{-c^{2}+d^{2}} \arctan\left(-\frac{\sqrt{-c^{2}+d^{2}}\left(d \cos\left(f x+e\right)+c\right)}{\left(c^{2}-d^{2}\right) \sin\left(f x+e\right)}\right)+\left(b^{2} c^{5}-2 a b c^{4} d+2 a b c^{2} d^{3}-a^{2} c d^{4}+\left(a^{2}-b^{2}\right) c^{3} d^{2}\right) \sin\left(f x+e\right)}{\left(c^{7}-2 c^{5} d^{2}+c^{3} d^{4}\right) f \cos\left(f x+e\right)+\left(c^{6} d-2 c^{4} d^{3}+c^{2} d^{5}\right) f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/2*(2*(a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + 2*(a^2*c^4*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + (a^2*c^4*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(e + fx)})^2}{(c + d\sec{(e + fx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)

[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**2, x)

Giac [A]
time = 0.57, size = 237, normalized size = 1.78

$$\frac{\frac{(fx+e)a^2}{c^2} + \frac{2\,(2\,abc^3 - 2\,a^2c^2d - b^2c^2d + a^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c + 2\,d) + \arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)}{\sqrt{-c^2 + d^2}}\right)\right)}{(c^4 - c^2d^2)\sqrt{-c^2 + d^2}} - \frac{2\,(b^2c^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 2\,abcd\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + a^2d^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right))}{(c^3 - cd^2)\left(c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 - d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 - c - d\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")

[Out] ((f*x + e)*a^2/c^2 + 2*(2*a*b*c^3 - 2*a^2*c^2*d - b^2*c^2*d + a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) - 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e) + a^2*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f

Mupad [B]
time = 10.20, size = 2500, normalized size = 18.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^2,x)
```

```
[Out] (2*a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^
5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4
*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^
2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (a^
2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a
^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^
8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (
a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7
*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2))/c
^2 + (a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^
4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b
^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*
b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (a^2*((32*(
2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d
^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*
a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*tan(
e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2
*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2))/c^2)/((64
*(a^6*d^5 - 2*a^5*b*c^5 - a^6*c*d^4 + 2*a^6*c^4*d + 4*a^4*b^2*c^5 - 3*a^6*c
^2*d^3 + 2*a^6*c^3*d^2 - 4*a^3*b^3*c^4*d - a^4*b^2*c*d^4 + a^4*b^2*c^4*d +
2*a^5*b*c^2*d^3 + 2*a^5*b*c^3*d^2 + a^2*b^4*c^3*d^2 - a^4*b^2*c^2*d^3 + 3*a
^4*b^2*c^3*d^2 - 6*a^5*b*c^4*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*(
(32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4
*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2
+ 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d -
 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (a^2*((32*(2*a^2*c^8*d
 - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^
5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3
 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a^2*tan(e/2 + (f*x)
/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*3
2i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2)*1i)/c^2 - (a^2*((32*t
an(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*
b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a
^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^
3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (a^2*((32*(2*a^2*c^8*d - a^
2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4
 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*
```

a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2)*1i)/c^2)))/(c^2*f) + (atan ((((a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c ^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^ 5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c ^3*d^3 - c^4*d^2) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6* d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2 *d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a* d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2 *a*c^2 + b*c*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) + ((a*d - b*c)* ((c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2* b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3* d^2) - (((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d ^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2* a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d^ 2) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^ 4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2*a*c^2 + b*c*d) *1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))/((64*(a^6*d^5 - 2*a^5*b*c^5 - a^6*c*d^4 + 2*a^6*c^4*d + 4*a^4*b^2*c^5 - 3*a^...

3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=237

$$\frac{a^2x}{c^3}-\frac{\left(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f}-\frac{d(bc-ad)^2}{2c^2\left(c^2-d^2\right)f(d}$$

```
[Out] a^2*x/c^3-(3*b^2*c^4*d-2*a*b*c^3*(2*c^2+d^2)+a^2*(6*c^4*d-5*c^2*d^3+2*d^5))
*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^
(5/2)/f-1/2*d*(-a*d+b*c)^2*sin(f*x+e)/c^2/(c^2-d^2)/f/(d+c*cos(f*x+e))^2-1/
2*(-a*d+b*c)*(3*a*d*(2*c^2-d^2)-b*c*(2*c^2+d^2))*sin(f*x+e)/c^2/(c^2-d^2)^2
/f/(d+c*cos(f*x+e))
```

Rubi [A]
time = 0.56, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4026, 3067, 3100, 2814, 2738, 214}

$$-\frac{\left(a^2(6c^4d-5c^2d^3+2d^5)-2abc^3(2c^2+d^2)+3b^2c^4d\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{a^2x}{c^3}-\frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)\sin(e+fx)}{2c^2f\left(c^2-d^2\right)^2(c\cos(e+fx)+d)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*x)/c^3 - ((3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^
2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(
c - d)^(5/2)*(c + d)^(5/2)*f) - (d*(b*c - a*d)^2*Sin[e + f*x])/(2*c^2*(c^2
- d^2)*f*(d + c*Cos[e + f*x])^2) - ((b*c - a*d)*(3*a*d*(2*c^2 - d^2) - b*c*
(2*c^2 + d^2))*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\cos(e+fx)(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^3}\,dx$$

$$= -\frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{\int \frac{-2c(bc-ad)^2+\left(b^2c^2d-2abc\left(2c^2-d^2\right)+a^2\left(2c^2d-d^3 \right.\right.}{(d+c\cos(e+f}}{2c^2\left(c^2-\right.}$$

$$= -\frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)}{2c^2\left(c^2-d^2\right)^2 f(d+c\cos(e+}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2\right.}{2c^2\left(c^2-d^2\right)^2 f(d+c\cos(}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2\right.}{2c^2\left(c^2-d^2\right)^2 f(d+c\cos(}$$

$$= \frac{a^2x}{c^3} + \frac{\left(4abc^5-6a^2c^4d-3b^2c^4d+2abc^3d^2+5a^2c^2d^3-2a^2d^5\right)\tanh^{-1}\left(\frac{\sqrt{c-d}}{\sqrt{}}\right.}{c^3(c-d)^{5/2}(c+d)^{5/2}f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 493 vs. 2(237) = 474.
time = 2.05, size = 493, normalized size = 2.08

$$\frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^2\left(\frac{4\left(3b^2c^4d-2abc^3\left(2c^2+d^2\right)+a^2\left(6c^4d-5c^2d^3+2d^5\right)\right)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^2}{\left(c^2-d^2\right)^{5/2}}+\frac{2a^2c^6e-6a^2c^2d^4e+4a^2d^6e+2a^2c^6fx-6a^2c^2d^4fx+4a^2d^6fx+8a^2cd\left(c^2-d^2\right)^2(e+fx)\cos(e+fx)+2a^2c^2\left(c^2-d^2\right)^2(e+fx)\cos(2(e+fx))+2b^2c^5d\sin(e+fx)-12abc^4d^2\sin(e+fx)+10a^2c^3d^3\sin(e+fx)+4b^2c^3d^3\sin(e+fx)-4a^2cd^5\sin(e+fx)+2b^2c^6\sin(2(e+fx))-8abc^5d\sin(2(e+fx))+6a^2c^4d^2\sin(2(e+fx))+b^2c^4d^2\sin(2(e+fx))+2abc^3d^3\sin(2(e+fx))-3a^2c^2d^4\sin(2(e+fx))}{\left(c^2-d^2\right)^2}\right)}{4c^3f(b+a\cos(e+fx))^2(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^2*((4*(3*b^2*c^4*d
- 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[((-c
 + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 - d^2
)^(5/2) + (2*a^2*c^6*e - 6*a^2*c^2*d^4*e + 4*a^2*d^6*e + 2*a^2*c^6*f*x - 6*
a^2*c^2*d^4*f*x + 4*a^2*d^6*f*x + 8*a^2*c*d*(c^2 - d^2)^2*(e + f*x)*Cos[e +
 f*x] + 2*a^2*c^2*(c^2 - d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^2*c^5*d*Si
n[e + f*x] - 12*a*b*c^4*d^2*Sin[e + f*x] + 10*a^2*c^3*d^3*Sin[e + f*x] + 4*
b^2*c^3*d^3*Sin[e + f*x] - 4*a^2*c*d^5*Sin[e + f*x] + 2*b^2*c^6*Sin[2*(e +
f*x)] - 8*a*b*c^5*d*Sin[2*(e + f*x)] + 6*a^2*c^4*d^2*Sin[2*(e + f*x)] + b^2
*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*c^2*d^4*
Sin[2*(e + f*x)])/(c^2 - d^2)^2))/(4*c^3*f*(b + a*Cos[e + f*x])^2*(c + d*Se
c[e + f*x])^3)
```

Maple [A]
time = 0.40, size = 386, normalized size = 1.63

method	result
derivativedivides	$\frac{\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2c\,d^3-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c^2d^2-a^2c\,d^3-2a^2d^4-8ab\,c^3\cdots\right.}{\cdots}\right.}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}\cdots}{\cdots}$
default	$\frac{\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2c\,d^3-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c^2d^2-a^2c\,d^3-2a^2d^4-8ab\,c^3\cdots\right.}{\cdots}\right.}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/c^3*((-1/2*(6*a^2*c^2*d^2+a^2*c*d^3-2*a^2*d^4-8*a*b*c^3*d-2*a*b*c^2*
d^2+2*b^2*c^4+b^2*c^3*d+2*b^2*c^2*d^2)*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+
1/2*e)^3+1/2*c*(6*a^2*c^2*d^2-a^2*c*d^3-2*a^2*d^4-8*a*b*c^3*d+2*a*b*c^2*d^2
+2*b^2*c^4-b^2*c^3*d+2*b^2*c^2*d^2)/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e))/(c*ta
n(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-1/2*(6*a^2*c^4*d-5*a^2*c^2
*d^3+2*a^2*d^5-4*a*b*c^5-2*a*b*c^3*d^2+3*b^2*c^4*d)/(c^4-2*c^2*d^2+d^4)/((c
+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))+2*a
^2/c^3*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(228) = 456$.
time = 3.08, size = 1433, normalized size = 6.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*x
 + e)^2 + 8*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*cos
(f*x + e) + 4*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*x -
 (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^
2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^5 - 3
*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3 + 5*a
^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sqrt(c^2
- d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2
- d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^
4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*
c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2
 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11
 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^
3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6
- c^3*d^8)*f), 1/2*(2*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^
6)*f*x*cos(f*x + e)^2 + 4*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*
c*d^7)*f*x*cos(f*x + e) + 2*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 -
a^2*d^8)*f*x + (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 -
 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a
^2*c^2*d^5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b
*c^4*d^3 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x +
 e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^4 + 2*a^2*c
*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*c^8 - 8*a*b
*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2 - b^2)*c^6
*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^
2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d
^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*
f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(e + fx))^2}{(c + d\sec(e + fx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(223) = 446.

time = 0.61, size = 658, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((4*a*b*c^5 - 6*a^2*c^4*d - 3*b^2*c^4*d + 2*a*b*c^3*d^2 + 5*a^2*c^2*d^3 - 2
*a^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*ta
n(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c
^5*d^2 + c^3*d^4)*sqrt(-c^2 + d^2)) + (f*x + e)*a^2/c^3 - (2*b^2*c^5*tan(1/
2*f*x + 1/2*e)^3 - 8*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - b^2*c^4*d*tan(1/2*f
*x + 1/2*e)^3 + 6*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*c^3*d^2*tan(1/
2*f*x + 1/2*e)^3 + b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^2*c^2*d^3*tan(1
/2*f*x + 1/2*e)^3 + 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 - 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^2*d^5*tan(1
/2*f*x + 1/2*e)^3 - 2*b^2*c^5*tan(1/2*f*x + 1/2*e) + 8*a*b*c^4*d*tan(1/2*f*
x + 1/2*e) - b^2*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e) + 6*a*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - b^2*c^3*d^2*tan(1/2*f*x + 1/2*
e) - 5*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e
) - 2*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e) +
 2*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*
x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 12.14, size = 2500, normalized size = 10.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^3,x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^3*(2*b^2*c^4 - 2*a^2*d^4 + a^2*c*d^3 + b^2*c^3*d + 6*a
^2*c^2*d^2 + 2*b^2*c^2*d^2 - 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c^2*d - c^3)*(
c + d)^2) - (tan(e/2 + (f*x)/2)*(2*a^2*d^4 - 2*b^2*c^4 + a^2*c*d^3 + b^2*c^
3*d - 6*a^2*c^2*d^2 - 2*b^2*c^2*d^2 + 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c + d
)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d
^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) - (2*a^2*atan(
((a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9
+ 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*
c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^
5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b
*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2
))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3
- 3*c^11*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9
```

+ 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))/c^3 - (a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 - (8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))/c^3)/((a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 48*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))*1i)/c^3 - (16*(4*a^6*d^9 - 8*a^5*b*c^9 - 2*a^6*c*d^8 + 12*a^6*c^8*d + 16*a^4*b^2*c^9 - 18*a^6*c^2*d^7 + 13*a^6*c^3*d^6 + 36*a^6*c^4*d^5 - 26*a^6*c^5*d^4 - 34*a^6*c^6*d^3 + 24*a^6*c^7*d^2 - 24*a^3*b^3*c^8*d + 6*a^4*b^2*c^8*d - 4*a^5*b*c^2*d^7 - 4*a^5*b*

```
c^3*d^6 + 4*a^5*b*c^4*d^5 + 4*a^5*b*c^6*d^3 + 12*a^5*b*c^7*d^2 + 9*a^2*b^4*
c^7*d^2 - 12*a^3*b^3*c^6*d^3 + 6*a^4*b^2*c^3*d^6 + 6*a^4*b^2*c^4*d^5 - 14*a
^4*b^2*c^5*d^4 - 12*a^4*b^2*c^6*d^3 + 46*a^4*b^2*c^7*d^2 - 40*a^5*b*c^8*d))
/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 -
3*c^11*d^2) + (a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4
*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10
*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 -
6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b
*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^...
```

3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=377

$$\frac{a^2x}{c^4}-\frac{\left(b^2c^4d(4c^2+d^2)-ab(4c^7+6c^5d^2)+a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4(c-d)^{7/2}(c+d)^{7/2}f}+$$

```
[Out] a^2*x/c^4-(b^2*c^4*d*(4*c^2+d^2)-a*b*(4*c^7+6*c^5*d^2)+a^2*(8*c^6*d-8*c^4*d
^3+7*c^2*d^5-2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^
4/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2-d^
2)/f/(d+c*cos(f*x+e))^3-1/6*d*(-a*d+b*c)*(-8*a*c^2*d+3*a*d^3+6*b*c^3-b*c*d^
2)*sin(f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(2*a*b*c*d*(18*c^4-5
*c^2*d^2+2*d^4)-a^2*d^2*(34*c^4-28*c^2*d^2+9*d^4)-b^2*(6*c^6+10*c^4*d^2-c^2
*d^4))*sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))
```

Rubi [A]
time = 1.37, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4026, 3127, 3110, 3100, 2814, 2738, 214}

$$-\frac{\left(-a^2d^2(34c^4-28c^2d^2+9d^4)+2abcd(18c^4-5c^2d^2+2d^4)-\left(b^2(6c^6+10c^4d^2-c^2d^4)\right)\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^3\left(c\cos(e+fx)+d\right)}-\frac{\left(a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7)-ab(4c^7+6c^5d^2)+b^2c^4d(4c^2+d^2)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4f(c-d)^{7/2}(c+d)^{7/2}}+\frac{a^2x}{c^4}+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}-\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{6c^3f\left(c^2-d^2\right)^2\left(c\cos(e+fx)+d\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*x)/c^4 - ((b^2*c^4*d*(4*c^2 + d^2) - a*b*(4*c^7 + 6*c^5*d^2) + a^2*(8*
c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/
2])/Sqrt[c + d]])/(c^4*(c - d)^(7/2)*(c + d)^(7/2)*f) + (d^2*(b + a*Cos[e +
 f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) - (d*(b*c
 - a*d)*(6*b*c^3 - 8*a*c^2*d - b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2
 - d^2)^2*f*(d + c*Cos[e + f*x])^2) - ((2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d
^4) - a^2*d^2*(34*c^4 - 28*c^2*d^2 + 9*d^4) - b^2*(6*c^6 + 10*c^4*d^2 - c^2
*d^4))*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^2}{(d+c\cos(e+fx))^4}\,dx$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))\left(-d(3bc-2ad)+\left(3bc^2-3acd-bd^2\right)\cos\ldots\right.}{(d+c\cos(e+fx))^3}}{3c\,(c^2-d^2)}$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\mathrm{s}\ldots}{6c^3\left(c^2-d^2\right)^2 f(d+c\cos(e+fx)\ldots}$$

$$= \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+3ad^3\right)\mathrm{s}\ldots}{6c^3\left(c^2-d^2\right)^2 f(d+c\cos(e+fx)\ldots}$$

$$= \frac{a^2x}{c^4} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+\ldots\right.}{6c^3\left(c^2-d^2\right)^2 f(d+c\cos(\ldots}$$

$$= \frac{a^2x}{c^4} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\left(6bc^3-8ac^2d-bcd^2+\ldots\right.}{6c^3\left(c^2-d^2\right)^2 f(d+c\cos(\ldots}$$

$$= \frac{a^2x}{c^4} + \frac{\left(4abc^7-8a^2c^6d-4b^2c^6d+6abc^5d^2+8a^2c^4d^3-b^2c^4d^3-7a^2c^2d^5+2a^2\ldots\right.}{c^4(c-d)^{7/2}(c+d)^{7/2}f}$$

Mathematica [A]
time = 3.31, size = 438, normalized size = 1.16

$$\frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^2\left(6a^2(e+fx)(d+c\cos(e+fx))^3+\frac{6\left(b^2c^4d\left(4c^2+d^2\right)-2abc^5\left(2c^2+3d^2\right)+a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)\right)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^3}{\left(c^2-d^2\right)^{7/2}}+\frac{2cd^2(bc-ad)^2\sin(e+fx)}{c^2-d^2}-\frac{cd\left(a^2d^2\left(12c^2-7d^2\right)+b^2\left(6c^4-c^2d^2\right)+ab\left(-18c^3d+8cd^3\right)\right)(d+c\cos(e+fx))\sin(e+fx)}{\left(c^2-d^2\right)^2}+\frac{c\left(-2abcd\left(18c^4-5c^2d^2+2d^4\right)+a^2d^2\left(36c^4-32c^2d^2+11d^4\right)+b^2\left(6c^6+10c^4d^2-c^2d^4\right)\right)(d+c\cos(e+fx))^2\sin(e+fx)}{\left(c^2-d^2\right)^3}\right)}{6c^4f(b+a\cos(e+fx))^2(c+d\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^2*(6*a^2*(e + f*x
)*(d + c*Cos[e + f*x])^3 + (6*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2 +
 3*d^2) + a^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[((-c + d)*
Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^(7/2
) + (2*c*d^2*(b*c - a*d)^2*Sin[e + f*x])/(c^2 - d^2) - (c*d*(a^2*d^2*(12*c^
2 - 7*d^2) + b^2*(6*c^4 - c^2*d^2) + a*b*(-18*c^3*d + 8*c*d^3))*(d + c*Cos[
```

```
e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-2*a*b*c*d*(18*c^4 - 5*c^2*d^2
+ 2*d^4) + a^2*d^2*(36*c^4 - 32*c^2*d^2 + 11*d^4) + b^2*(6*c^6 + 10*c^4*d^2
 - c^2*d^4))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*
(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^4)
```

Maple [A]
time = 0.60, size = 635, normalized size = 1.68

method	result
derivativedivides	$2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12abd\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+6b^2c^4d^2+b^2c^3d^3\right)c\left(\tan^5\left(\frac{fx}{2}\right.\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}\right.$
default	$2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12abd\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+6b^2c^4d^2+b^2c^3d^3\right)c\left(\tan^5\left(\frac{fx}{2}\right.\right.}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/c^4*((-1/2*(12*a^2*c^4*d^2+4*a^2*c^3*d^3-6*a^2*c^2*d^4-a^2*c*d^5+2*a
^2*d^6-12*a*b*c^5*d-6*a*b*c^4*d^2-4*a*b*c^3*d^3+2*b^2*c^6+2*b^2*c^5*d+6*b^2
*c^4*d^2+b^2*c^3*d^3)*c/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^
5+2/3*(18*a^2*c^4*d^2-11*a^2*c^2*d^4+3*a^2*d^6-18*a*b*c^5*d-2*a*b*c^3*d^3+3
*b^2*c^6+7*b^2*c^4*d^2)*c/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)^3-1/2*(12*a^2*c^4*d^2-4*a^2*c^3*d^3-6*a^2*c^2*d^4+a^2*c*d^5+2*a^2*d^6-12*
a*b*c^5*d+6*a*b*c^4*d^2-4*a*b*c^3*d^3+2*b^2*c^6-2*b^2*c^5*d+6*b^2*c^4*d^2-b
^2*c^3*d^3)*c/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(c*tan(1/
2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^3-1/2*(8*a^2*c^6*d-8*a^2*c^4*d^3
+7*a^2*c^2*d^5-2*a^2*d^7-4*a*b*c^7-6*a*b*c^5*d^2+4*b^2*c^6*d+b^2*c^4*d^3)/(
c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+
1/2*e)/((c+d)*(c-d))^(1/2)))+2*a^2/c^4*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. $2(370) = 740$.

time = 4.92, size = 2394, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*c
^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d^5
 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^2*c^9*d^2 - 4*a^
2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e) +
12*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*d^11)
*f*x - 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10 - 4*(2
*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^8*d^2 -
 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b^2)*c^7*
d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d^6 + 2*a^
2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(f*x + e)^2
 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^9 - 4*(2*a^
2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5 +
8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4 -
 (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3 -
 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^2
- (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(2
*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2*d
^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2)*
c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c
^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 -
4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^
6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d
^7 - 4*c^6*d^9 + c^4*d^11)*f), 1/6*(6*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7
*d^4 - 4*a^2*c^5*d^6 + a^2*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^2*c^10*d - 4
*a^2*c^8*d^3 + 6*a^2*c^6*d^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e
)^2 + 18*(a^2*c^9*d^2 - 4*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2
*c*d^10)*f*x*cos(f*x + e) + 6*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7
- 4*a^2*c^2*d^9 + a^2*d^11)*f*x + 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*
c^2*d^8 + 2*a^2*d^10 - 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4
*a*b*c^10 + 6*a*b*c^8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)
```

```
*c^9*d + (8*a^2 - b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7
*d^3 - 7*a^2*c^4*d^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b
^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*
d^7 + 2*a^2*c*d^9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*
x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^
2 - d^2)*sin(f*x + e))) + (2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5
+ 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4
 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3
 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^
2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*
(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2
*d^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2
)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4
*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5
- 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*
d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8
*d^7 - 4*c^6*d^9 + c^4*d^11)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. 2(362) = 724.
time = 0.64, size = 1201, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a*b*c^7 - 8*a^2*c^6*d - 4*b^2*c^6*d + 6*a*b*c^5*d^2 + 8*a^2*c^4*d
^3 - b^2*c^4*d^3 - 7*a^2*c^2*d^5 + 2*a^2*d^7)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^
2 + d^2)) + 3*(f*x + e)*a^2/c^4 - (6*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*
b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 6*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^
2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 54*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 +
12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^2*c^5*d^3*tan(1/2*f*x + 1/2*e)
```

```
^5 - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^2*c^5*d^3*tan(1/2*f*x + 1
/2*e)^5 - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*c^4*d^4*tan(1/2*f*x
+ 1/2*e)^5 + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^2*c^3*d^5*tan(1/2
*f*x + 1/2*e)^5 - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*c^3*d^5*tan
(1/2*f*x + 1/2*e)^5 - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^2*c*d^7*t
an(1/2*f*x + 1/2*e)^5 + 6*a^2*d^8*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c^8*tan(1
/2*f*x + 1/2*e)^3 + 72*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72*a^2*c^6*d^2*ta
n(1/2*f*x + 1/2*e)^3 - 16*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 64*a*b*c^5*d
^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 28*b^2
*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 56
*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*d^8*tan(1/2*f*x + 1/2*e)^3 + 6
*b^2*c^8*tan(1/2*f*x + 1/2*e) - 36*a*b*c^7*d*tan(1/2*f*x + 1/2*e) + 6*b^2*c
^7*d*tan(1/2*f*x + 1/2*e) + 36*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 54*a*b*c^
6*d^2*tan(1/2*f*x + 1/2*e) + 12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) + 60*a^2*c
^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e) + 27*b^2*
c^5*d^3*tan(1/2*f*x + 1/2*e) - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 6*a*b*c
^4*d^4*tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 45*a^2*
c^3*d^5*tan(1/2*f*x + 1/2*e) - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e) - 3*b^2*
c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e) + 15*a^2*
c*d^7*tan(1/2*f*x + 1/2*e) + 6*a^2*d^8*tan(1/2*f*x + 1/2*e))/((c^9 - 3*c^7*
d^2 + 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*
e)^2 - c - d)^3))/f
```

Mupad [B]
time = 15.07, size = 2500, normalized size = 6.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^4,x)
```

```
[Out] (2*a^2*atan(((a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c
*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11
+ 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 +
156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*
a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^
9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*
c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a
^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13
*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5
*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d
^3 - 5*c^15*d^2) + (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*
a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^
2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2
*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^1
```

1*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6
 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8
*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*
d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3
 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5
*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*
d^3 - 5*c^18*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*
d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^1
4*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*
d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9
 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^
14*d^3 - 5*c^15*d^2)))*1i)/c^4))/c^4 + (a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c
^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c
^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^
6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d
^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*
b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68
*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10
- 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^
2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11
- c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^
5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) - (a^2*((8*(4*a^2*c^21 - 16*a^2*
c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11
- 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 -
 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 1
2*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*
c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^1
8*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a
*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*
c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 -
c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d
^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a^2*tan(e/2 + (f*x)/2)*(8*c^
21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^1
0 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^
5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d
^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^1
2*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4))/c^4)/((16*(4*a^6
*d^13 - 8*a^5*b*c^13 - 2*a^6*c*d^12 + 16*a^6*c^12*d + 16*a^4*b^2*c^13 - 26*
a^6*c^2*d^11 + 11*a^6*c^3*d^10 + 70*a^6*c^4*d^9 - 34*a^6*c^5*d^8 - 110*a^6*
c^6*d^7 + 66*a^6*c^7*d^6 + 110*a^6*c^8*d^5 - 64*a^6*c^9*d^4 - 64*a^6*c^10*d
^3 + 48*a^6*c^11*d^2 - 32*a^3*b^3*c^12*d + 8*a^4*b^2*c^12*d + 12*a^5*b*c^4*
d^9 + 12*a^5*b*c^5*d^8 - 40*a^5*b*c^6*d^7 - 28*a^5*b*c^7*d^6 + 28*a^5*b*c^8
*d^5 + 12*a^5*b*c^9*d^4 - 44*a^5*b*c^10*d^3 + 12*a^5*b*c^11*d^2 + a^2*b^4*c
^7*d^6 + 8*a^2*b^4*c^9*d^4 + 16*a^2*b^4*c^11*d^2 - 12*a^3*b^3*c^8*d^5 - 56*
a^3*b^3*c^10*d^3 - 2*a^4*b^2*c^3*d^10 - 2*a^4*b^2*c^4*d^9 - 2*a^4*b^2*c^6*d

```
^7 + 22*a^4*b^2*c^7*d^6 + 18*a^4*b^2*c^8*d^5 + 10*a^4*b^2*c^9*d^4 - 22*a^4*
b^2*c^10*d^3 + 104*a^4*b^2*c^11*d^2 - 56*a^5*b*c^12*d))/(c^19*d + c^20 - c^
9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 +
10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^2*((8*tan(e/2 + (
f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2
*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*
d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5
 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c...
```

3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=254

$$\frac{a^3x}{c^3}-\frac{(bc-ad)\left(2abcd(4c^2-d^2)-b^2c^2(c^2+2d^2)-a^2(6c^4-5c^2d^2+2d^4)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f}+$$

```
[Out] a^3*x/c^3-(-a*d+b*c)*(2*a*b*c*d*(4*c^2-d^2)-b^2*c^2*(c^2+2*d^2)-a^2*(6*c^4-
5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(
c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)^2*(b+a*cos(f*x+e))*sin(f*x+e)/c/(c^
2-d^2)/f/(d+c*cos(f*x+e))^2+1/2*(-a*d+b*c)^2*(5*a*c^2-2*a*d^2-3*b*c*d)*sin(
f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))
```

Rubi [A]
time = 0.79, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4026, 2871, 3100, 2814, 2738, 214}

$$\frac{a^3x}{c^3}-\frac{(bc-ad)\left(-\left(a^2(6c^4-5c^2d^2+2d^4)\right)+2abcd(4c^2-d^2)-b^2c^2(c^2+2d^2)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}+\frac{(bc-ad)^2\left(5ac^2-2ad^2-3bcd\right)\sin(e+fx)}{2c^2f\left(c^2-d^2\right)^2\left(c\cos(e+fx)+d\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*x)/c^3 - ((b*c - a*d)*(2*a*b*c*d*(4*c^2 - d^2) - b^2*c^2*(c^2 + 2*d^2)
 - a^2*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/
Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) + ((b*c - a*d)^2*(b + a*C
os[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) + ((b
*c - a*d)^2*(5*a*c^2 - 3*b*c*d - 2*a*d^2)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^
2*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^3}\,dx$$

$$= \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2} + \frac{\int \frac{5ab^2c^2-4a^2bcd-2b^3cd+a^3d^2+(b^3c^2-2a^3c}{}}{}$$

$$= \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\,(5ac^2-3bcd-2ad^2)\,\mathrm{s}}{2c^2\,(c^2-d^2)^2\,f(d+c\cos(e+}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\,(5ac^2-3bcd-}{2c^2\,(c^2-d^2)^2\,f(d+c}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\,(5ac^2-3bcd-}{2c^2\,(c^2-d^2)^2\,f(d+c}$$

$$= \frac{a^3x}{c^3} + \frac{(bc-ad)\,(6a^2c^4+b^2c^4-8abc^3d-5a^2c^2d^2+2b^2c^2d^2+2abcd^3+2a^2d^4}{c^3(c-d)^{5/2}(c+d)^{5/2}f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 517 vs. 2(254) = 508.
time = 2.21, size = 517, normalized size = 2.04

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-4*(-9*a*b^2*c^4*d + 3*a^2*b*c^3*(2*c^2 + d^2) + b^3*c^3*(c^2 + 2*d^2) +
a^3*(-6*c^4*d + 5*c^2*d^3 - 2*d^5))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]])/(c^2 - d^2)^(5/2) + (2*a^3*c^6*e - 6*a^3*c^2*d^4*e + 4*a^3*d
^6*e + 2*a^3*c^6*f*x - 6*a^3*c^2*d^4*f*x + 4*a^3*d^6*f*x + 8*a^3*c*d*(c^2 -
 d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^3*(c^3 - c*d^2)^2*(e + f*x)*Cos[2*(e +
 f*x)] + 2*b^3*c^6*Sin[e + f*x] + 6*a*b^2*c^5*d*Sin[e + f*x] - 18*a^2*b*c^4
*d^2*Sin[e + f*x] - 8*b^3*c^4*d^2*Sin[e + f*x] + 10*a^3*c^3*d^3*Sin[e + f*x
] + 12*a*b^2*c^3*d^3*Sin[e + f*x] - 4*a^3*c*d^5*Sin[e + f*x] + 6*a*b^2*c^6*
Sin[2*(e + f*x)] - 12*a^2*b*c^5*d*Sin[2*(e + f*x)] - 3*b^3*c^5*d*Sin[2*(e +
 f*x)] + 6*a^3*c^4*d^2*Sin[2*(e + f*x)] + 3*a*b^2*c^4*d^2*Sin[2*(e + f*x)]
+ 3*a^2*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^3*c^2*d^4*Sin[2*(e + f*x)])/((c^2
- d^2)^2*(d + c*Cos[e + f*x])^2))/(4*c^3*f)
```

Maple [A]
time = 0.51, size = 458, normalized size = 1.80

method	result
derivativedivides	$\frac{2\left(-\frac{\left(6a^3c^2d^2+a^3c\,d^3-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^3c^2d^2-\right.}{}\right.}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\right.}$
default	$\frac{2\left(-\frac{\left(6a^3c^2d^2+a^3c\,d^3-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^3c^2d^2-\right.}{}\right.}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/c^3*((-1/2*(6*a^3*c^2*d^2+a^3*c*d^3-2*a^3*d^4-12*a^2*b*c^3*d-3*a^2*b
*c^2*d^2+6*a*b^2*c^4+3*a*b^2*c^3*d+6*a*b^2*c^2*d^2-b^3*c^4-4*b^3*c^3*d)*c/(
c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*c*(6*a^3*c^2*d^2-a^3*c*d^3-2*
a^3*d^4-12*a^2*b*c^3*d+3*a^2*b*c^2*d^2+6*a*b^2*c^4-3*a*b^2*c^3*d+6*a*b^2*c^
2*d^2+b^3*c^4-4*b^3*c^3*d)/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x
+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-1/2*(6*a^3*c^4*d-5*a^3*c^2*d^3+2*a^
3*d^5-6*a^2*b*c^5-3*a^2*b*c^3*d^2+9*a*b^2*c^4*d-b^3*c^5-2*b^3*c^3*d^2)/(c^4
-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)
*(c-d))^(1/2)))+2*a^3/c^3*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(244) = 488$.
time = 3.80, size = 1653, normalized size = 6.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*f*x*cos(f*x
 + e)^2 + 8*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7)*f*x*cos
(f*x + e) + 4*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3*d^8)*f*x -
 (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 + 3*a*b^2)
*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2*d^5 + (6*
a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)*c^5*d^2)*c
os(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)*c^6*d - 3*
(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + e))*sqrt(c
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e
)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7
 - (9*a^2*b + 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3
)*c^4*d^4 - (7*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^
3*c^2*d^6 - (4*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^
3)*c^5*d^3 - (3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 -
3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 +
 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c
^3*d^8)*f), 1/2*(2*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*
f*x*cos(f*x + e)^2 + 4*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d
^7)*f*x*cos(f*x + e) + 2*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3
*d^8)*f*x + (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3
 + 3*a*b^2)*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^
2*d^5 + (6*a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)
*c^5*d^2)*cos(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)
*c^6*d - 3*(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x +
 e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7 - (9*a^2*b +
5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3)*c^4*d^4 - (7
*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^3*c^2*d^6 - (4
*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^3)*c^5*d^3 - (
3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*
c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c
^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(e + fx))^3}{(c + d\sec(e + fx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(238) = 476.
time = 0.63, size = 818, normalized size = 3.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/c^3 + (6*a^2*b*c^5 + b^3*c^5 - 6*a^3*c^4*d - 9*a*b^2*c^4*d +
 3*a^2*b*c^3*d^2 + 2*b^3*c^3*d^2 + 5*a^3*c^2*d^3 - 2*a^3*d^5)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*
tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(
-c^2 + d^2)) - (6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)^3 - b^3*c^5*tan(1/2*f*x +
1/2*e)^3 - 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^4*d*tan(1/2*f*
x + 1/2*e)^3 - 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*c^3*d^2*tan(1/2*f
*x + 1/2*e)^3 + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*a*b^2*c^3*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^3*c^2*d^3
*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^3
*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^5*tan(1/2*f*x + 1/2*e) - b^3*c^5*ta
n(1/2*f*x + 1/2*e) + 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^4*d*ta
n(1/2*f*x + 1/2*e) + 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^3*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^3*d^2*ta
n(1/2*f*x + 1/2*e) + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 5*a^3*c^2*d^3*tan
(1/2*f*x + 1/2*e) - 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 6*a*b^2*c^2*d^3*
tan(1/2*f*x + 1/2*e) + 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e) + 2*a^3*d^5*tan(1/2
*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*t
an(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 14.50, size = 2500, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^3,x)
```

```
[Out] (atan(((((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*
c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 +
32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*
c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3
 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^
3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b
```

^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + ((a*d - b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*1i)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - ((a*d - b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c -

```
d)^5)^(1/2)*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^
2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^
6 + 10*c^9*d^4 - 5*c^11*d^2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a
^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3
- 8*a*b*c^3*d)*1i)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^
4 - 5*c^11*d^2)))/((16*(4*a^9*d^9 - 12*a^8*b*c^9 - 2*a^9*c*d^8 + 12*a^9*c^8
*d + a^3*b^6*c^9 + 12*a^5*b^4*c^9 - 2*a^6*b^3*c^9 + 36*a^7*b^2*c^9 - 18*a^9
*c^2*d^7 + 13*a^9*c^3*d^6 + 36*a^9*c^4*d^5 - 26*a^9*c^5*d^4 - 34*a^9*c^6*d^
3 + 24*a^9*c^7*d^2 - 18*a^4*b^5*c^8*d - 118*a^6*b^3*c^8*d + 18*a^7*b^2*c^8*
d - 6*a^8*b*c^2*d^7 - 6*a^8*b*c^3*d^6 + 6*a^8*b*c^4*d^5 + 6*a^8*b*c^6*d^3 +
 18*a^8*b*c^7*d^2 + 4*a^3*b^6*c^5*d^4 + 4*a^3*b^6*c^7*d^2 - 36*a^4*b^5*c^6*
d^3 + 12*a^5*b^4*c^5*d^4 + 111*a^5*b^4*c^7*d^2 - 4*a^6*b^3*c^2*d^7 - 4*a^6*
b^3*c^3*d^6 + 10*a^6*b^3*c^4*d^5 + 6*a^6*b^3*c^5*d^4 - 68*a^6*b^3*c^6*d^3 +
 18*a^7*b^2*c^3*d^6 + 18*a^7*b^2*c^4*d^5 - 45*a^7*b^2*c^5*d^4 - 36*a^7*b^2*
c^6*d^3 + 126*a^7*b^2*c^7*d^2 - 60*a^8*b*c^8*d))/(c^12*d + c^13 - c^6*d^7 -
 c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 -...
```

3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=412

$$\frac{a^3x}{c^4}-\frac{\left(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7)\right)\tanh^{-1}\left(\sqrt{\right.}{c^4\sqrt{c-d}\,\sqrt{c+d}\,(c^2-d^2)^3 f}$$

```
[Out] a^3*x/c^4-1/3*d*(-a*d+b*c)*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2-d^2)/f/(d+c
*cos(f*x+e))^3+1/6*(-a*d+b*c)^2*(-8*a*c^2*d+3*a*d^3+3*b*c^3+2*b*c*d^2)*sin(
f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(-a*d+b*c)*(b^2*c^2*d*(13*c
^2+2*d^2)-a*b*c*(18*c^4+17*c^2*d^2-5*d^4)+a^2*(34*c^4*d-28*c^2*d^3+9*d^5))*
sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))-(3*a*b^2*c^4*d*(4*c^2+d^2)-b^
3*c^5*(c^2+4*d^2)-a^2*b*(6*c^7+9*c^5*d^2)+a^3*(8*c^6*d-8*c^4*d^3+7*c^2*d^5-
2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^4/(c^2-d^2)^3
/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4026, 3068, 3110, 3100, 2814, 2738, 214}

$$\frac{a^3x}{c^4}-\frac{(bc-ad)\left(a^2(34c^4d-28c^2d^3+9d^5)-abc(18c^4+17c^2d^2-5d^4)+b^2c^2d(13c^2+2d^2)\right)\sin(e+fx)}{6c^3f\,(c^2-d^2)^3\,(c\cos(e+fx)+d)}-\frac{\left(a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7)-a^2b(6c^7+9c^5d^2)+3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4f\sqrt{c-d}\,\sqrt{c+d}\,(c^2-d^2)^3}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\,(c^2-d^2)\,(c\cos(e+fx)+d)^3}+\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{6c^3f\,(c^2-d^2)^2\,(c\cos(e+fx)+d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*x)/c^4 - ((3*a*b^2*c^4*d*(4*c^2 + d^2) - b^3*c^5*(c^2 + 4*d^2) - a^2*b
*(6*c^7 + 9*c^5*d^2) + a^3*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTa
nh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^4*Sqrt[c - d]*Sqrt[c + d
]*(c^2 - d^2)^3*f) - (d*(b*c - a*d)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(3
*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) + ((b*c - a*d)^2*(3*b*c^3 - 8*a*c^
2*d + 2*b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/(6*c^3*(c^2 - d^2)^2*f*(d + c*Cos[
e + f*x])^2) - ((b*c - a*d)*(b^2*c^2*d*(13*c^2 + 2*d^2) - a*b*c*(18*c^4 + 1
7*c^2*d^2 - 5*d^4) + a^2*(34*c^4*d - 28*c^2*d^3 + 9*d^5))*Sin[e + f*x])/(6*
c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \int \frac{\cos(e+fx)(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^4}\,dx$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{\int \frac{(b+a\cos(e+fx))((3bc-2ad)(bc-ad)-}{}}{}$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d+2bc\right.}{6c^3\left(c^2-d^2\right)^2 f(d+c}$$

$$= -\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d+2bc\right.}{6c^3\left(c^2-d^2\right)^2 f(d+c}$$

$$= \frac{a^3x}{c^4} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d\right.}{6c^3\left(c^2-d^2\right)^2 f(}$$

$$= \frac{a^3x}{c^4} - \frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^3} + \frac{(bc-ad)^2\left(3bc^3-8ac^2d\right.}{6c^3\left(c^2-d^2\right)^2 f(}$$

$$= \frac{a^3x}{c^4} - \frac{\left(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c\right.}{c^4\sqrt{c-d}\,\sqrt{c+d}\,\left(c^2-d\right.}$$

Mathematica [A]
time = 3.77, size = 459, normalized size = 1.11

$$\frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^3\left(6a^3(e+fx)(d+c\cos(e+fx))^3-\frac{6\left(-3ab^2c^4d(4c^2+d^2)+b^3c^5(c^2+4d^2)+a^2b(6c^7+9c^5d^2)+a^3(-8c^6d+8c^4d^3-7c^2d^5+2d^7)\right)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^3}{(c^2-d^2)^{7/2}}-\frac{2cd(bc-ad)^3\sin(e+fx)}{c^2-d^2}+\frac{c(bc-ad)^2(3bc^3-12ac^2d+2bcd^2+7ad^3)(d+c\cos(e+fx))\sin(e+fx)}{(c^2-d^2)^2}+\frac{c\left(-b^3c^3d(13c^2+2d^2)+3ab^2c^2(6c^4+10c^2d^2-d^4)-3a^2bcd(18c^4-5c^2d^2+2d^4)+a^3(36c^4d^2-32c^2d^4+11d^6)\right)(d+c\cos(e+fx))^2\sin(e+fx)}{(c^2-d^2)^3}\right)}{6c^4f(b+a\cos(e+fx))^3(c+d\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^3*(6*a^3*(e + f*x)*
(d + c*Cos[e + f*x])^3 - (6*(-3*a*b^2*c^4*d*(4*c^2 + d^2) + b^3*c^5*(c^2 +
4*d^2) + a^2*b*(6*c^7 + 9*c^5*d^2) + a^3*(-8*c^6*d + 8*c^4*d^3 - 7*c^2*d^5
+ 2*d^7))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e
 + f*x])^3)/(c^2 - d^2)^(7/2) - (2*c*d*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d
```

```
^2) + (c*(b*c - a*d)^2*(3*b*c^3 - 12*a*c^2*d + 2*b*c*d^2 + 7*a*d^3)*(d + c*
Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-(b^3*c^3*d*(13*c^2 + 2*d^2
)) + 3*a*b^2*c^2*(6*c^4 + 10*c^2*d^2 - d^4) - 3*a^2*b*c*d*(18*c^4 - 5*c^2*d
^2 + 2*d^4) + a^3*(36*c^4*d^2 - 32*c^2*d^4 + 11*d^6))*(d + c*Cos[e + f*x])^
2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[
e + f*x])^4)
```

Maple [A]

time = 0.75, size = 785, normalized size = 1.91

method	result
derivativedivides	$\frac{\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\,b^2c^5}{2(c-d)\left(c^3+3c^2d+3c\,d^2+\right.}}{}}{}$
default	$\frac{\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\,b^2c^5}{2(c-d)\left(c^3+3c^2d+3c\,d^2+\right.}}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*a^3/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^3*c^4*d^2+4*a^
3*c^3*d^3-6*a^3*c^2*d^4-a^3*c*d^5+2*a^3*d^6-18*a^2*b*c^5*d-9*a^2*b*c^4*d^2-
6*a^2*b*c^3*d^3+6*a*b^2*c^6+6*a*b^2*c^5*d+18*a*b^2*c^4*d^2+3*a*b^2*c^3*d^3-
b^3*c^6-6*b^3*c^5*d-2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c/(c-d)/(c^3+3*c^2*d+3*c*d
^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^3*c^4*d^2-11*a^3*c^2*d^4+3*a^3*d^6-2
7*a^2*b*c^5*d-3*a^2*b*c^3*d^3+9*a*b^2*c^6+21*a*b^2*c^4*d^2-7*b^3*c^5*d-3*b^
3*c^3*d^3)*c/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*(12*a
^3*c^4*d^2-4*a^3*c^3*d^3-6*a^3*c^2*d^4+a^3*c*d^5+2*a^3*d^6-18*a^2*b*c^5*d+9
*a^2*b*c^4*d^2-6*a^2*b*c^3*d^3+6*a*b^2*c^6-6*a*b^2*c^5*d+18*a*b^2*c^4*d^2-3
*a*b^2*c^3*d^3+b^3*c^6-6*b^3*c^5*d+2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c/(c+d)/(c^
3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/
2*f*x+1/2*e)^2-c-d)^3-1/2*(8*a^3*c^6*d-8*a^3*c^4*d^3+7*a^3*c^2*d^5-2*a^3*d^
7-6*a^2*b*c^7-9*a^2*b*c^5*d^2+12*a*b^2*c^6*d+3*a*b^2*c^4*d^3-b^3*c^7-4*b^3*
c^5*d^2)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*ta
n(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(404) = 808$.
time = 4.18, size = 2808, normalized size = 6.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*c
^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d^5
 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^3*c^9*d^2 - 4*a^
3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e) +
12*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*d^11)
*f*x + 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2*a^3 +
 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^4*d^6 +
 (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 + 3*a*b^2
)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*cos(f*x +
e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d + 4*(2*a^3
+ 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^2)*c^6*d^4)
*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b^3)*c^8*d^2
+ 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8*a^3 - 3*a*b^
2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c
^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3
)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*
a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^
2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b
^2)*c^9*d^2 + (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (
21*a^2*b - 2*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*
(b^3*c^11 + 6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 +
(20*a^3 + 21*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b
^2)*c^6*d^5 + (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f
*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d
^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^
6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8
```

```
+ c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9
 + c^4*d^11)*f), 1/6*(6*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c
^5*d^6 + a^3*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^3*c^10*d - 4*a^3*c^8*d^3 +
 6*a^3*c^6*d^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 18*(a^3*
c^9*d^2 - 4*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*c
os(f*x + e) + 6*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^
9 + a^3*d^11)*f*x - 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3
 + 4*(2*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b
^2)*c^4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*
a^3 + 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^
3)*cos(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*
d + 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*
b^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b +
b^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8
*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^3*c^10*d + 6*a*
b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3)*c^8*d^3 +
(26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*a^3 + 39*a*b
^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^2*b*c^4*d^7
- 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b^2)*c^9*d^2
+ (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (21*a^2*b - 2
*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(b^3*c^11 +
6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 + (20*a^3 + 21
*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b^2)*c^6*d^5
+ (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f*x + e))*sin
(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*
x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos
(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*
f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)
*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. 2(396) = 792.
time = 0.67, size = 1572, normalized size = 3.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(3*(6*a^2*b*c^7 + b^3*c^7 - 8*a^3*c^6*d - 12*a*b^2*c^6*d + 9*a^2*b*c^5*d^2 + 4*b^3*c^5*d^2 + 8*a^3*c^4*d^3 - 3*a*b^2*c^4*d^3 - 7*a^3*c^2*d^5 + 2*a^3*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^2 + d^2)) + 3*(f*x + e)*a^3/c^4 - (18*a*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 3*b^3*c^8*tan(1/2*f*x + 1/2*e)^5 - 54*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 12*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 27*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 81*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 9*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^5 + 6*a^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*b^2*c^8*tan(1/2*f*x + 1/2*e)^3 + 108*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 28*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 48*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 96*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 84*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 12*b^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 56*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^3*d^8*tan(1/2*f*x + 1/2*e)^3 + 18*a*b^2*c^8*tan(1/2*f*x + 1/2*e) + 3*b^3*c^8*tan(1/2*f*x + 1/2*e) - 54*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e) + 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e) - 12*b^3*c^7*d*tan(1/2*f*x + 1/2*e) + 36*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) - 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e) + 36*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 27*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 60*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e) + 81*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 6*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e) - 9*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e) + 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) - 6*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e) - 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*e) - 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*b^3*c^3*d^5*tan(1/2*f*x + 1/2*e) - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e) + 15*a^3*c*d^7*tan(1/2*f*x + 1/2*e) + 6*a^3*d^8*tan(1/2*f*x + 1/2*e))/((c^9 - 3*c^7*d^2 + 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f

Mupad [**B**]

time = 16.09, size = 2500, normalized size = 6.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^4,x)

[Out] ((tan(e/2 + (f*x)/2)^5*(b^3*c^6 - 2*a^3*d^6 - 6*a*b^2*c^6 + a^3*c*d^5 + 6*b^3*c^5*d + 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 - 12*a^3*c^4*d^2 + 2*b^3*c^3*d^3 + 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 - 18*a*b^2*c^4*d^2 + 6*a^2*b*c^3*d^3 + 9*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d + 18*a^2*b*c^5*d))/((c^3*d - c^4)*(c + d)^3) + (4*tan(e/2 + (f*x)/2)^3*(7*b^3*c^5*d - 9*a*b^2*c^6 - 3*a^3*d^6 + 11*a^3*c^2*d^4 - 18*a^3*c^4*d^2 + 3*b^3*c^3*d^3 - 21*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + 27*a^2*b*c^5*d))/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) - (tan(e/2 + (f*x)/2)*(2*a^3*d^6 + b^3*c^6 + 6*a*b^2*c^6 + a^3*c*d^5 - 6*b^3*c^5*d - 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 + 12*a^3*c^4*d^2 - 2*b^3*c^3*d^3 + 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 18*a*b^2*c^4*d^2 - 6*a^2*b*c^3*d^3 + 9*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d - 18*a^2*b*c^5*d))/((c + d)*(3*c^5*d - c^6 + c^3*d^3 - 3*c^4*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) - (2*a^3*atan(((a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^3*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4 + (8*tan(e/2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 - 8*a^6*c^13*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6*c^3*d^11 + 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^10*d^4 + 48*a^6*c^11*d^3 + 44*a^6*c^12*d^2 + 16*b^6*c^10*d^4 + 8*b^6*c^12*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^11*d^3 - 160*a^3*b^3*c^13*d + 36*a^5*b*c^5*d^9

- 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^11*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^10*d^4 + 210*a^2*b^4*c^12*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^11*d^3 - 12*a^4*b^2*c^4*d^10 - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^10*d^4 + 300*a^4*b^2*c^12*d^2 - 24*a*b^5*c^13*d - 96*a^5*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))/c^4 - (a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^7 + 54*a*b^2*c^15*d^6 - 54*a*b^2*c^16*d^5 - 66*a*b^2*c^17*d^4 + 66*a*b^2*c^18*d^3 + 24*a*b^2*c^19*d^2 + 18*a^2*b*c^12*d^9 - 18*a^2*b*c^13*d^8 - 42*a^2*b*c^14*d^7 + 42*a^2*b*c^15*d^6 + 18*a^2*b*c^16*d^5 - 18*a^2*b*c^17*d^4 + 18*a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a^3*tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i)/(c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i)/c^4 - (8*tan(e/2 + (f*x)/2)*(4*a^6*c^14 + 8*a^6*d^14 + b^6*c^14 - 8*a^6*c*d^13 - 8*a^6*c^13*d + 12*a^2*b^4*c^14 + 36*a^4*b^2*c^14 - 48*a^6*c^2*d^12 + 48*a^6*c^3*d^11 + 117*a^6*c^4*d^10 - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^1...

3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=622

$$\frac{a^3x}{c^5}-\frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40c^6d^3+6}{4c^5\sqrt{c-d}\,\sqrt{c+d}\,(c^2-d^2)^4 f}$$

```
[Out] a^3*x/c^5+1/4*d^2*(b+a*cos(f*x+e))^3*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+
e))^4-1/12*d*(-11*a*c^2*d+4*a*d^3+8*b*c^3-b*c*d^2)*(b+a*cos(f*x+e))^2*sin(f
*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^3-1/24*(-a*d+b*c)*(2*a*b*c*d*(32*c
^4+c^2*d^2+2*d^4)-a^2*d^2*(58*c^4-35*c^2*d^2+12*d^4)-b^2*(12*c^6+25*c^4*d^2
-2*c^2*d^4))*sin(f*x+e)/c^4/(c^2-d^2)^3/f/(d+c*cos(f*x+e))^2-1/24*(b^3*c^3*
d*(68*c^4+39*c^2*d^2-2*d^4)+a^2*b*c*d*(272*c^6+10*c^4*d^2+49*c^2*d^4-16*d^6
)-3*a*b^2*c^2*(24*c^6+84*c^4*d^2-5*c^2*d^4+2*d^6)-a^3*(212*c^6*d^2-210*c^4*
d^4+139*c^2*d^6-36*d^8))*sin(f*x+e)/c^4/(c^2-d^2)^4/f/(d+c*cos(f*x+e))-1/4*
(15*a*b^2*c^6*d*(4*c^2+3*d^2)-3*a^2*b*c^5*(8*c^4+24*c^2*d^2+3*d^4)-b^3*c^5*
(4*c^4+27*c^2*d^2+4*d^4)+a^3*(40*c^8*d-40*c^6*d^3+63*c^4*d^5-36*c^2*d^7+8*d
^9))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^5/(c^2-d^2)^4/f/
(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 1.23, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4026, 3127, 3126, 3110, 3100, 2814, 2738, 214}

$$\frac{a^3x}{c^5}-\frac{(bc-ad)\left(-a^2d^2(58c^4-35c^2d^2+12d^4)+2abcd(32c^4+c^2d^2+2d^4)-(b^2(12c^6+25c^4d^2-2c^2d^4))\right)\sin(e+fx)}{24c^4f(c^2-d^2)^3(c\cos(e+fx)+d)^2}-\frac{(-(a^3(212c^6d^2-210c^4d^4+139c^2d^6-36d^8))+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2-5c^2d^4+2d^6)+b^3c^3d(68c^4+39c^2d^2-2d^4))\sin(e+fx)}{24c^4f(c^2-d^2)^4(c\cos(e+fx)+d)}-\frac{(a^3(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9)-3a^2bc^5(8c^4+24c^2d^2+3d^4)+15ab^2c^6d(4c^2+3d^2)-b^3c^5(4c^4+27c^2d^2+4d^4))\tanh^{-1}\left(\frac{\sqrt{c-d}\tan(\frac{1}{2}(e+fx))}{\sqrt{c+d}}\right)}{4c^5f\sqrt{c-d}\sqrt{c+d}(c^2-d^2)^4}+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf(c^2-d^2)(c\cos(e+fx)+d)^4}-\frac{d(-11ac^2d+4ad^3+8bc^3-bcd^2)\sin(e+fx)(a\cos(e+fx)+b)^2}{12c^2f(c^2-d^2)^2(c\cos(e+fx)+d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*x)/c^5 - ((15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^
2*d^2 + 3*d^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*
c^6*d^3 + 63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f
*x)/2])/Sqrt[c + d]])/(4*c^5*Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)^4*f) + (d^
2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e + f*
x])^4) - (d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e + f*x])
^2*Sin[e + f*x])/(12*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])^3) - ((b*c -
a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*c^2*d^2 +
 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(24*c^4*(c^
2 - d^2)^3*f*(d + c*Cos[e + f*x])^2) - ((b^3*c^3*d*(68*c^4 + 39*c^2*d^2 - 2
*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^4 - 16*d^6) - 3*a*b^2*c^
2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*(212*c^6*d^2 - 210*c^4*d^
4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/(24*c^4*(c^2 - d^2)^4*f*(d + c*Cos
[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
```

```
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^3}{(d+c\cos(e+fx))^5}\,dx$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} + \frac{\int \frac{(b+a\cos(e+fx))^2(-d(4bc-3ad)+(4bc^2-4acd-bd^2)}{(d+c\cos(e+fx))}}{4c\left(c^2-d^2\right.}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\left(b+a\,\mathrm{co}\right.}{12c^2\left(c^2-d^2\right)^2f(d+c\cos}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\left(b+a\,\mathrm{co}\right.}{12c^2\left(c^2-d^2\right)^2f(d+c\cos}$$

$$= \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\left(b+a\,\mathrm{co}\right.}{12c^2\left(c^2-d^2\right)^2f(d+c\cos}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(}{12c^2\left(c^2-d^2\right)^2f(d}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\left(c^2-d^2\right)f(d+c\cos(e+fx))^4} - \frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(}{12c^2\left(c^2-d^2\right)^2f(d}$$

$$= \frac{a^3x}{c^5} - \frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2}{4c^5\sqrt{c-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1285 vs. $2(622) = 1244$.
time = 6.82, size = 1285, normalized size = 2.07

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*(e + f*x)*(d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3
)/(c^5*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((-24*a^2*b*c^9 -
 4*b^3*c^9 + 40*a^3*c^8*d + 60*a*b^2*c^8*d - 72*a^2*b*c^7*d^2 - 27*b^3*c^7*
d^2 - 40*a^3*c^6*d^3 + 45*a*b^2*c^6*d^3 - 9*a^2*b*c^5*d^4 - 4*b^3*c^5*d^4 +
 63*a^3*c^4*d^5 - 36*a^3*c^2*d^7 + 8*a^3*d^9)*ArcTanh[((-c + d)*Tan[(e + f*
x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e
+ f*x])^3)/(4*c^5*Sqrt[c^2 - d^2]*(-c^2 + d^2)^4*f*(b + a*Cos[e + f*x])^3*(
c + d*Sec[e + f*x])^5) + ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e
+ f*x])^3*(b^3*c^3*d^2*Sin[e + f*x] - 3*a*b^2*c^2*d^3*Sin[e + f*x] + 3*a^2*
```

```
b*c*d^4*Sin[e + f*x] - a^3*d^5*Sin[e + f*x]))/(4*c^4*(c^2 - d^2)*f*(b + a*C
os[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((d + c*Cos[e + f*x])^2*Sec[e + f*
x]^2*(a + b*Sec[e + f*x])^3*(-8*b^3*c^5*d*Sin[e + f*x] + 36*a*b^2*c^4*d^2*S
in[e + f*x] - 48*a^2*b*c^3*d^3*Sin[e + f*x] + b^3*c^3*d^3*Sin[e + f*x] + 20
*a^3*c^2*d^4*Sin[e + f*x] - 15*a*b^2*c^2*d^4*Sin[e + f*x] + 27*a^2*b*c*d^5*
Sin[e + f*x] - 13*a^3*d^6*Sin[e + f*x]))/(12*c^4*(c^2 - d^2)^2*f*(b + a*Cos
[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((d + c*Cos[e + f*x])^3*Sec[e + f*x]
^2*(a + b*Sec[e + f*x])^3*(12*b^3*c^7*Sin[e + f*x] - 108*a*b^2*c^6*d*Sin[e
+ f*x] + 216*a^2*b*c^5*d^2*Sin[e + f*x] + 25*b^3*c^5*d^2*Sin[e + f*x] - 120
*a^3*c^4*d^3*Sin[e + f*x] + 9*a*b^2*c^4*d^3*Sin[e + f*x] - 165*a^2*b*c^3*d^
4*Sin[e + f*x] - 2*b^3*c^3*d^4*Sin[e + f*x] + 131*a^3*c^2*d^5*Sin[e + f*x]
- 6*a*b^2*c^2*d^5*Sin[e + f*x] + 54*a^2*b*c*d^6*Sin[e + f*x] - 46*a^3*d^7*S
in[e + f*x]))/(24*c^4*(c^2 - d^2)^3*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e +
 f*x])^5) + ((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3*(
72*a*b^2*c^8*Sin[e + f*x] - 288*a^2*b*c^7*d*Sin[e + f*x] - 68*b^3*c^7*d*Sin
[e + f*x] + 240*a^3*c^6*d^2*Sin[e + f*x] + 252*a*b^2*c^6*d^2*Sin[e + f*x] +
 24*a^2*b*c^5*d^3*Sin[e + f*x] - 39*b^3*c^5*d^3*Sin[e + f*x] - 280*a^3*c^4*
d^4*Sin[e + f*x] - 15*a*b^2*c^4*d^4*Sin[e + f*x] - 69*a^2*b*c^3*d^5*Sin[e +
 f*x] + 2*b^3*c^3*d^5*Sin[e + f*x] + 195*a^3*c^2*d^6*Sin[e + f*x] + 6*a*b^2
*c^2*d^6*Sin[e + f*x] + 18*a^2*b*c*d^7*Sin[e + f*x] - 50*a^3*d^8*Sin[e + f*
x]))/(24*c^4*(c^2 - d^2)^4*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(603) = 1206$.
time = 1.08, size = 1345, normalized size = 2.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/c^5*((-1/8*(80*a^3*c^6*d^2+40*a^3*c^5*d^3-40*a^3*c^4*d^4-15*a^3*c^3*
d^5+32*a^3*c^2*d^6+4*a^3*c*d^7-8*a^3*d^8-96*a^2*b*c^7*d-72*a^2*b*c^6*d^2-96
*a^2*b*c^5*d^3-15*a^2*b*c^4*d^4+24*a*b^2*c^8+36*a*b^2*c^7*d+144*a*b^2*c^6*d
^2+51*a*b^2*c^5*d^3+24*a*b^2*c^4*d^4-4*b^3*c^8-32*b^3*c^7*d-21*b^3*c^6*d^2-
32*b^3*c^5*d^3-4*b^3*c^4*d^4)*c/(c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d^4)*t
an(1/2*f*x+1/2*e)^7+1/24*c*(720*a^3*c^6*d^2+120*a^3*c^5*d^3-520*a^3*c^4*d^4
-69*a^3*c^3*d^5+320*a^3*c^2*d^6+12*a^3*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d-216
*a^2*b*c^6*d^2-480*a^2*b*c^5*d^3+27*a^2*b*c^4*d^4+216*a*b^2*c^8+108*a*b^2*c
^7*d+1008*a*b^2*c^6*d^2+81*a*b^2*c^5*d^3+120*a*b^2*c^4*d^4-12*b^3*c^8-224*b
^3*c^7*d-39*b^3*c^6*d^2-224*b^3*c^5*d^3-12*b^3*c^4*d^4)/(c^3+3*c^2*d+3*c*d^
2+d^3)/(c-d)^2*tan(1/2*f*x+1/2*e)^5-1/24*c*(720*a^3*c^6*d^2-120*a^3*c^5*d^3
-520*a^3*c^4*d^4+69*a^3*c^3*d^5+320*a^3*c^2*d^6-12*a^3*c*d^7-72*a^3*d^8-864
*a^2*b*c^7*d+216*a^2*b*c^6*d^2-480*a^2*b*c^5*d^3-27*a^2*b*c^4*d^4+216*a*b^2
*c^8-108*a*b^2*c^7*d+1008*a*b^2*c^6*d^2-81*a*b^2*c^5*d^3+120*a*b^2*c^4*d^4+
12*b^3*c^8-224*b^3*c^7*d+39*b^3*c^6*d^2-224*b^3*c^5*d^3+12*b^3*c^4*d^4)/(c-
d)^3/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/8*(80*a^3*c^6*d^2-40*a^3*c^5*d^
3-40*a^3*c^4*d^4+15*a^3*c^3*d^5+32*a^3*c^2*d^6-4*a^3*c*d^7-8*a^3*d^8-96*a^2
```

```
*b*c^7*d+72*a^2*b*c^6*d^2-96*a^2*b*c^5*d^3+15*a^2*b*c^4*d^4+24*a*b^2*c^8-36
*a*b^2*c^7*d+144*a*b^2*c^6*d^2-51*a*b^2*c^5*d^3+24*a*b^2*c^4*d^4+4*b^3*c^8-
32*b^3*c^7*d+21*b^3*c^6*d^2-32*b^3*c^5*d^3+4*b^3*c^4*d^4)*c/(c+d)/(c^4-4*c^
3*d+6*c^2*d^2-4*c*d^3+d^4)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*ta
n(1/2*f*x+1/2*e)^2-c-d)^4-1/8*(40*a^3*c^8*d-40*a^3*c^6*d^3+63*a^3*c^4*d^5-3
6*a^3*c^2*d^7+8*a^3*d^9-24*a^2*b*c^9-72*a^2*b*c^7*d^2-9*a^2*b*c^5*d^4+60*a*
b^2*c^8*d+45*a*b^2*c^6*d^3-4*b^3*c^9-27*b^3*c^7*d^2-4*b^3*c^5*d^4)/(c^8-4*c
^6*d^2+6*c^4*d^4-4*c^2*d^6+d^8)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f
*x+1/2*e)/((c+d)*(c-d))^(1/2)))+2*a^3/c^5*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2162 vs. $2(614) = 1228$.
time = 4.83, size = 4386, normalized size = 7.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(48*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5
*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 192*(a^3*c^13*d - 5*a^3*c
^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f
*x*cos(f*x + e)^3 + 288*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 1
0*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 192*(a^
3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^
11 - a^3*c*d^13)*f*x*cos(f*x + e) + 48*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a
^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x + 3*(63*a^3*c
^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a
^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c
^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a
^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^
2*b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^
9*d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10
```

- 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b + 3*b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*cos(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6*a^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3)*c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*x + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a^3*c*d^12 - 4*(6*a^2*b + b^3)*c^10*d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^5 - 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^3*c^12*d^2 + 18*a*b^2*c^11*d^3 - 116*a^3*c^3*d^11 + 24*a^3*c*d^13 - (150*a^2*b + 41*b^3)*c^10*d^4 + 77*(2*a^3 + 3*a*b^2)*c^9*d^5 - (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a*b^2)*c^7*d^7 + (165*a^2*b + 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9 + (72*a*b^2*c^14 - 18*a^2*b*c^5*d^9 + 50*a^3*c^4*d^10 - 4*(72*a^2*b + 17*b^3)*c^13*d + 60*(4*a^3 + 3*a*b^2)*c^12*d^2 + (312*a^2*b + 29*b^3)*c^11*d^3 - (520*a^3 + 267*a*b^2)*c^10*d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 + 21*a*b^2)*c^8*d^6 + (87*a^2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d^8)*cos(f*x + e)^3 + (12*b^3*c^14 + 108*a*b^2*c^13*d + 104*a^3*c^3*d^11 - (648*a^2*b + 203*b^3)*c^12*d^2 + 15*(40*a^3 + 51*a*b^2)*c^11*d^3 + (339*a^2*b + 47*b^3)*c^10*d^4 - (1189*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^3)*c^8*d^6 + (997*a^3 + 84*a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8*(64*a^3 + 3*a*b^2)*c^5*d^9)*cos(f*x + e)^2 + (8*b^3*c^13*d + 72*a*b^2*c^12*d^2 - 407*a^3*c^4*d^10 + 84*a^3*c^2*d^12 - 8*(66*a^2*b + 19*b^3)*c^11*d^3 + 8*(65*a^3 + 93*a*b^2)*c^10*d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 + 759*a*b^2)*c^8*d^6 + (471*a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)*c^6*d^8 - 3*(9*a^2*b + 4*b^3)*c^5*d^9)*cos(f*x + e))*sin(f*x + e))/((c^19 - 5*c^17*d^2 + 10*c^15*d^4 - 10*c^13*d^6 + 5*c^11*d^8 - c^9*d^10)*f*cos(f*x + e)^4 + 4*(c^18*d - 5*c^16*d^3 + 10*c^14*d^5 - 10*c^12*d^7 + 5*c^10*d^9 - c^8*d^11)*f*cos(f*x + e)^3 + 6*(c^17*d^2 - 5*c^15*d^4 + 10*c^13*d^6 - 10*c^11*d^8 + 5*c^9*d^10 - c^7*d^12)*f*cos(f*x + e)^2 + 4*(c^16*d^3 - 5*c^14*d^5 + 10*c^12*d^7 - 10*c^10*d^9 + 5*c^8*d^11 - c^6*d^13)*f*cos(f*x + e) + (c^15*d^4 - 5*c^13*d^6 + 10*c^11*d^8 - 10*c^9*d^10 + 5*c^7*d^12 - c^5*d^14)*f),
1/24*(24*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 96*(a^3*c^13*d - 5*a^3*c^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f*x*cos(f*x + e)^3 + 144*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 10*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 96*(a^3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^11 - a^3*c*d^13)*f*x*cos(f*x + e) + 24*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x - 3*(63*a^3*c^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^2*

```
b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^9*
d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10 -
 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b + 3*
b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*co
s(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6*a
^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3)*
c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*x
+ e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(e + fx))^3}{(c + d\sec(e + fx))^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3173 vs. $2(603) = 1206$.
time = 0.77, size = 3173, normalized size = 5.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(24*a^2*b*c^9 + 4*b^3*c^9 - 40*a^3*c^8*d - 60*a*b^2*c^8*d + 72*a^2*
b*c^7*d^2 + 27*b^3*c^7*d^2 + 40*a^3*c^6*d^3 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^
5*d^4 + 4*b^3*c^5*d^4 - 63*a^3*c^4*d^5 + 36*a^3*c^2*d^7 - 8*a^3*d^9)*(pi*fl
oor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*
e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^13 - 4*c^11*d^2 + 6*c^9
*d^4 - 4*c^7*d^6 + c^5*d^8)*sqrt(-c^2 + d^2)) + 12*(f*x + e)*a^3/c^5 - (72*
a*b^2*c^11*tan(1/2*f*x + 1/2*e)^7 - 12*b^3*c^11*tan(1/2*f*x + 1/2*e)^7 - 28
8*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^7 - 108*a*b^2*c^10*d*tan(1/2*f*x + 1/2*
e)^7 - 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^7 + 240*a^3*c^9*d^2*tan(1/2*f*x +
 1/2*e)^7 + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 324*a*b^2*c^9*d^2*ta
n(1/2*f*x + 1/2*e)^7 + 189*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 - 600*a^3*c^8
*d^3*tan(1/2*f*x + 1/2*e)^7 - 504*a^2*b*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 89
1*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 183*b^3*c^8*d^3*tan(1/2*f*x + 1/2*
e)^7 + 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 459*a^2*b*c^7*d^4*tan(1/2*f
*x + 1/2*e)^7 + 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 183*b^3*c^7*d^4*
tan(1/2*f*x + 1/2*e)^7 + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 513*a^2*b
*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 189*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e)^7
```

- 189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 249*a^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 63*a*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 - 291*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 45*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 - 72*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^7 + 273*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^7 + 12*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e)^7 - 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^7 + 24*a^3*d^11*tan(1/2*f*x + 1/2*e)^7 - 216*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^5 + 12*b^3*c^11*tan(1/2*f*x + 1/2*e)^5 + 864*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^5 + 108*a*b^2*c^10*d*tan(1/2*f*x + 1/2*e)^5 + 212*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^5 - 720*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 684*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 - 197*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^5 + 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 - 600*a^2*b*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 + 819*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^5 + 1360*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 + 141*a^2*b*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 + 861*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 - 27*b^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^5 - 1051*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 237*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 807*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 197*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^5 - 1029*a^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 507*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 39*a*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 212*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^5 + 759*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 - 27*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 - 120*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 + 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^5 + 473*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 380*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e)^5 - 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^5 + 72*a^3*d^11*tan(1/2*f*x + 1/2*e)^5 + 216*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^3 + 12*b^3*c^11*tan(1/2*f*x + 1/2*e)^3 - 864*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^3 + 108*a*b^2*c^10*d*tan(1/2*f*x + 1/2*e)^3 - 212*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^3 + 720*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 - 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 + 684*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 - 197*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^3 + 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 600*a^2*b*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 819*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 + 27*b^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^3 - 1360*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 + 141*a^2*b*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 861*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 27*b^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^3 - 1051*a^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 + 237*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 - 807*a*b^2*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 + 197*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^3 + 1029*a^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 507*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 - 39*a*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 212*b^3*c^5*d^6*tan(1/2*f*x + 1/2*e)^3 + 759*a^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 + 27*a^2*b*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 - 120*a*b^2*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 - 12*b^3*c^4*d^7*tan(1/2*f*x + 1/2*e)^3 - 473*a^3*c^3*d^8*tan(1/2*f*x + 1/2*e)^3 - 380*a^3*c^2*d^9*tan(1/2*f*x + 1/2*e)^3 + 84*a^3*c*d^10*tan(1/2*f*x + 1/2*e)^3 + 72*a^3*d^11*tan(1/2*f*x + 1/2*e)^3 - 72*a*b^2*c^11*tan(1/2*f*x + 1/2*e) - 12*b^3*c^11*tan(1/2*f*x + 1/2*e) + 288*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e) - 10

```
8*a*b^2*c^10*d*tan(1/2*f*x + 1/2*e) + 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e) -
240*a^3*c^9*d^2*tan(1/2*f*x + 1/2*e) + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*
e) - 324*a*b^2*c^9*d^2*tan(1/2*f*x + 1/2*e) + 189*b^3*c^9*d^2*tan(1/2*f*x +
 1/2*e) - 600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e) + 504*a^2*b*c^8*d^3*tan(1/2*
f*x + 1/2*e) - 891*a*b^2*c^8*d^3*tan(1/2*f*x + ...
```

Mupad [B]
time = 16.35, size = 2500, normalized size = 4.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^5,x)
```

```
[Out] (atan(((((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 128
*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^1
8 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4
*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^
6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 484
8*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3
+ 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4
 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5
*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 +
 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*
b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^1
4*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11
+ 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12
640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*
a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4
*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(c^22*d + c^23 -
c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^
10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 +
21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (((32*a^3*c^27 + 16*b^3*c^27 + 96
*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^
11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*
a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9
 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22
*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14
*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^
18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c
^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*
c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9
+ 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*
c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 3
6*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^
```

17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1
200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^2
4*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)/(c^26*d + c
^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21
*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^2
2*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (tan(e/2 + (f*x)/2)*((c +
d)^9*(c - d)^9)^(1/2)*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d
+ 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3
*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2
*c^8*d)*(128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024
*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d
^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584
*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2))/
(16*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d
^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)*(c^22*d + c^23
 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13
*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5
 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)))*((c + d)^9*(c - d)^9)^(1/2)*(4*
b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3
*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d
^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d))/(8*(c^23 - c^5*d
^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^
8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)))*(4*b^3*c^9 - 8*a^3*d^9 + 24*a
^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3
+ 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*
a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)*1i)/(8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c
^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^
19*d^4 - 9*c^21*d^2)) + (((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(
64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d
+ 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^
15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c
^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a
^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1
024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 +
 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13
*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*...

3.198 $\int \sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=320

$$-\frac{2(a-b)\sqrt{a+b}\,d\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{bf}$$

```
[Out] -2*(a-b)*d*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/
(a-b))^(1/2)/b/f+2*(b*(c-d)+a*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/
2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f
*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A]
time = 0.19, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4001, 3869, 4090, 3917, 4089}

$$\frac{2\sqrt{a+b}\,(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{2d(a-b)\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a + b]*(b*(c - d) +
 a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
+ f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4001

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[Csc[e + f*x]*((b*c + a*d + b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]])
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\,dx=(ac)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx+\int \frac{\sec(e+fx)(bc+ad+\ldots}{\sqrt{a+b\sec(e\ldots}}$$

$$=-\frac{2\sqrt{a+b}\,c\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\ldots\right.\right.}{f\ldots}$$

$$=-\frac{2(a-b)\sqrt{a+b}\,d\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+f\ldots}}{\sqrt{a+b}}\ldots\right.\right.}{b\ldots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 17.79, size = 913, normalized size = 2.85

$$\frac{2d\cos(e+fx)\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\sin(e+fx)}{f(d+c\cos(e+fx))}+\frac{2\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\left(a\sqrt{\frac{-a+b}{a+b}}\,d\tan\left(\frac{1}{2}(e+fx)\right)+b\sqrt{\frac{-a+b}{a+b}}\,d\tan\left(\frac{1}{2}(e+fx)\right)-2a\sqrt{\frac{-a+b}{a+b}}\,d\tan^3\left(\frac{1}{2}(e+fx)\right)+a\sqrt{\frac{-a+b}{a+b}}\,d\tan^5\left(\frac{1}{2}(e+fx)\right)-b\sqrt{\frac{-a+b}{a+b}}\,d\tan^5\left(\frac{1}{2}(e+fx)\right)+2iac\Pi\left(-\frac{a+b}{a-b};i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a+b}{a-b}\right)\sqrt{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{a+b}}+2iac\Pi\left(-\frac{a+b}{a-b};i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a+b}{a-b}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\sqrt{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{a+b}}-i(a-b)dE\left(i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a+b}{a-b}\right)\sqrt{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}\left(1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{a+b}}-i(a-b)(c-d)F\left(i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a+b}{a-b}\right)\sqrt{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}\left(1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{a+b}}\right)}{\sqrt{\frac{-a+b}{a+b}}\,f\sqrt{b+a\cos(e+fx)}\,(d+c\cos(e+fx))\sec^{\frac{3}{2}}(e+fx)\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}}\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^{3/2}\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*d*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])*Sin[e + f*x
])/(f*(d + c*Cos[e + f*x])) + (2*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*
x])*(a*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] + b*Sqrt[(-a + b)/(a + b)]
*d*Tan[(e + f*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^3 + a*S
qrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*d*Tan
[(e + f*x)/2]^5 + (2*I)*a*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)
/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
+ (2*I)*a*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f
*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b
)] - I*(a - b)*d*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*
Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a
- b)*(c - d)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt
[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(Sqrt[(-a
 + b)/(a + b)]*f*Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])*Sec[e + f*x]
^(3/2)*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)*(1 + T
an[(e + f*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*
x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. $2(293) = 586$.

time = 3.54, size = 1372, normalized size = 4.29

method	result	size
default	Expression too large to display	1372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2
*(-cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+
e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(
a+b))^(1/2))*a*c+cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e
)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f
*x+e),((a-b)/(a+b))^(1/2))*a*d+cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
```

```
*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos
(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c+cos(f*x+e)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*d-cos(f*x+e)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*
sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-co
s(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)
/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))
^(1/2))*b*d+2*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b
)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x
+e),-1,((a-b)/(a+b))^(1/2))*a*c-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f
*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/s
in(f*x+e),((a-b)/(a+b))^(1/2))*a*c+(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*co
s(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e
),((a-b)/(a+b))^(1/2))*a*d*sin(f*x+e)+(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a
*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*
x+e),((a-b)/(a+b))^(1/2))*b*c*sin(f*x+e)+(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin
(f*x+e),((a-b)/(a+b))^(1/2))*b*d*sin(f*x+e)-(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+c
os(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e
))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*d*sin(f*x+e)+2*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*Ellipti
cPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a*c+cos(f*x+e)^2*a*d
-cos(f*x+e)*a*d+cos(f*x+e)*b*d-d*b)/sin(f*x+e)^5/(a*cos(f*x+e)+b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=220

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e}{a-b}}}{cf}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/c/f+2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),
2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e
)/c/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4011, 3869, 4058}

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(c*f) + (2*(b*c - a*d)*EllipticPi
[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt
[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e
+ f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4011

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[
```

```
e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{a\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} + \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,d}{c}$$

$$= -\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{cf}$$

Mathematica [A]
time = 7.29, size = 225, normalized size = 1.02

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(-\left((a-b)c(c+d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)+2a(c^2-d^2)\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)+2d(-bc+ad)\Pi\left(\frac{c-d}{c+d};\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{a+b\sec(e+fx)}}{c(c-d)(c+d)f(b+a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(-((a - b)*c*(c + d)*EllipticF[Arc
Sin[Tan[(e + f*x)/2]], (a - b)/(a + b)]) + 2*a*(c^2 - d^2)*EllipticPi[-1, A
rcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d*(-(b*c) + a*d)*EllipticPi[(
c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[
e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(206) = 412$.
time = 2.74, size = 443, normalized size = 2.01

method	result

default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,(\cos(fx+e)+1)^2(-1+\cos(fx+e))\left(\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+
e))*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c^2+Ellipt
icF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c*d-EllipticF((-1+cos
(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c^2-EllipticF((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e)
,-1,((a-b)/(a+b))^(1/2))*a*c^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,(
(a-b)/(a+b))^(1/2))*a*d^2-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+
d),((a-b)/(a+b))^(1/2))*a*d^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)
/(c+d),((a-b)/(a+b))^(1/2))*b*c*d)/(a*cos(f*x+e)+b)/sin(f*x+e)^2/c/(c-d)/(c
+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec(e + fx)}}{c + d\sec(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{c+\frac{d}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=380

$$-\frac{2(a-b)\sqrt{a+b}\,(3bc+4ad)\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{}}{3bf}$$

```
[Out] -2/3*(a-b)*(4*a*d+3*b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(
1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/3*(a*b*(6*c-4*d)-b^2*(3*c-d)+3*a^2*d)*cot(
f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a
+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/
f-2*a*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((
a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*
x+e))/(a-b))^(1/2)/f+2/3*b*d*(a+b*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.30, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4003, 4143, 4006, 3869, 3917, 4089}

$$\frac{2\sqrt{a+b}\,(3a^2d+ab(6c-4d)-b^2(3c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bf}-\frac{2(a-b)\sqrt{a+b}\,(4ad+3bc)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bf}-\frac{2ac\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c + 4*a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) + (2*Sqrt[a +
 b]*(a*b*(6*c - 4*d) - b^2*(3*c - d) + 3*a^2*d)*Cot[e + f*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*b*f) - (2*
a*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*b*d*Sqrt[a + b*Sec[e + f*x
]]*Tan[e + f*x])/(3*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f} + \frac{2}{3}\int \frac{\frac{3a^2c}{2}+\frac{1}{2}(6abc-$$

$$= \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f} + \frac{2}{3}\int \frac{\frac{3a^2c}{2}+\left(-\frac{1}{2}b(3b\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(3bc+4ad)\cot(e+fx)E\Big(\sin^{-1}\Big(\frac{\sqrt{a+}}{}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(3bc+4ad)\cot(e+fx)E\Big(\sin^{-1}\Big(\frac{\sqrt{a+}}{}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6063 vs. 2(380) = 760.
time = 24.50, size = 6063, normalized size = 15.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2339 vs. 2(345) = 690.
time = 3.53, size = 2340, normalized size = 6.16

method	result	size
default	Expression too large to display	2340

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(-1+cos(f*x+e))^2*(3*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a
*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*
x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c+4*cos(f*x+e)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+
cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d-6*cos(f*x+e)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1
/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*
```

b*c-4*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d-cos(f*x+e)^2*b^2*d-3*cos(f*x+e)^3*a*b*c-cos(f*x+e)^3*a*b*d+3*cos(f*x+e)^2*a*b*c-4*cos(f*x+e)^2*a*b*d+5*cos(f*x+e)*a*b*d+b^2*d-4*cos(f*x+e)^3*a^2*d+4*cos(f*x+e)^2*a^2*d-3*cos(f*x+e)^2*b^2*c+3*cos(f*x+e)*b^2*c+3*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c+4*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d-6*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c-4*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+4*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d+3*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c-6*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+3*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c-3*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d-3*cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c-cos(f*x+e)^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*d+4*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d+3*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c-6*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+3*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c-3*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d-3*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c-cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*

```
x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*d)*((a*cos(f*x+e)+b)/c
os(f*x+e))^(1/2)*(cos(f*x+e)+1)^2/(a*cos(f*x+e)+b)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e))*sqrt(b*sec(f
*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (e + fx))^{\frac{3}{2}} (c + d \sec (e + fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)*(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{\cos (e + f\,x)}\right)^{3/2} \left(c + \frac{d}{\cos (e + f\,x)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
[Out] int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=326

$$\frac{2b\sqrt{a+b}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{df}$$

```
[Out] 2*b*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/d/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/c/f-2*(-a*d+b*c)^2*EllipticPi(1/2*(1-sec(f*x+e))^(
1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1
/2)*tan(f*x+e)/c/d/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4013, 4006, 3869, 3917, 4058}

$$-\frac{2(bc-ad)^2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cdf(c+d)\sqrt{-\tan^2(e+fx)}\,\sqrt{a+b\sec(e+fx)}}-\frac{2a\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{cf}+\frac{2b\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*a*Sqrt[a + b]*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(c*f) - (2*(b*c - a*d)^2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - S
ec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Ta
n[e + f*x])/(c*d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Dist[1/(c*d), Int[(a^2*d + b^2*c*Csc[e + f*x])/S
qrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)^2/(c*d), Int[Csc[e + f*x
]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{\int \frac{a^2d+b^2c\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{cd} - \frac{(bc-ad)^2\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,d$$

$$= -\frac{2(bc-ad)^2\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

$$= \frac{2b\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e}{a+}}}{df}$$

Mathematica [A]
time = 8.81, size = 230, normalized size = 0.71

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)^2c(c+d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right)-2\left(a^2(c^2-d^2)\,\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right)+(bc-ad)^2\Pi\left(\frac{c-d}{c+d};\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right)\right)\right)\sqrt{a+b\sec(e+fx)}}{c(c-d)(c+d)f(b+a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)^2*c*(c + d)*EllipticF[Ar
cSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(a^2*(c^2 - d^2)*EllipticPi[-1
, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + (b*c - a*d)^2*EllipticPi[(c
- d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*Sqrt[a + b*Sec[e
 + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A]
time = 2.87, size = 581, normalized size = 1.78

method	result
default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}(\cos(fx+e)+1)^2\left(\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)a^2c^2+\text{E}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*c^2+EllipticF((-1+cos(f*
x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*c*d-2*EllipticF((-1+cos(f*x+e))/s
in(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c^2-2*EllipticF((-1+cos(f*x+e))/sin(f*x+
e),((a-b)/(a+b))^(1/2))*a*b*c*d+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*b^2*c^2+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*b^2*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2
))*a^2*c^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*
a^2*d^2-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(
1/2))*a^2*d^2+4*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a
+b))^(1/2))*a*b*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a
-b)/(a+b))^(1/2))*b^2*c^2)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)^2/c/
(c+d)/(c-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec{(e+fx)})^{\frac{3}{2}}}{c+d\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c+\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=442

$$-\frac{2(a-b)\sqrt{a+b}\,(35abc+23a^2d+9b^2d)\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{15bf}$$

```
[Out] -2/15*(a-b)*(23*a^2*d+35*a*b*c+9*b^2*d)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+
b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/15*(a^2*b*(45*c-23*d)-a*b^2
*(35*c-17*d)+b^3*(5*c-9*d)+15*a^3*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))
^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*a^2*c*cot(f*x+e)*EllipticPi((a
+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(
b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+2/5*b*d*(a+
b*sec(f*x+e))^(3/2)*tan(f*x+e)/f+2/15*b*(8*a*d+5*b*c)*(a+b*sec(f*x+e))^(1/2
)*tan(f*x+e)/f
```

Rubi [A]
time = 0.43, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4003, 4141, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(23a^2d+35abc+9b^2d)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bf}-\frac{2a^2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}+\frac{2\sqrt{a+b}\,(15a^3d+a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bf}+\frac{2b(8ad+5bc)\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{15f}+\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*c + 23*a^2*d + 9*b^2*d)*Cot[e + f*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(15*b
*f) + (2*Sqrt[a + b]*(a^2*b*(45*c - 23*d) - a*b^2*(35*c - 17*d) + b^3*(5*c
- 9*d) + 15*a^3*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/(15*b*f) - (2*a^2*Sqrt[a + b]*c*Cot[e + f*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*
x]))/(a - b))])/f + (2*b*(5*b*c + 8*a*d)*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f
*x])/(15*f) + (2*b*d*(a + b*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
```

```
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{2}{5}\int \sqrt{a+b\sec(e+}$$

$$= \frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\,\tan(e+fx)}{15f} + \frac{2bd(a+b}{}$$

$$= \frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\,\tan(e+fx)}{15f} + \frac{2bd(a+b}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(35abc+23a^2d+9b^2d)\cot(e+fx)E\Big(\sin}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(35abc+23a^2d+9b^2d)\cot(e+fx)E\Big(\sin}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7138 vs. $2(442) = 884$.
time = 25.49, size = 7138, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3284 vs. $2(403) = 806$.
time = 3.45, size = 3285, normalized size = 7.43

method	result	size
default	Expression too large to display	3285

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -2/15/f*(cos(f*x+e)+1)^2*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(23*cos(f*x+e)^4*a^3*d-23*cos(f*x+e)^3*a^3*d+9*cos(f*x+e)^3*b^3*d-6*cos(f*x+e)^2*b^3*d+5*cos(f*x+e)^3*b^3*c-5*cos(f*x+e)*b^3*c+35*cos(f*x+e)^4*a^2*b*c+11*cos(f*x+e)^4*a^2*b*d+5*cos(f*x+e)^4*a*b^2*c+9*cos(f*x+e)^4*a*b^2*d-35*cos(f*x+e)^3*a^2*b*c+23*cos(f*x+e)^3*a^2*b*d+35*cos(f*x+e)^3*a*b^2*c+5*cos(f*x+e)^3*a*b^2*d-34*cos(f*x+e)^2*a^2*b*d-40*cos(f*x+e)^2*a*b^2*c-14*cos(f*x+e)*a*b^2*d-15*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*c+15*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d+5*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*c+9*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d-23*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d-9*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d+30*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3*c-15*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*c+15*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d+5*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*c+9*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d-23*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d-9*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d+30*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3*c-3*b^3*d+45*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e)

```
)/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*c+23*cos(f*x+e)^3*sin(f*x+e)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*E
llipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*d+35*cos(f*x
+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f
*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1
/2))*a*b^2*c+17*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
(a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*d-35*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*Ellipti
cE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*c-23*cos(f*x+e)^3*
sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+
1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a
^2*b*d-35*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos
(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*a*b^2*c-9*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+
cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*d+45*cos(f*x+e)^2*sin(f*x
+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b
))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*c+
23*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)
+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*a^2*b*d+35*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*
x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*c+17*cos(f*x+e)^2*sin(f*x+e)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/
2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*d-35*cos
(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*sec(f*x + e)^3 + a^2*c + (b^2*c + 2*a*b*d)*sec(f*x + e)^2 +
 (2*a*b*c + a^2*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(e + fx))^{\frac{5}{2}} (c + d\sec(e + fx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(5/2)*(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2\sqrt{a+b}\,d\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{bf}$$

```
[Out] 2*d*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/a/f
```

Rubi [A]
time = 0.08, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4006, 3869, 3917}

$$\frac{2d\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
 b))])/(a*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\,d\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e}{a+b}}}{bf}$$

Mathematica [A]

time = 2.45, size = 145, normalized size = 0.70

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-c+d)F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)+2c\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sec(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]],
 (a - b)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A]

time = 2.90, size = 215, normalized size = 1.03

method	result
default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}(\cos(fx+e)+1)^2(-1+\cos(fx+e))\left(\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\right.\right.}{f(a\cos(fx+e)+b)\sin(fx+e)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+
e))*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c-EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi((-1+cos(f*x+
e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*c)/(a*cos(f*x+e)+b)/sin(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec{\left(e + f x \right)}}{\sqrt{a + b \sec{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral((c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2), x)
```

3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=216

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{a+b}{a};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{acf}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/a/c/f-2*d*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+
d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/c/(c+
d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4015, 3869, 4058}

$$-\frac{2d\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\operatorname{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\,\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*c*f) - (2*d*EllipticPi[(2*d)/(
c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*
Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*
Sqrt[-Tan[e + f*x]^2])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4015

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Dist[1/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x],
x] - Dist[d/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*
```

```
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\,\mathrm{s}}}{c}$$

$$= -\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx}}{\sqrt{a+b}}\right.\right.}{a\mathrm{c}}$$

Mathematica [A]
time = 27.70, size = 251, normalized size = 1.16

$$-\frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\,(d+c\cos(e+fx))\left(c(c+d)F\left(\mathrm{ArcSin}\left(\tan\left(\tfrac{1}{2}(e+fx)\right)\right)|\tfrac{a-b}{a+b}\right)-2\left((c^2-d^2)\,\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\tfrac{1}{2}(e+fx)\right)\right)|\tfrac{a-b}{a+b}\right)+d^2\Pi\left(\tfrac{c-d}{c+d};\mathrm{ArcSin}\left(\tan\left(\tfrac{1}{2}(e+fx)\right)\right)|\tfrac{a-b}{a+b}\right)\right)\right)\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\,\sec^{\frac{3}{2}}(e+fx)\sqrt{1+\sec(e+fx)}}{c(c-d)(c+d)f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\,\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(d + c*Cos[e +
f*x])*(c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(
(c^2 - d^2)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + d^2
*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*S
qrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*
x]])/(c*(c - d)*(c + d)*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]]
*(c + d*Sec[e + f*x]))
```

Maple [A]
time = 3.18, size = 318, normalized size = 1.47

method	result
default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}(\cos(fx+e)+1)^2(-1+\cos(fx+e))\left(c^2\,\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+
e))*(c^2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))+d*Ellipt
icF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c-2*c^2*EllipticPi((-1+
cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))+2*EllipticPi((-1+cos(f*x+e))
/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*d^2-2*EllipticPi((-1+cos(f*x+e))/sin(f*
x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d^2)/(a*cos(f*x+e)+b)/sin(f*x+e)^2/c/
(c+d)/(c-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=376

$$\frac{2(bc-ad)\cot(e+fx)E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx)}{a-b}}}{ab\sqrt{a+b}\,f}$$

```
[Out] 2*(-a*d+b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/
2)/a/b/f/(a+b)^(1/2)-2*(-a*d+b*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+s
ec(f*x+e))/(a-b))^(1/2)/a/b/f/(a+b)^(1/2)-2*c*cot(f*x+e)*EllipticPi((a+b*se
c(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-
sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b*(-a*d+b*
c)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4008, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2f}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2(bc-ad)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abf\sqrt{a+b}}+\frac{2(bc-ad)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*c - a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(a*b*Sqrt[a + b]*f) - (2*(b*c - a*d)*Cot[e + f
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b
))])/(a*b*Sqrt[a + b]*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*
f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x
]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2b(bc-ad)\tan(e+fx)}{a\,(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}(a^2-b^2)c+\frac{1}{2}a(bc-ad)\sec(e+fx)+\frac{1}{2}b(bc-ad)\,\mathrm{se}}{\sqrt{a+b\sec(e+fx)}}}{a\,(a^2-b^2)}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{a\,(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}(a^2-b^2)c+\left(\frac{1}{2}a(bc-ad)-\frac{1}{2}b(bc-ad)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}}{a\,(a^2-b^2)}$$

$$= \frac{2(bc-ad)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e}{a+b}}}{ab\sqrt{a+b}\,f}$$

$$= \frac{2(bc-ad)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e}{a+b}}}{ab\sqrt{a+b}\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.39, size = 1491, normalized size = 3.97

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]*(c + d*Sec[e + f*x])*((2*(-(b*c) + a*d
)*Sin[e + f*x])/(a*(a^2 - b^2)) - (2*(-(b^2*c*Sin[e + f*x]) + a*b*d*Sin[e +
 f*x]))/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(d + c*Cos[e + f*x])*(a +
 b*Sec[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*
(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2
]^2)/(1 + Tan[(e + f*x)/2]^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/
2] + b^2*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - a^2*Sqrt[(-a + b)/(a +
 b)]*d*Tan[(e + f*x)/2] - a*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] - 2
*a*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^3 + 2*a^2*Sqrt[(-a + b)/(a +
 b)]*d*Tan[(e + f*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5
 - b^2*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - a^2*Sqrt[(-a + b)/(a +
 b)]*d*Tan[(e + f*x)/2]^5 + a*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5
 - (2*I)*a^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(
a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*c
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[
(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (2*I)*a^2*c*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a +
```

```
b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a
*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*c*Elliptic
Pi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]],
(a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a +
b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-(b*
c) + a*d)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a
+ b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a
 + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(2
*b*c + a*(c - d))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)
*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*S
qrt[(-a + b)/(a + b)]*(a^2 - b^2)*f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x
])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(
e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2009 vs. $2(347) = 694$.
time = 3.34, size = 2010, normalized size = 5.35

method	result	size
default	Expression too large to display	2010

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*4^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE
((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c-cos(f*x+e
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))
^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)
*a*b*d-cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(
f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*sin(f*x+e)*a*b*c+cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos
(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+EllipticF((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(
f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*a^2*d-cos(f*x+e)*a^2*d+cos(f*x+e)*a*b
*c-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2
)*a^2*d+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))
^(1/2)*b^2*c+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2)
)*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e
)+1)/(a+b))^(1/2)*a^2*c-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(
a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)
```

```
/(cos(f*x+e)+1)/(a+b))^(1/2)*b^2*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a
-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+
e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*a^2*c-cos(f*x+e)^2*a*b*c-cos(f*x+e)^2*a*b
*d+cos(f*x+e)*a*b*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2
))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+
e)+1)/(a+b))^(1/2)*a*b*c+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))
^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos
(f*x+e)+1)/(a+b))^(1/2)*a*b*d+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(
a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)
/(cos(f*x+e)+1)/(a+b))^(1/2)*a*b*c-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a
-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+
e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*a*b*d-2*cos(f*x+e)*sin(f*x+e)*EllipticPi(
(-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*b^2*c+cos(f*x+e)^2*
a^2*d+cos(f*x+e)^2*b^2*c-cos(f*x+e)*b^2*c-cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos
(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d+cos(f*x+e)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*E
llipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*b^2*c+2
*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)
+1)/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/
2))*sin(f*x+e)*a^2*c-cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f
*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d)/(a*cos(f*x+e)+b)/sin(
f*x+e)/a/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^2*sec(f*x + e)^2
+ 2*a*b*sec(f*x + e) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d\sec{(e + fx)}}{(a + b\sec{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+\frac{d}{\cos(e+f\,x)}}{{\left(a+\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2),x)
[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2), x)
```

3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$\frac{2(7a^2bc-3b^3c-4a^3d)\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1-}{}}}{3a^2(a-b)b(a+b)^{3/2}f}$$

```
[Out] 2/3*(-4*a^3*d+7*a^2*b*c-3*b^3*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+se
c(f*x+e))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2/3*(-3*a^3*d+6*a^2*b*c+a^
2*b*d-a*b^2*c-3*b^3*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/
(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f
*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^3/f+2/3*b*(-a*d+b*c
)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+2/3*b*(-4*a^3*d+7*a^2*b*c
-3*b^3*c)*tan(f*x+e)/a^2/(a^2-b^2)^2/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4008, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3f}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}+\frac{2(-4a^3d+7a^2bc-3b^3c)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2bf(a-b)(a+b)^{3/2}}-\frac{2(-3a^3d+6a^2bc+a^2bd-ab^2c-3b^3c)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2bf(a-b)(a+b)^{3/2}}+\frac{2b(-4a^3d+7a^2bc-3b^3c)\tan(e+fx)}{3a^2f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*f) - (2*(6*a^2*b*c - a*b^2*c - 3*b^3*c - 3*a^3*d + a^2*b*d)*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a
- b))])/(a^3*f) + (2*b*(b*c - a*d)*Tan[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*
Sec[e + f*x])^(3/2)) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Tan[e + f*x])/(
3*a^2*(a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \frac{2b(bc-ad)\tan(e+fx)}{3a\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2-b^2\right)c+\frac{3}{2}a(bc-ad)\sec(e+fx)-\frac{1}{2}b(bc-ad}{(a+b\sec(e+fx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{3a\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{2b\left(7a^2bc-3b^3c-4a^3d\right)\tan(e+fx)}{3a^2\left(a^2-b^2\right)^2f\sqrt{a+b\sec(e+fx)}} +$$

$$= \frac{2b(bc-ad)\tan(e+fx)}{3a\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{2b\left(7a^2bc-3b^3c-4a^3d\right)\tan(e+fx)}{3a^2\left(a^2-b^2\right)^2f\sqrt{a+b\sec(e+fx)}} +$$

$$= \frac{2\left(7a^2bc-3b^3c-4a^3d\right)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{3a^2(a-b)b(a+b)^{3/2}f}$$

$$= \frac{2\left(7a^2bc-3b^3c-4a^3d\right)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{3a^2(a-b)b(a+b)^{3/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.79, size = 2083, normalized size = 4.21

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^3*Sec[e + f*x]^2*(c + d*Sec[e + f*x])*((2*(-7*a^2*b*c
 + 3*b^3*c + 4*a^3*d)*Sin[e + f*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*c*Sin[e
 + f*x] - a*b^2*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[e + f*x])^2)
 - (2*(-8*a^2*b^2*c*Sin[e + f*x] + 4*b^4*c*Sin[e + f*x] + 5*a^3*b*d*Sin[e +
 f*x] - a*b^3*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[e + f*x]))))
```

```
/(f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)) + (2*(b + a*Cos[e + f*
x])^(5/2)*Sec[e + f*x]^(3/2)*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e +
f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(7*a^3*b*Sqrt[(
-a + b)/(a + b)]*c*Tan[(e + f*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*c*Ta
n[(e + f*x)/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 3*b^4*
Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 4*a^4*Sqrt[(-a + b)/(a + b)]*d*
Tan[(e + f*x)/2] - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] - 14*a
^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^3 + 6*a*b^3*Sqrt[(-a + b)/(a
 + b)]*c*Tan[(e + f*x)/2]^3 + 8*a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/
2]^3 + 7*a^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 7*a^2*b^2*Sqrt
[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*c*
Tan[(e + f*x)/2]^5 + 3*b^4*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 4*
a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 + 4*a^3*b*Sqrt[(-a + b)/(a
+ b)]*d*Tan[(e + f*x)/2]^5 - (6*I)*a^4*c*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]
^2)/(a + b)] + (12*I)*a^2*b^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e +
f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
b)] - (6*I)*b^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
 + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqr
t[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (6*I)*a^
4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e
+ f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2
]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (12
*I)*a^2*b^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e +
 f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
 b)] - (6*I)*b^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan
[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)
/(a + b)] + I*(a - b)*(-7*a^2*b*c + 3*b^3*c + 4*a^3*d)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e
+ f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 +
b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-4*a*b^2*c - 6*b^3*c + 3*a^3*(c
 - d) + a^2*b*(9*c + d))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e
+ f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x
)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
))/(3*a^2*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*f*(d + c*Cos[e + f*x])*(a +
b*Sec[e + f*x])^(5/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^
2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) - b*(1 + Tan[(e +
 f*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5709 vs. $2(456) = 912$.

time = 3.25, size = 5710, normalized size = 11.54

method	result	size
default	Expression too large to display	5710

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^3*sec(f*x + e)^3
+ 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec{\left(e + f x \right)}}{\left(a + b \sec{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+\frac{d}{\cos(e+f\,x)}}{{\left(a+\frac{b}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2), x)
```

3.207 $\int \sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=389

$$-\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\mathrm{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(}{(c+d)(a+b\sec(}}}{\sqrt{a+b}\,f}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/((a+b)/(c+d))^(1/2)-2*cot(f
*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sec(f*
x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e
))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((
-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4017, 4021, 4067}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\mathrm{ArcSin}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f\sqrt{\frac{a+b}{c+d}}}-\frac{2\sqrt{c+d}\,\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{a(c+d)}{(a+b)c};\mathrm{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*f) + (2*
Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c +
d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a + b*
Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[
e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[(a + b)/(c + d)]*f)
```

Rule 4017

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
```

```
d*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]
]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0]
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4067

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}\,dx = c\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx + d\int \frac{\sec(e+fx)\sqrt{a+b\mathrm{se}}}{\sqrt{c+d\sec(e+}}$$

$$= -\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d}}{\sqrt{c+d}\,\sqrt{a+b}}\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.54, size = 39925, normalized size = 102.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 2.44, size = 543, normalized size = 1.40

method	result
default	$\frac{2\left(2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)bd+2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)ac-\mathrm{EllipticF}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b
),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*d+2*EllipticPi((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+
b))^(1/2))*a*c-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(
1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d+EllipticF((-1+cos(f*x+
e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c-Ell
ipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*b*d)*cos(f*x+e)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*((d+c*cos(f
*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^2*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d
))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)/(-1+cos(f*x+e))/(d+c
*cos(f*x+e))/(a*cos(f*x+e)+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec{(e + fx)}}\,\sqrt{c + d\sec{(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(}{(c+d)(a+b\sec(e}}}{\sqrt{a+b}\,cf}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+
b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*s
ec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/c/f/(a+b)^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {4021}

$$-\frac{2\sqrt{c+d}\,\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\Pi\left(\frac{a(c+d)}{(a+b)c};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{cf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)}{(a+b)}\right.}{}$$

Mathematica [A]
time = 5.46, size = 336, normalized size = 1.70

$$\frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)\left((a+b)cF\left(\mathrm{ArcSin}\left(\frac{\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}}{\sqrt{2}}\right)|\frac{2(bc-ad)}{(a+b)(c-d)}\right)-a(c+d)\Pi\left(\frac{bc-ad}{ac+bc};\mathrm{ArcSin}\left(\frac{\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}}{\sqrt{2}}\right)|\frac{2(bc-ad)}{(a+b)(c-d)}\right)\right)\sqrt{a+b\sec(e+fx)}\sin^2\left(\frac{1}{2}(e+fx)\right)}{(a+b)cf\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e + 
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*((a + b)*c*EllipticF
[ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*
d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))] - a*(c + d)*EllipticPi[(b*
c - a*d)/(a*c + b*c), ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))])*Sqrt
[a + b*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/((a + b)*c*f*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A]
time = 2.64, size = 352, normalized size = 1.78

method	result
default	$\frac{2\left(2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)a-\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)a+\mathrm{EllipticF}\right.}{f(-1+\cos(f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-
b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(f*x+e))*((a
-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a+EllipticF((-
1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2
))*b)*cos(f*x+e)*sin(f*x+e)^2*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)
*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e)
```

```
)^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e)
)/(a*cos(f*x+e)+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(e + f x \right)}}}{\sqrt{c + d \sec{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/sqrt(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=598

$$-\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(}{(c+d)(a+b\sec(}}}{\sqrt{a+b}\,c^2f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+
b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*s
ec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/c^2/f/(a+b)^
(1/2)-2*d*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/
2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1+sec(f*x+e))*(
a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)
/f/(c+d)^(1/2)/(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)-2*
(a-b)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)
/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(
a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)/(-a*d+b*c)/f/(c+
d)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4024, 4021, 4071, 4069, 4079}

$$-\frac{2\sqrt{c+d}\,\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{a(c+d)}{(a+b)c};\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{c^2f\sqrt{a+b}}-\frac{2d(a-b)\sqrt{a+b}\,\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{cf(c-d)\sqrt{c+d}\,(bc-ad)}-\frac{2d\sqrt{a+b}\,\cot(e+fx)(\sec(e+fx)+1)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{cf(c-d)\sqrt{c+d}\,\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c^2*f) -
 (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a
 + b)*(c + d*Sec[e + f*x]))])/(c*(c - d)*Sqrt[c + d]*f*Sqrt[-(((b*c - a*d)*
```

```
(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]))))]) - (2*(a - b)*Sqrt[a +
 b]*d*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]*(c +
 d*Sec[e + f*x]))/(c*(c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4024

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[
c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e
+ f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4071

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(3/2), x_Symbol] :> Dist[(a - b)/(c -
d), Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x
], x] + Dist[(b*c - a*d)/(c - d), Int[Csc[e + f*x]*((1 + Csc[e + f*x])/(Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(3/2))), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
 0]
```

Rule 4079

```
Int[(sec[(e_.) + (f_.)*(x_)]*((A_) + (B_.)*sec[(e_.) + (f_.)*(x_)]))/(Sqrt[
(a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sec[(e_.) + (f_.)*(x_)]
)^(3/2)), x_Symbol] :> Simp[2*A*(1 + Sec[e + f*x])*(Sqrt[(b*c - a*d)*((1 -
Sec[e + f*x])/((a + b)*(c + d*Sec[e + f*x])))]/(f*(b*c - a*d)*Rt[(c + d)/(a
 + b), 2]*Tan[e + f*x]*Sqrt[(-(b*c - a*d))*((1 + Sec[e + f*x])/((a - b)*(c
+ d*Sec[e + f*x])))]))*EllipticE[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*
Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]])], (a + b)*((c - d)/((a - b)*(c + d)
))], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

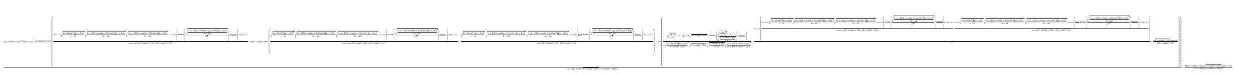
Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)}{(a+b)}\cdots}{\cdots}$$

$$= -\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)}{(a+b)}\cdots}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1708 vs. 2(598) = 1196.
time = 9.31, size = 1708, normalized size = 2.86



Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*((4*b*c*(
b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e
+ f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt
[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]],
```

```
(2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqr
t[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(a*c + b*d)
*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*
Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a -
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c
- a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2
)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Cs
c[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/(
(a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*S
qrt[d + c*Cos[e + f*x]])) + 2*a*d*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e +
 f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*
Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a
 + b)*(c + d))]])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x]
)]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b
)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]]) - (2*(b*c - a*d)*(
((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)
*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[Arc
Sin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sq
rt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c
+ d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt
[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c),
ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*
Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/((c - d)*(c + d
)*f*Sqrt[b + a*Cos[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)) + (2*d*(d + c*Cos[
e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f*x])/((-c^2 + d^2)*f*(c + d*Sec
[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2846 vs. $2(553) = 1106$.

time = 2.74, size = 2847, normalized size = 4.76

method	result	size
default	Expression too large to display	2847

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(-((a-b)/(a+b))^(1/2)*b*c*d-sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)
/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)
/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*
b*c^2-((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(
f*x+e)+1)/(c+d))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*
x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*d^2*sin(f*x+e)+((a*cos(f*x+e)+b)/(c
os(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*Ell
ipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*b*d^2*sin(f*x+e)-2*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2
)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b
))^(1/2))*a*c^2*sin(f*x+e)+2*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*
((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticPi((-1+cos(f*x+e))*((
a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))
^(1/2))*a*d^2*sin(f*x+e)+cos(f*x+e)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))*((
a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e
)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1
/2)*b*c*d+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(
c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+
c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c^2*sin(f*x+e)+((a-b)/(a+b))^(1
/2)*b*d^2+cos(f*x+e)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/
2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e
)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c^2-cos(f
*x+e)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/
2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c^2-cos(f*x+e)*sin(f*x+e
)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*
x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*d^2+cos(f*x+e)*sin(f*x+e)*EllipticE((-1
+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+
e)+1)/(c+d))^(1/2)*b*d^2-2*cos(f*x+e)*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+
b))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/
(cos(f*x+e)+1)/(c+d))^(1/2)*a*c^2+2*cos(f*x+e)*sin(f*x+e)*EllipticPi((-1+co
s(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/(
(a-b)/(a+b))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos
(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*d^2+sin(f*x+e)*EllipticF((-1+cos(f*x
+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*co
s(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c
+d))^(1/2)*a*c*d-sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)
/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c*d-sin(f*x+e
)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*
```

```
x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c*d+sin(f*x+e)*EllipticE((-1+cos(f*x+e)
)*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f
*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d)
)^(1/2)*b*c*d+cos(f*x+e)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))
^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f
*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c*d-c
os(f*x+e)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+
e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))
^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c*d-cos(f*x+e)*sin(f
*x+e)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*co
s(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c*d+((a-b)/(a+b))^(1/2)*cos(f*x+e)^
2*a*c*d-((a-b)/(a+b))^(1/2)*cos(f*x+e)*a*c*d+((a-b)/(a+b))^(1/2)*cos(f*x+e)
*b*c*d+((a-b)/(a+b))^(1/2)*cos(f*x+e)*a*d^2-((a-b)/(a+b))^(1/2)*cos(f*x+e)*
b*d^2-((a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a*d^2)*cos(f*x+e)*((a*cos(f*x+e)+b)/
cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(d+c*cos(f
*x+e))/(a*cos(f*x+e)+b)/((a-b)/(a+b))^(1/2)/(c-d)/(c+d)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(e + f x \right)}}}{\left(c + d \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{{\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=899

$$\frac{2(a-b)\sqrt{a+b}\,d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+}{(a-b)(d+c\cos(e+}}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)^2f}$$

```
[Out] 2/3*d^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))/(c
+d*sec(f*x+e))^(1/2)+2/3*(a-b)*d*(-7*a*c^2*d+3*a*d^3+6*b*c^3-2*b*c*d^2)*(d+
c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)
/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^
(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c
)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/(
c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)+2/3*(b*c^2*(3*c^2+3*c*d-2*d^2)-a*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c
*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/
(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(
1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)
*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(c
-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2
)-2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e
))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*
x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b
*sec(f*x+e))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2)
```

Rubi [A]

time = 1.52, antiderivative size = 899, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4027, 3127, 3132, 2890, 3077, 2897, 3075}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*d*(6*b*c^3 - 7*a*c^2*d - 2*b*c*d^2 + 3*a*d^3)*Sqrt[-
(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos
[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[
e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a -
```

```
b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c
 - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a
+ b]*(b*c^2*(3*c^2 + 3*c*d - 2*d^2) - a*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^
3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))
]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*
(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^
(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
+ f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*
(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d
 + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e
 + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*
x]]) + (2*d^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
```

```
 f*x]))))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
```

```
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)\sqrt{b+a\cos(e+fx)}}{(d+c\cos(e+fx))^{5/2}}\,d}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+fx}\right.}{}$$

$$= \frac{2d^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a\sqrt{d+c\cos(e+fx}\right.}{c^3\sqrt{b-}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}\,(}{c^3\sqrt{c+}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx}{(a+b)(d+c\cos(e+fx}}}{3c^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1990 vs. $2(899) = 1798$.
time = 6.87, size = 1990, normalized size = 2.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*Sin
[e + f*x])/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) - (2*(6*b*c^3*d*Sin[e +
 f*x] - 7*a*c^2*d^2*Sin[e + f*x] - 2*b*c*d^3*Sin[e + f*x] + 3*a*d^4*Sin[e +
 f*x]))/(3*c*(b*c - a*d)*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Se
c[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*Sqrt[a + b*
Sec[e + f*x]]*((4*(b*c - a*d)*(3*b^2*c^4 - 3*a*b*c^3*d - a^2*c^2*d^2 + b^2*
c^2*d^2 - a*b*c*d^3 + a^2*d^4)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-
a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*E
```

```
llipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/
((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*
c - a*d)*(3*a*b*c^4 - 3*a^2*c^3*d + 6*b^2*c^3*d - 7*a*b*c^2*d^2 - a^2*c*d^3
 - 2*b^2*c*d^3 + 4*a*b*d^4)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sq
rt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a
 - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*El
lipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
 - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/(
(a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[
((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), A
rcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(6*a*b*c^3*d - 7*a
^2*c^2*d^2 - 2*a*b*c*d^3 + 3*a^2*d^4)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[
(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a +
b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/
((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e +
f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a
 + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*
d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c
+ d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(
d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF
[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)
]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)
*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*
Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x
])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*
c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a
*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
 b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d
+ c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(3*c*(c - d
)^2*(c + d)^2*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*(c + d*Sec[e + f*x])^(
5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15723 vs. $2(820) = 1640$.

time = 2.71, size = 15724, normalized size = 17.49

method	result	size
default	Expression too large to display	15724

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(e + f x \right)}}}{\left(c + d \sec{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(5/2),x)
[Out] \text{Hanged}
```

3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=744

$$-\frac{2(a-b)\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}\,(d+c\cos(e+fx))^{3/2}\,\mathrm{csc}}{c(c-d)\sqrt{c+d}\,f\sqrt{b+a\cos(e+fx)}\,\sqrt{}}$$

```
[Out] -2*(a-b)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f
*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(
1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))
^(1/2)/c/(c-d)/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-
2*(b*c-a*(2*c-d))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(
b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)
/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e
)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*se
c(f*x+e))^(1/2)/c^2/(c-d)/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x
+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+
a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+
b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/
(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))
^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d
*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 744, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4027, 2877, 2890, 3077, 2897, 3075}

$$-\frac{2\sqrt{a+b}\,(bc-a(2c-d))\csc(e+fx)\sqrt{a+b\sec(e+fx)}\,(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{c^2 f(c-d)\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}-\frac{2a\sqrt{a+b}\csc(e+fx)\sqrt{a+b\sec(e+fx)}\,(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\,\Pi\left(\frac{(a+b)c}{a(c+d)};\mathrm{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{c^2 f\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}-\frac{2(a-b)\sqrt{a+b}\csc(e+fx)\sqrt{a+b\sec(e+fx)}\,(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{cf(c-d)\sqrt{c+d}\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
 + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
 c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c*(c
 - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2
*Sqrt[a + b]*(b*c - a*(2*c - d))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((
a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
```

```
- b)*(d + c*Cos[e + f*x]))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellipt
icF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*C
os[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x
]])/(c^2*(c - d)*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e +
f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*
(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d
 + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((
a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[
a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqr
t[a + b*Sec[e + f*x]])/(c^2*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])
```

Rule 2877

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
```

```
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{3/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

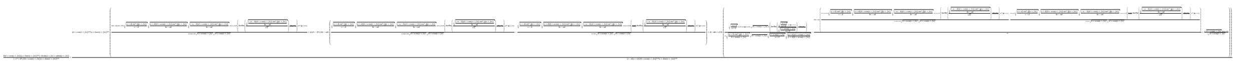
$$= \frac{\left(a^2\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}\,dx}{c^2\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}} + \cdots$$

$$= -\frac{2a\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}\cdots}{c^2\sqrt{c\cdots}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+\cdots}{(a-b)(d+c\cos(e+\cdots}}\cdots}{c(c-d)\sqrt{\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1750 vs. $2(744) = 1488$.

time = 9.55, size = 1750, normalized size = 2.35



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(3/2)*(-(b*c*Sin[e + f*x]) + a
*d*Sin[e + f*x]))/((-c^2 + d^2)*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])
^(3/2)) + ((d + c*Cos[e + f*x])^(3/2)*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c -
 a*d)*(a*b*c - b^2*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c +
 d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d
 + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[
ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(a^2*c - b^2
*c)*(b*c - a*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcS
in[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqr
t[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c +
 d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot
[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[(
(-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2
*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-(a*b*c) + a^2*d)*((Sqrt[(-a
+ b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[A
rcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(
a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e +
f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e
 + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e
 + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x
)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Si
n[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[
e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[(
(c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b
)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Ellipt
```

```
icPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]
*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos
[e + f*x]]))))/((c - d)*(c + d)*f*(b + a*Cos[e + f*x])^(3/2)*(c + d*Sec[e +
 f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4301 vs. $2(675) = 1350$.
time = 2.60, size = 4302, normalized size = 5.78

method	result	size
default	Expression too large to display	4302

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*(-((a-b)/(a+b))^(1/2)*b^2*c^2+((a-b)/(a+b))^(1/2)*a*b*c*d-EllipticF((-
1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2
))*sin(f*x+e)*cos(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+
c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*c*d-((a-b)/(a+b))^(1/2)*cos(f
*x+e)^2*a*b*c*d-((a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*c*d+((a-b)/(a+b))^(1/2
)*cos(f*x+e)^2*a*b*c^2+((a-b)/(a+b))^(1/2)*cos(f*x+e)*a^2*c*d-((a-b)/(a+b))
^(1/2)*cos(f*x+e)*a*b*c^2+((a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b*d^2-((a-b)/(a+
b))^(1/2)*cos(f*x+e)*b^2*c*d-((a-b)/(a+b))^(1/2)*a*b*d^2+((a-b)/(a+b))^(1/2
)*b^2*c*d+((a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^2*d^2-((a-b)/(a+b))^(1/2)*cos(
f*x+e)*a^2*d^2+((a-b)/(a+b))^(1/2)*cos(f*x+e)*b^2*c^2+2*EllipticPi((-1+cos(
f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a
-b)/(a+b))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*
((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*c^2-2*EllipticPi((-1+cos(
f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a
-b)/(a+b))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*
((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*d^2-EllipticF((-1+cos(f*x
+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*
x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f
*x+e)+1)/(c+d))^(1/2)*a^2*c^2-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)
/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(
cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b^
2*c^2+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/
2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*d^2+EllipticE((-1+cos(
f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin
(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(co
s(f*x+e)+1)/(c+d))^(1/2)*b^2*c^2+2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))
^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*sin
```

```
(f*x+e)*cos(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(
f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*c^2-2*EllipticPi((-1+cos(f*x+e))*((
a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))
^(1/2))*sin(f*x+e)*cos(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)
*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*d^2-EllipticF((-1+cos(f*
x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f
*x+e)*cos(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*
x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*c^2-EllipticF((-1+cos(f*x+e))*((a-b)/
(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*cos(f*x
+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*
x+e)+1)/(c+d))^(1/2)*b^2*c^2+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/
sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*cos(f*x+e)*((a*cos(f
*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d)
)^(1/2)*a^2*d^2+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((
a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*cos(f*x+e)*((a*cos(f*x+e)+b)/(cos
(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b^2*c
^2-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*
((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*c*d+2*((a*cos(f*x+e)+b)/(
cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*El
lipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*a*b*c^2*sin(f*x+e)-((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/
2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticF((-1+cos(f*x+e))*
((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c*d*sin
(f*x+e)+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-
d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(
1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a^2*c*d-EllipticE((-1+co
s(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*s
in(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(
cos(f*x+e)+1)/(c+d))^(1/2)*a*b*c^2-EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^
(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)
+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/
2)*a*b*d^2+((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/
(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/s
in(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c*d*sin(f*x+e)+2*EllipticF((
-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/
2))*sin(f*x+e)*cos(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d
+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*b*c^2-EllipticF((-1+cos(f*x+e)
)*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(e + fx)})^{\frac{3}{2}}}{(c + d\sec{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\left(c + \frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2), x)
```

3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=919

$$-\frac{2(a-b)\sqrt{a+b}\,(3bc^3-7ac^2d+bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+}{(a-b)(d+c\cos(e+}}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f}$$

```
[Out] -2/3*d*(-a*d+b*c)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(
f*x+e))/(c+d*sec(f*x+e))^(1/2)-2/3*(a-b)*(-7*a*c^2*d+3*a*d^3+3*b*c^3+b*c*d^
2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))
^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)
/c^2/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e
))^(1/2)-2/3*(b^2*c^3*(3*c+d)-2*a*b*c^2*(3*c^2+2*c*d-d^2)+a^2*d*(9*c^3-2*c^
2*d-6*c*d^2+3*d^3))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)
*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x
+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*
sec(f*x+e))^(1/2)/c^3/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/
2)/(c+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi(
(c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)
*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos
(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d
+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 1.45, antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4027, 3068, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*c^3 - 7*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sqrt[-(((
b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*
c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e
+ f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e +
 f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)
```

```
*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*
(b^2*c^3*(3*c + d) - 2*a*b*c^2*(3*c^2 + 2*c*d - d^2) + a^2*d*(9*c^3 - 2*c^2
*d - 6*c*d^2 + 3*d^3))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d
+ c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d +
c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin
[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3
*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/(
(a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a
 - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Ellip
ticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*
Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*c - a*d)*Sqrt[a + b*Sec[e + f*x]]*Sin[e
 + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
```

```
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
```

] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+\right.}{}$$

$$= -\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a^2\sqrt{d+c\cos(e+\right.}{c^3\sqrt{}}$$

$$= -\frac{2a\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}}{c^3\sqrt{c-}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(3bc^3-7ac^2d+bcd^2+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx}{(a+b)(d+c\cos(e+fx)}}}{3c}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1960 vs. $2(919) = 1838$.
time = 6.71, size = 1960, normalized size = 2.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((2*(-(b*c*
d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x]
)^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e +
f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f
*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(
5/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(3*a*b*c^3 + a
^2*c^2*d - 4*b^2*c^2*d + a*b*c*d^2 - a^2*d^3)*Sqrt[((c + d)*Cot[(e + f*x)/2
]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
```

```
 f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(
e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]]) + 4*(b*c - a*d)*(3*a^2*c^3 - 3*b^2*c^3 + 4*a*b*c^2*d + a^2*c*d^2 -
b^2*c*d^2 - 4*a*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((
c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)
*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Ellipti
cF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c +
 d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin
[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-3*a*b*c^3 + 7*a^2*c^2
*d - a*b*c*d^2 - 3*a^2*d^3)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/
2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e
 + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*
(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqr
t[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d +
 c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c
+ (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[
e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sq
rt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]]
, (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*S
qrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c +
 d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin
[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e
+ f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(3*c*(c - d)^2*(c + d
)^2*f*(b + a*Cos[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13059 vs. $2(840) = 1680$.
time = 2.52, size = 13060, normalized size = 14.21

method	result	size
default	Expression too large to display	13060

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(e + fx)})^{\frac{3}{2}}}{(c + d\sec{(e + fx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(5/2),x)
[Out] \text{Hanged}
```

3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=1122

$$\frac{2(a-b)\sqrt{a+b}\,(2abcd(35c^4-8c^2d^2+5d^4)-a^2d^2(58c^4-41c^2d^2+15d^4)-b^2(15c^6+19c^4d^2-2c^2d^4))\sqrt{-\cdots}}{15c^3(c\cdots}$$

```
[Out] 2/5*d^2*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d
+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)-2/15*d*(-13*a*c^2*d+5*a*d^3+10*b*c^
3-2*b*c*d^2)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-d^2)^2/f/(d+c*cos(f
*x+e))/(c+d*sec(f*x+e))^(1/2)+2/15*(a-b)*(2*a*b*c*d*(35*c^4-8*c^2*d^2+5*d^4
)-a^2*d^2*(58*c^4-41*c^2*d^2+15*d^4)-b^2*(15*c^6+19*c^4*d^2-2*c^2*d^4))*(d+
c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)
/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^
(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c
)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(
c-d)^3/(c+d)^(5/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)-2/15*(b^2*c^3*(15*c^3+10*c^2*d+9*c*d^2-2*d^3)-2*a*b*c^2*(15*c^4+20*c^3
*d-4*c^2*d^2-4*c*d^3+5*d^4)+a^2*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*
c*d^4-15*d^5))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a
*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))
^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f
*x+e))^(1/2)/c^4/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2)-2*a*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)
^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/
(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+
e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*co
s(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^4/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^
(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 2.16, antiderivative size = 1122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {4027, 3127, 3126, 3132, 2890, 3077, 2897, 3075}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a*b*c*d*(35*c^4 - 8*c^2*d^2 + 5*d^4) - a^2*d^2*(5
8*c^4 - 41*c^2*d^2 + 15*d^4) - b^2*(15*c^6 + 19*c^4*d^2 - 2*c^2*d^4))*Sqrt[
```

```
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b
*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[
a + b]*(b^2*c^3*(15*c^3 + 10*c^2*d + 9*c*d^2 - 2*d^3) - 2*a*b*c^2*(15*c^4 +
 20*c^3*d - 4*c^2*d^2 - 4*c*d^3 + 5*d^4) + a^2*d*(60*c^5 - 2*c^4*d - 66*c^3
*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*
x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]
))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[
d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[
e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e +
 f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1
- Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 +
Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*
Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*Sqrt[c + d]*f*Sqrt[b
+ a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])*S
qrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x
])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(10*b*c^3 - 13*a*c^2*d - 2*b*c*d^2 +
5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d
+ c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
```

```
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{7/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+\right.}{}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2d(10bc^3-13ac^2d}{15c^2\,(c^2-d^2)}$$

$$= \frac{2d^2(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} - \frac{2d(10bc^3-13ac^2d}{15c^2\,(c^2-d^2)}$$

$$= -\frac{2a\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}}{c^4\sqrt{c-}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(2abcd(35c^4-8c^2d^2+5d^4)-a^2d^2(58c^4-41c^2d^2+15d^4)-b}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2385 vs. $2(1122) = 2244$.
time = 7.52, size = 2385, normalized size = 2.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((-2*(-(b
*c*d^2*Sin[e + f*x]) + a*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos[e
 + f*x])^3) - (4*(5*b*c^3*d*Sin[e + f*x] - 8*a*c^2*d^2*Sin[e + f*x] - b*c*d
^3*Sin[e + f*x] + 4*a*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*Cos[e
 + f*x])^2) + (2*(15*b^2*c^6*Sin[e + f*x] - 70*a*b*c^5*d*Sin[e + f*x] + 58*
a^2*c^4*d^2*Sin[e + f*x] + 19*b^2*c^4*d^2*Sin[e + f*x] + 16*a*b*c^3*d^3*Sin
[e + f*x] - 41*a^2*c^2*d^4*Sin[e + f*x] - 2*b^2*c^2*d^4*Sin[e + f*x] - 10*a
*b*c*d^5*Sin[e + f*x] + 15*a^2*d^6*Sin[e + f*x]))/(15*c^2*(b*c - a*d)*(c^2
- d^2)^3*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x
])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])
```

```
^(3/2)*((4*(b*c - a*d)*(-15*a*b^2*c^6 + 5*a^2*b*c^5*d + 25*b^3*c^5*d + 13*a
^3*c^4*d^2 - 38*a*b^2*c^4*d^2 + 25*a^2*b*c^3*d^3 + 7*b^3*c^3*d^3 - 18*a^3*c
^2*d^4 - 11*a*b^2*c^2*d^4 + 2*a^2*b*c*d^5 + 5*a^3*d^6)*Sqrt[((c + d)*Cot[(e
 + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d
))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d +
c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-15*a^2*b*c^6 + 15*b^3*c^6 + 15*a^3*c^5*d
 - 55*a*b^2*c^5*d + 33*a^2*b*c^4*d^2 + 19*b^3*c^4*d^2 + 13*a^3*c^3*d^3 + 35
*a*b^2*c^3*d^3 - 70*a^2*b*c^2*d^4 - 2*b^3*c^2*d^4 + 4*a^3*c*d^5 - 12*a*b^2*
c*d^5 + 20*a^2*b*d^6)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c
 + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*
(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b
)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c +
d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[
Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2
]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[
b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(15*a*b^2*c^6 - 70*a^2*b
*c^5*d + 58*a^3*c^4*d^2 + 19*a*b^2*c^4*d^2 + 16*a^2*b*c^3*d^3 - 41*a^3*c^2*
d^4 - 2*a*b^2*c^2*d^4 - 10*a^2*b*c*d^5 + 15*a^3*d^6)*((Sqrt[(-a + b)/(a + b
)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt
[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (
2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/
(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a
 + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))])
 - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c
- d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*S
qrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e +
 f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)
/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
- ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b
+ a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Co
s[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c -
 a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f
*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(
a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]])
)))/(15*c^2*(c - d)^3*(c + d)^3*(-(b*c) + a*d)*f*(b + a*Cos[e + f*x])^(3/2)
*(c + d*Sec[e + f*x])^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 39417 vs. 2(1037) = 2074.
time = 3.48, size = 39418, normalized size = 35.13

method	result	size
default	Expression too large to display	39418

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^4*sec(f*x +
 e)^4 + 4*c*d^3*sec(f*x + e)^3 + 6*c^2*d^2*sec(f*x + e)^2 + 4*c^3*d*sec(f*x
 + e) + c^4), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(7/2),x)
```

```
[Out] \text{Hanged}
```

3.214 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=891

$$-\frac{2(a-b)\sqrt{a+b}\,(7ac^2-4bcd-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}\,(d \cdots}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\,\mathrm{co}\cdots}}$$

```
[Out] 2/3*(-a*d+b*c)^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/(d+c*cos(f
*x+e))/(c+d*sec(f*x+e))^(1/2)-2/3*(a-b)*(7*a*c^2-3*a*d^2-4*b*c*d)*(d+c*cos(
f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)
^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*
(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+c
os(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)^2
/(c+d)^(3/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/3*(b^2*c^2*(
c+3*d)-a*b*c*(7*c^2+4*c*d-3*d^2)+a^2*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c*co
s(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+
b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2
)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1
+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)
^2/(c+d)^(3/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(d+c*c
os(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(
a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e
))^(1/2)/c^3/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 1.34, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4027, 2871, 3132, 2890, 3077, 2897, 3075}

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{a+b\sec(e+fx)}\,a^2}{c^3\sqrt{c+d}\,f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}+\frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}-\frac{2(a-b)\sqrt{a+b}(7ac^2-4bdc-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{a+b\sec(e+fx)}}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}+\frac{2\sqrt{a+b}\left((9c^3-2dc^2-6d^2c+3d^3)a^2-bc(7c^2+4dc-3d^2)a+b^2c^2(c+3d)\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{a+b\sec(e+fx)}}{3c^3(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(7*a*c^2 - 4*b*c*d - 3*a*d^2)*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(3*c^2*(c - d)^2*(c + d)^(3/2)*f*Sqrt[b + a*Cos[e
```

```
+ f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^2*c^2*(c + 3*d) - a*b
*c*(7*c^2 + 4*c*d - 3*d^2) + a^2*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sqrt[
-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(3*c^3*(c - d)^2*(c + d)^(3/2)*f*S
qrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sqrt
[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-
(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*C
os[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[
(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]
])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^3*Sq
rt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c -
a*d)^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos
[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
```

```
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
```

```
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{5/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e+ \right.}{}$$

$$= \frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}} + \frac{\left(a^3\sqrt{d+c\cos(e+ \right.}{c^3\sqrt{}}$$

$$= -\frac{2a^2\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+fx}{(a-b)(d+c\cos(e+fx)}}}{c^3\sqrt{}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(7ac^2-4bcd-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2026 vs. 2(891) = 1782.
time = 6.74, size = 2026, normalized size = 2.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*Sin[e + f*x
] - 2*a*b*c*d*Sin[e + f*x] + a^2*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c
*Cos[e + f*x])^2) + (2*(7*a*b*c^3*Sin[e + f*x] - 7*a^2*c^2*d*Sin[e + f*x] -
 4*b^2*c^2*d*Sin[e + f*x] + a*b*c*d^2*Sin[e + f*x] + 3*a^2*d^3*Sin[e + f*x]
))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c
+ d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*(a + b*Sec[e + f*x])
^(5/2)*((4*(b*c - a*d)*(2*a^2*b*c^3 + b^3*c^3 + a^3*c^2*d - 8*a*b^2*c^2*d +
 2*a^2*b*c*d^2 + 3*b^3*c*d^2 - a^3*d^3)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(
c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e
```

```
 + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*
x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]
) + 4*(b*c - a*d)*(3*a^3*c^3 - 7*a*b^2*c^3 + 4*b^3*c^2*d + a^3*c*d^2 + 3*a*
b^2*c*d^2 - 4*a^2*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[
((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a -
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Ellip
ticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a
+ b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcS
in[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqr
t[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sq
rt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-7*a^2*b*c^3 + 7*a^3
*c^2*d + 4*a*b^2*c^2*d - a^2*b*c*d^2 - 3*a^3*d^3)*((Sqrt[(-a + b)/(a + b)]*
(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-
a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(
b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b
+ a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a +
b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) -
(2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d
)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt
[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*
x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)
/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]
^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (
(b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a
*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e
 + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*
d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)
/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c
) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))
/(3*c*(c - d)^2*(c + d)^2*f*(b + a*Cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15921 vs. $2(812) = 1624$.
time = 2.66, size = 15922, normalized size = 17.87

method	result	size
default	Expression too large to display	15922

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*sqrt(b*sec(f*x + e
) + a)*sqrt(d*sec(f*x + e) + c)/(d^3*sec(f*x + e)^3 + 3*c*d^2*sec(f*x + e)^
2 + 3*c^2*d*sec(f*x + e) + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(5/2),x)
[Out] \text{Hanged}
```

3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=1150

$$\frac{2(a-b)\sqrt{a+b}\,(b^2c^2d(29c^2+3d^2)-abc(35c^4+34c^2d^2-5d^4)+a^2(58c^4d-41c^2d^3+15d^5))\sqrt{-\frac{(bc-ad)\cdots}{(a+b)(\cdots}}\cdots}{15c^3(c-\cdots}$$

```
[Out] -2/5*d*(-a*d+b*c)*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2
-d^2)/f/(d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)+2/15*(-a*d+b*c)*(-13*a*c^
2*d+5*a*d^3+5*b*c^3+3*b*c*d^2)*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-d
^2)^2/f/(d+c*cos(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/15*(a-b)*(b^2*c^2*d*(29*c
^2+3*d^2)-a*b*c*(35*c^4+34*c^2*d^2-5*d^4)+a^2*(58*c^4*d-41*c^2*d^3+15*d^5))
*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(
1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a
+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c
^3/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))
^(1/2)+2/15*(b^3*c^4*(5*c^2+24*c*d+3*d^2)-a*b^2*c^3*(35*c^3+42*c^2*d+21*c*d
^2-2*d^3)+a^2*b*c^2*(45*c^4+48*c^3*d+c^2*d^2-8*c*d^3+10*d^4)-a^3*d*(60*c^5-
2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5))*(d+c*cos(f*x+e))^(3/2)*csc(
f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-co
s(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(
d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^4/(c-d)^3/(c+d)^(5/2)/(-a*d
+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(d+c*cos(f*x+e)
)^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2
)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(
a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/
c^4/f/(c+d)^(1/2)/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 2.36, antiderivative size = 1150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {4027, 3068, 3126, 3132, 2890, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(b^2*c^2*d*(29*c^2 + 3*d^2) - a*b*c*(35*c^4 + 34*c^2
*d^2 - 5*d^4) + a^2*(58*c^4*d - 41*c^2*d^3 + 15*d^5))*Sqrt[-(((b*c - a*d)*(
```

```
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
+ Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqr
t[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
qrt[a + b*Sec[e + f*x]])/(15*c^3*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(5
*c^2 + 24*c*d + 3*d^2) - a*b^2*c^3*(35*c^3 + 42*c^2*d + 21*c*d^2 - 2*d^3) +
 a^2*b*c^2*(45*c^4 + 48*c^3*d + c^2*d^2 - 8*c*d^3 + 10*d^4) - a^3*d*(60*c^5
 - 2*c^4*d - 66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a
*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d
)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])
^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])
/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d
))]*Sqrt[a + b*Sec[e + f*x]])/(15*c^4*(c - d)^3*(c + d)^(5/2)*(b*c - a*d)*f
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sq
rt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt
[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c
*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*
x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(c^4*
Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(b*
c - a*d)*(b + a*Cos[e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(5*c*(
c^2 - d^2)*f*(d + c*Cos[e + f*x])^2*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*(5*b*c^3 - 13*a*c^2*d + 3*b*c*d^2 + 5*a*d^3)*Sqrt[a + b*Sec[e + f*x]]*S
in[e + f*x])/(15*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e
+ f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
```

```
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
```

```
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{7/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d\,\cdots\right.}{\cdots}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-\cdots}{\cdots}$$

$$= -\frac{2d(bc-ad)(b+a\cos(e+fx))\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{5c\,(c^2-d^2)\,f(d+c\cos(e+fx))^2\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-\cdots}{\cdots}$$

$$= -\frac{2a^2\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad)(1+\cos(e+fx\cdots}{(a-b)(d+c\cos(e+fx\cdots}}\cdots}{c^4\sqrt{\cdots}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(b^2c^2d(29c^2+3d^2)-abc(35c^4+34c^2d^2-5d^4)+a^2(58c^4d-\cdots}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2344 vs. $2(1150) = 2300$.

time = 7.37, size = 2344, normalized size = 2.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((-2*(b^2*c
^2*d*Sin[e + f*x] - 2*a*b*c*d^2*Sin[e + f*x] + a^2*d^3*Sin[e + f*x]))/(5*c^
2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) + (2*(5*b^2*c^4*Sin[e + f*x] - 21*a*b
*c^3*d*Sin[e + f*x] + 16*a^2*c^2*d^2*Sin[e + f*x] + 3*b^2*c^2*d^2*Sin[e + f
*x] + 5*a*b*c*d^3*Sin[e + f*x] - 8*a^2*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^
2)^2*(d + c*Cos[e + f*x])^2) + (2*(35*a*b*c^5*Sin[e + f*x] - 58*a^2*c^4*d*S
in[e + f*x] - 29*b^2*c^4*d*Sin[e + f*x] + 34*a*b*c^3*d^2*Sin[e + f*x] + 41*
a^2*c^2*d^3*Sin[e + f*x] - 3*b^2*c^2*d^3*Sin[e + f*x] - 5*a*b*c*d^4*Sin[e +
 f*x] - 15*a^2*d^5*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^3*(d + c*Cos[e + f*x]
))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(7/2)) + ((d + c*Cos[e
```

```
+ f*x])^(7/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(10*a
^2*b*c^5 + 5*b^3*c^5 + 13*a^3*c^4*d - 48*a*b^2*c^4*d + 15*a^2*b*c^3*d^2 + 2
7*b^3*c^3*d^2 - 18*a^3*c^2*d^3 - 16*a*b^2*c^2*d^3 + 7*a^2*b*c*d^4 + 5*a^3*d
^5)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])
*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a -
 b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c
 - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a
*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(15*a^3*c^5 - 35*a
*b^2*c^5 + 23*a^2*b*c^4*d + 29*b^3*c^4*d + 13*a^3*c^3*d^2 - 5*a*b^2*c^3*d^2
 - 75*a^2*b*c^2*d^3 + 3*b^3*c^2*d^3 + 4*a^3*c*d^4 + 8*a*b^2*c*d^4 + 20*a^2*
b*d^5)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Co
s[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[(
(-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2
*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[
b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x
)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*
(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a
*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f
*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*(-35*a^2*b*c^5 + 58*a^3*c^4*d + 29*a*b^
2*c^4*d - 34*a^2*b*c^3*d^2 - 41*a^3*c^2*d^3 + 3*a*b^2*c^2*d^3 + 5*a^2*b*c*d
^4 + 15*a^3*d^5)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d +
 c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])
/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d))])/
(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos
[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d
)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f
*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b
)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c -
 a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*C
os[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e
+ f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2
)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
 - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a
- b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*
c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[
e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin
[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]]))))/(15*c^2*(c - d)^3*(c + d)^3*f*(b
 + a*Cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32282 vs. $2(1065) = 2130$.
time = 3.40, size = 32283, normalized size = 28.07

method	result	size
default	Expression too large to display	32283

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(7/2),x)
```

```
[Out] \text{Hanged}
```

3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=1428

$$2(a-b)\sqrt{a+b}\,(2b^3c^3d(133c^4+62c^2d^2-3d^4)+2a^2bcd(406c^6+73c^4d^2+132c^2d^4-35d^6)-ab^2c^2(245c^6+$$

```
[Out] 2/7*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c/(c^2-d^2)/f/
(d+c*cos(f*x+e))^3/(c+d*sec(f*x+e))^(1/2)-2/35*d*(-19*a*c^2*d+7*a*d^3+14*b*
c^3-2*b*c*d^2)*(b+a*cos(f*x+e))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^2/(c^2-
d^2)^2/f/(d+c*cos(f*x+e))^2/(c+d*sec(f*x+e))^(1/2)-2/105*(2*a*b*c*d*(91*c^4
-2*c^2*d^2+7*d^4)-a^2*d^2*(162*c^4-101*c^2*d^2+35*d^4)-b^2*(35*c^6+67*c^4*d
^2-6*c^2*d^4))*sin(f*x+e)*(a+b*sec(f*x+e))^(1/2)/c^3/(c^2-d^2)^3/f/(d+c*cos
(f*x+e))/(c+d*sec(f*x+e))^(1/2)+2/105*(a-b)*(2*b^3*c^3*d*(133*c^4+62*c^2*d^
2-3*d^4)+2*a^2*b*c*d*(406*c^6+73*c^4*d^2+132*c^2*d^4-35*d^6)-a*b^2*c^2*(245
*c^6+852*c^4*d^2+41*c^2*d^4+14*d^6)-a^3*(582*c^6*d^2-485*c^4*d^4+392*c^2*d^
6-105*d^8))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*co
s(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d)
)^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1
/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+
e))^(1/2)/c^4/(c-d)^4/(c+d)^(7/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+
d*sec(f*x+e))^(1/2)+2/105*(b^3*c^4*(35*c^4+231*c^3*d+67*c^2*d^2+57*c*d^3-6*
d^4)-a*b^2*c^3*(245*c^5+413*c^4*d+439*c^3*d^2+53*c^2*d^3-12*c*d^4+14*d^5)+a
^2*b*c^2*(315*c^6+497*c^5*d+219*c^4*d^2-73*c^3*d^3+208*c^2*d^4+56*c*d^5-70*
d^6)-a^3*d*(525*c^7+57*c^6*d-699*c^5*d^2+214*c^4*d^3+672*c^3*d^4-280*c^2*d^
5-210*c*d^6+105*d^7))*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/
2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f
*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(a+
b*sec(f*x+e))^(1/2)/c^5/(c-d)^4/(c+d)^(7/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(
1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*Ellipti
cPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(
a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b)^(1/2)*(-(-a*d+b*c)*(1
-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b
)/(d+c*cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/c^5/f/(c+d)^(1/2)/(b+a*cos
(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 3.71, antiderivative size = 1428, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {4027, 3127, 3126, 3132, 2890, 3077, 2897, 3075}

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*b^3*c^3*d*(133*c^4 + 62*c^2*d^2 - 3*d^4) + 2*a^2*
b*c*d*(406*c^6 + 73*c^4*d^2 + 132*c^2*d^4 - 35*d^6) - a*b^2*c^2*(245*c^6 +
852*c^4*d^2 + 41*c^2*d^4 + 14*d^6) - a^3*(582*c^6*d^2 - 485*c^4*d^4 + 392*c
^2*d^6 - 105*d^8))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*
Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[ArcSin[(Sq
rt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^4*(
c - d)^4*(c + d)^(7/2)*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*
Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 +
 57*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*
d^3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 7
3*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d -
699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^7
))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]
*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(
d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b
 + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c -
d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(105*c^5*(c - d)^4*(c + d)
^(7/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (
2*a^2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos
[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c
)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*S
qrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*
Sec[e + f*x]])/(c^5*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e
 + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e +
f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3*Sqrt[c + d*Sec[e + f*x]]) -
 (2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])*Sq
rt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(35*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e +
 f*x])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*(2*a*b*c*d*(91*c^4 - 2*c^2*d^2 + 7*
d^4) - a^2*d^2*(162*c^4 - 101*c^2*d^2 + 35*d^4) - b^2*(35*c^6 + 67*c^4*d^2
- 6*c^2*d^4))*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(105*c^3*(c^2 - d^2)^3
*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
```

<u>Rule 2890</u>

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
```

```
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
```

```
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\cos^2(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{9/2}}\,dx}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{7c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} + \frac{\left(2\sqrt{d+c\cos(e\right.}{}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{7c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d(14bc^3-19ac^2}{35}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{7c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d(14bc^3-19ac^2}{35}$$

$$= \frac{2d^2(b+a\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}\,\sin(e+fx)}{7c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3\sqrt{c+d\sec(e+fx)}} - \frac{2d(14bc^3-19ac^2}{35}$$

$$= -\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx}{(a-b)(d+c\cos(e+fx)}}}{c^5\sqrt{c}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(2b^3c^3d(133c^4+62c^2d^2-3d^4)+2a^2bcd(406c^6+73c^4d^2+1}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2979 vs. 2(1428) = 2856.

time = 8.36, size = 2979, normalized size = 2.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*
c^2*d^2*Sin[e + f*x] - 2*a*b*c*d^3*Sin[e + f*x] + a^2*d^4*Sin[e + f*x]))/(7
*c^3*(c^2 - d^2)*(d + c*Cos[e + f*x])^4) + (2*(-14*b^2*c^4*d*Sin[e + f*x] +
 43*a*b*c^3*d^2*Sin[e + f*x] - 29*a^2*c^2*d^3*Sin[e + f*x] + 2*b^2*c^2*d^3*
Sin[e + f*x] - 19*a*b*c*d^4*Sin[e + f*x] + 17*a^2*d^5*Sin[e + f*x]))/(35*c^
3*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^3) + (2*(35*b^2*c^6*Sin[e + f*x] - 224
*a*b*c^5*d*Sin[e + f*x] + 234*a^2*c^4*d^2*Sin[e + f*x] + 67*b^2*c^4*d^2*Sin
```

```
[e + f*x] + 52*a*b*c^3*d^3*Sin[e + f*x] - 209*a^2*c^2*d^4*Sin[e + f*x] - 6*
b^2*c^2*d^4*Sin[e + f*x] - 20*a*b*c*d^5*Sin[e + f*x] + 71*a^2*d^6*Sin[e + f
*x]))/(105*c^3*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^2) + (2*(245*a*b^2*c^8*Si
n[e + f*x] - 812*a^2*b*c^7*d*Sin[e + f*x] - 266*b^3*c^7*d*Sin[e + f*x] + 58
2*a^3*c^6*d^2*Sin[e + f*x] + 852*a*b^2*c^6*d^2*Sin[e + f*x] - 146*a^2*b*c^5
*d^3*Sin[e + f*x] - 124*b^3*c^5*d^3*Sin[e + f*x] - 485*a^3*c^4*d^4*Sin[e +
f*x] + 41*a*b^2*c^4*d^4*Sin[e + f*x] - 264*a^2*b*c^3*d^5*Sin[e + f*x] + 6*b
^3*c^3*d^5*Sin[e + f*x] + 392*a^3*c^2*d^6*Sin[e + f*x] + 14*a*b^2*c^2*d^6*S
in[e + f*x] + 70*a^2*b*c*d^7*Sin[e + f*x] - 105*a^3*d^8*Sin[e + f*x]))/(105
*c^3*(b*c - a*d)*(c^2 - d^2)^4*(d + c*Cos[e + f*x])))))/(f*(b + a*Cos[e + f*
x])^2*(c + d*Sec[e + f*x])^(9/2)) + ((d + c*Cos[e + f*x])^(9/2)*Sec[e + f*x
]^2*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(-70*a^2*b^2*c^8 - 35*b^4*c^
8 - 77*a^3*b*c^7*d + 427*a*b^3*c^7*d + 162*a^4*c^6*d^2 - 522*a^2*b^2*c^6*d^
2 - 298*b^4*c^6*d^2 + 348*a^3*b*c^5*d^3 + 666*a*b^3*c^5*d^3 - 263*a^4*c^4*d
^4 - 586*a^2*b^2*c^4*d^4 - 51*b^4*c^4*d^4 + 127*a^3*b*c^3*d^5 + 59*a*b^3*c^
3*d^5 + 136*a^4*c^2*d^6 + 26*a^2*b^2*c^2*d^6 - 14*a^3*b*c*d^7 - 35*a^4*d^8)
*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*
x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)
*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c -
a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Co
s[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-105*a^3*b*c^8 + 245
*a*b^3*c^8 + 105*a^4*c^7*d - 567*a^2*b^2*c^7*d - 266*b^4*c^7*d + 190*a^3*b*
c^6*d^2 + 586*a*b^3*c^6*d^2 + 162*a^4*c^5*d^3 + 706*a^2*b^2*c^5*d^3 - 124*b
^4*c^5*d^3 - 1261*a^3*b*c^4*d^4 - 83*a*b^3*c^4*d^4 + 145*a^4*c^3*d^5 - 223*
a^2*b^2*c^3*d^5 + 6*b^4*c^3*d^5 + 548*a^3*b*c^2*d^6 + 20*a*b^3*c^2*d^6 - 28
*a^4*c*d^7 + 84*a^2*b^2*c*d^7 - 140*a^3*b*d^8)*((Sqrt[((c + d)*Cot[(e + f*x
)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(
e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Si
n[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[
e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
 a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos
[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c -
a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*
x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2
*(245*a^2*b^2*c^8 - 812*a^3*b*c^7*d - 266*a*b^3*c^7*d + 582*a^4*c^6*d^2 + 8
52*a^2*b^2*c^6*d^2 - 146*a^3*b*c^5*d^3 - 124*a*b^3*c^5*d^3 - 485*a^4*c^4*d^
4 + 41*a^2*b^2*c^4*d^4 - 264*a^3*b*c^3*d^5 + 6*a*b^3*c^3*d^5 + 392*a^4*c^2*
d^6 + 14*a^2*b^2*c^2*d^6 + 70*a^3*b*c*d^7 - 105*a^4*d^8)*((Sqrt[(-a + b)/(a
 + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(
Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]
], (2*(b*c - a*d))/((-a + b)*(c + d))]])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]
```

```
^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x]
)/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]
))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)
/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d
)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc
[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x
)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x
]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)
*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b
*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e
 + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75467 vs. $2(1337) = 2674$.
time = 4.48, size = 75468, normalized size = 52.85

method	result	size
default	Expression too large to display	75468

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*sqrt(b*sec(f*x + e
) + a)*sqrt(d*sec(f*x + e) + c)/(d^5*sec(f*x + e)^5 + 5*c*d^4*sec(f*x + e)^
4 + 10*c^2*d^3*sec(f*x + e)^3 + 10*c^3*d^2*sec(f*x + e)^2 + 5*c^4*d*sec(f*x
 + e) + c^5), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(9/2),x)
```

```
[Out] \text{Hanged}
```

3.217 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=652

$$-\frac{2c(c+d)\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\mathrm{ArcSin}\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(}{(c-d)(a+b\sec(}}}{a(a+b)f\sqrt{c+d\sec(e}}$$

[Out] -2*c*(c+d)*cot(f*x+e)*EllipticPi(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e))^(3/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*((a+b)*(-a*d+b*c)*(-1+sec(f*x+e))*(c+d*sec(f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(1/2)/a/(a+b)/f/(c+d*sec(f*x+e))^(1/2)+2*d*(c+d)*cot(f*x+e)*EllipticPi(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e))^(3/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(-(a+b)*(a*d-b*c)*(-1+sec(f*x+e))*(c+d*sec(f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(1/2)/b/(a+b)/f/(c+d*sec(f*x+e))^(1/2)+2*(-a*d+b*c)*cot(f*x+e)*EllipticF(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*((-a*d+b*c)*(-1+sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/b/f/((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)

Rubi [F]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

[In] Int[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]

[Out] Defer[Int][(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]], x]

Rubi steps

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.69, size = 49385, normalized size = 75.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 2.80, size = 491, normalized size = 0.75

method	result
default	$\frac{2\left(2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)d^2+2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)c^2-\mathrm{EllipticF}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b
),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*d^2+2*EllipticPi((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+
b))^(1/2))*c^2-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*c^2+2*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))
^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c*d-EllipticF((-1+cos(f*
x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d^2)*
cos(f*x+e)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*((a*cos(f*x+e)+b)/
(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2
)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(a*c
os(f*x+e)+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec{(e + fx)})^{\frac{3}{2}}}{\sqrt{a + b\sec{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**(3/2)/sqrt(a + b*sec(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2), x)
```

3.218 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e}{(a+b)(c+d\sec(e}}}{a\sqrt{c+d}\,f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+
d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*s
ec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(
1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/f/(c+d)^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {4021}

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\,\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\Pi\left(\frac{(a+b)c}{a(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{af\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x])
)/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/(
(a - b)*(c + d*Sec[e + f*x]))))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{(a+b)c}{a(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)}{(a-b)}\right.}{}$$

Mathematica [A]
time = 3.39, size = 325, normalized size = 1.64

$$\frac{4\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(e+fx)\right)}{a-b}}\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\csc(e+fx)\left(a(c+d)F\left(\text{ArcSin}\left(\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{2bc-2ad}}\right)|\frac{2(-bc+ad)}{(a-b)(c+d)}\right)-(a+b)c\Pi\left(\frac{-bc+ad}{a(c+d)};\text{ArcSin}\left(\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{2bc-2ad}}\right)|\frac{2(-bc+ad)}{(a-b)(c+d)}\right)\right)\sqrt{c+d\sec(e+fx)}\sin^2\left(\frac{1}{2}(e+fx)\right)}{a(c+d)f\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Sqrt[((a + b)*Cot[(e + f*x)/2]^2)/(a - b)]*Sqrt[((c + d)*(b + a*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*(a*(c + d)*EllipticF[Ar
cSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a*d)
]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] - (a + b)*c*EllipticPi[(-(b*c) +
a*d)/(a*(c + d)), ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2
]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))])*Sqrt[c + d*S
ec[e + f*x]]*Sin[(e + f*x)/2]^2)/(a*(c + d)*f*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A]
time = 2.74, size = 352, normalized size = 1.78

method	result
default	$\frac{2\left(2\,\text{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)c-\text{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)c+\text{EllipticF}\right.}{f(-1+\cos(f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-
b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c-EllipticF((-1+cos(f*x+e))*((a
-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c+EllipticF((-
1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2
))*d)*cos(f*x+e)*sin(f*x+e)^2*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)
*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e)
```

```
)^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e)
)/(a*cos(f*x+e)+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sec{\left(e + f x \right)}}}{\sqrt{a + b \sec{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2),x)
[Out] int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2), x)
```

3.219 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=398

$$-\frac{2\sqrt{c+d}\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(}{(c+d)(a+b\sec(}}}{a\sqrt{a+b}\,cf}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+
b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*s
ec(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/a/c/f/(a+b)^
(1/2)-2*b*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/
2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))
*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/a/(-a*d+b*c)/f/(c+d)^(
1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4023, 4021, 4069}

$$-\frac{2b\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{af\sqrt{c+d}\,(bc-ad)}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{a(c+d)}{(a+b)c};\text{ArcSin}\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{acf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x
]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/
((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(a*Sqrt[a + b]*c*f) -
 (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f
*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4021

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
```

```
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4023

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqr
t[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0]
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \frac{b\int \frac{\sec(e+fx}{\sqrt{a+b\sec(e+fx)}\,\sqrt{c}}}{a}$$

$$= -\frac{2\sqrt{c+d}\,\cot(e+fx)\Pi\left(\frac{a(c+d)}{(a+b)c};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+a}}{\sqrt{c+d}\,\sqrt{a+b}}\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.49, size = 249, normalized size = 0.63

$$\frac{4i\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\,\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left(F\left(i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\,\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)-2\Pi\left(-\frac{a+b}{a-b};i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\,\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\right)\sec(e+fx)}{\sqrt{\frac{-a+b}{a+b}}\,f\sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] ((4*I)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*(EllipticF[I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a -
b)*(c + d))] - 2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))])*Sec[e + f*x]
)/(Sqrt[(-a + b)/(a + b)]*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x
]])
```

Maple [A]
time = 3.10, size = 292, normalized size = 0.73

method	result
default	$-\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)-2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\cos(fx+e)}{f(-1+\cos(fx+e))(d+c\cos(fx+e))(a\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/f*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f
*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))*cos(f*x+e)*((a
*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)
/(c+d))^(1/2)*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*((a*cos(f*x+
e)+b)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)/(
(a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\,\sqrt{c+d\sec{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos{(e+f\,x)}}}\,\sqrt{c+\frac{d}{\cos{(e+f\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=622

$$-\frac{2(a-b)\sqrt{a+b}\,d^2\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(}{(a+b)(c+d\sec(}}}{c(c-d)\sqrt{c+d}\,(bc-ad)^2f}$$

```
[Out] -2*(a-b)*d^2*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^
(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+
e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c/(c-d)/(-a*d+b*c)^
2/f/(c+d)^(1/2)-2*(2*c-d)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e
))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*(c+d*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*
x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/c^2/
(c-d)/(-a*d+b*c)/f/(c+d)^(1/2)-2*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec
(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sec(f*x
+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*s
ec(f*x+e)))^(1/2)/a/c^2/f/(c+d)^(1/2)
```

Rubi [A]

time = 0.89, antiderivative size = 763, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4027, 3133, 2890, 3077, 2897, 3075}

$$-\frac{2d\sqrt{a+b}\,(2c-d)\csc(e+fx)\sqrt{a+b\sec(e+fx)}\,(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\,\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{c^2f(c-d)\sqrt{c+d}\,(bc-ad)\sqrt{a\cos(e+fx)+b}\,\sqrt{c+d\sec(e+fx)}}-\frac{2\sqrt{a+b}\,\csc(e+fx)\sqrt{a+b\sec(e+fx)}\,(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\Pi\left(\frac{(a+b)c}{a(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\,\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{ac^2f\sqrt{c+d}\,\sqrt{a\cos(e+fx)+b}\,\sqrt{c+d\sec(e+fx)}}-\frac{2d^2(a-b)\sqrt{a+b}\,\csc(e+fx)\sqrt{a+b\sec(e+fx)}\,(c\cos(e+fx)+d)^{3/2}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{b+a\cos(e+fx)}}{\sqrt{a+b}\,\sqrt{d+c\cos(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{cf(c-d)\sqrt{c+d}\,(bc-ad)^2\sqrt{a\cos(e+fx)+b}\,\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*d^2*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b
)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*
(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticE[A
rcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e
+ f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(
c*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*S
ec[e + f*x]]) - (2*Sqrt[a + b]*(2*c - d)*d*Sqrt[-(((b*c - a*d)*(1 - Cos[e +
 f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f
*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f
*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sq
rt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[a + b*S
```

```
ec[e + f*x]])/(c^2*(c - d)*Sqrt[c + d]*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x
]]*Sqrt[c + d*Sec[e + f*x]]) - (2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[
e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e
+ f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e
+ f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*C
os[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sqrt[a + b*Sec[e + f*x]])/(a*c^2*Sqrt[c + d]*f*Sqrt[b + a*
Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3133

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Dist[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int[(b +
a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{}{\sqrt{b+a\cos}}}{\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e}}$$

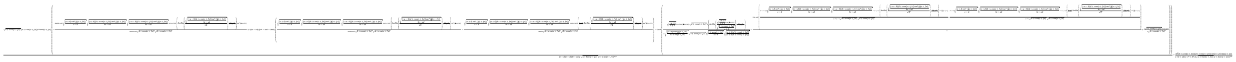
$$= \frac{\left(\sqrt{d+c\cos(e+fx)}\,\sqrt{a+b\sec(e+fx)}\right)\int \frac{\sqrt{d+c\cos}}{\sqrt{b+a\cos}}}{c^2\sqrt{b+a\cos(e+fx)}\,\sqrt{c+d\sec(e+fx)}}$$

$$= -\frac{2\sqrt{a+b}\,\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-\frac{(bc-ad}{(a-b)(}}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,d^2\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\,\sqrt{-}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1761 vs. 2(622) = 1244.

time = 9.69, size = 1761, normalized size = 2.83



Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]^2*((-4*b*
c*d*(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b
 + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*C
os[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin
[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d
)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(b*c^2
 - a*c*d - 2*b*d^2)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c +
 d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d
 + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[
ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)
*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f
*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sq
rt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]]
, (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b
+ a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) - 2*a*d^2*((Sqrt[(-a + b)/(a +
 b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sq
rt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]],
 (2*(b*c - a*d))/((-a + b)*(c + d))])/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2
)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/
(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))
]) - (2*(b*c - a*d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(
c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e
 + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/
2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*
x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]
) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(
b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c
 - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*
x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
 f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))
/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]
```

```
]))))/((c - d)*(c + d)*(b*c - a*d)*f*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e
+ f*x])^(3/2)) + (2*d^2*(b + a*Cos[e + f*x])*(d + c*Cos[e + f*x])*Sec[e + f
*x]*Tan[e + f*x])/((-(b*c) + a*d)*(-c^2 + d^2)*f*Sqrt[a + b*Sec[e + f*x]]*(
c + d*Sec[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3450 vs. 2(577) = 1154.
time = 2.90, size = 3451, normalized size = 5.55

method	result	size
default	Expression too large to display	3451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/f*(-((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos
(f*x+e)+1)/(c+d))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f
*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d^3*sin(f*x+e)-2*((a*cos(f*x+e)+b)
/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-
d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*d^3*sin(f*x+e)-2*((a*cos(f*x+e)+b)/(
cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)
/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c^3*sin(f*x+e)+((a*cos(f*x+e)+b)/(cos(
f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d)
)^(1/2))*b*c^3*sin(f*x+e)-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/
sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*cos(f*x+e)
*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))
/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c^3+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^
(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a
*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)
/(c+d))^(1/2)*b*c^3+((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos
(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b)
)^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*c*d^2*sin(f*x+e)-Elli
pticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*c
os(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c*d^2+2*((a*cos(f*x+e)+b)/(cos(f*x
+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))
^(1/2)/((a-b)/(a+b))^(1/2))*a*c^2*d*sin(f*x+e)+2*((a*cos(f*x+e)+b)/(cos(f*x
+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))
^(1/2)/((a-b)/(a+b))^(1/2))*b*c*d^2*sin(f*x+e)-((a*cos(f*x+e)+b)/(cos(f*x+e
```

```
)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*EllipticF((
-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/
2))*a*c^2*d*sin(f*x+e)-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*
x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x
+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c*d^2+E
llipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/
(c+d))^(1/2))*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+
c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c^2*d+EllipticE((-1+cos(f*x+e))
*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)
*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))
/(cos(f*x+e)+1)/(c+d))^(1/2)*a*c*d^2-((a-b)/(a+b))^(1/2)*b*d^3+((a*cos(f*x+
e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(
1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/
(a-b)/(c+d))^(1/2))*a*d^3*sin(f*x+e)+((a-b)/(a+b))^(1/2)*b*c*d^2-cos(f*x+e)
^2*((a-b)/(a+b))^(1/2)*a*c*d^2+cos(f*x+e)*((a-b)/(a+b))^(1/2)*a*c*d^2-cos(f
*x+e)*((a-b)/(a+b))^(1/2)*b*c*d^2+EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(
1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a*
cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/
(c+d))^(1/2)*a*d^3-EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e)
,((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)+b)/(
cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*
d^3-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b
),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a*cos(f*
x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))
^(1/2)*a*d^3-EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b
)*(c-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)+b)/(cos(f*
x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*b*c*d^2+
2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((
c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)
+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/
2)*a*c^2*d+2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+
b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a
*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)
/(c+d))^(1/2)*b*c*d^2-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x
+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)+b
)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)
*a*c^2*d-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)*(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(b*d^2*sec(f*x +
 e)^3 + a*c^2 + (2*b*c*d + a*d^2)*sec(f*x + e)^2 + (b*c^2 + 2*a*c*d)*sec(f*
x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec(e + fx)}\,(c + d\sec(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{\cos(e + f\,x)}}\,\left(c + \frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)), x)
```

3.221 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt[3]{d+c\cos(e+fx)}\,\sqrt[3]{a+b\sec(e+fx)}\,\text{Int}\left(\frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}},x\right)}{\sqrt[3]{b+a\cos(e+fx)}\,\sqrt[3]{c+d\sec(e+fx)}}$$

```
[Out] (d+c*cos(f*x+e))^(1/3)*(a+b*sec(f*x+e))^(1/3)*Unintegrable((b+a*cos(f*x+e))
^(1/3)/(d+c*cos(f*x+e))^(1/3),x)/(b+a*cos(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1
/3)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(1/3)*(a + b*Sec[e + f*x])^(1/3)*Defer[Int][(b + a*Co
s[e + f*x])^(1/3)/(d + c*Cos[e + f*x])^(1/3), x])/((b + a*Cos[e + f*x])^(1/
3)*(c + d*Sec[e + f*x])^(1/3))
```

Rubi steps

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \frac{\left(\sqrt[3]{d+c\cos(e+fx)}\,\sqrt[3]{a+b\sec(e+fx)}\right)\int \frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}\,dx}{\sqrt[3]{b+a\cos(e+fx)}\,\sqrt[3]{c+d\sec(e+fx)}}$$

Mathematica [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3), x)
```

3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Mathematica [A]
time = 48.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + b \sec{\left(e + f x \right)}}}{\left(c + d \sec{\left(e + f x \right)}\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(4/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(4/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3),x)
[Out] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3), x)
```

3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Mathematica [A]
time = 78.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(7/3),x)
[Out] \text{Hanged}
```

3.224 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$

Optimal. Leaf size=89

$$\frac{(d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}},x\right)}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

```
[Out] (d+c*cos(f*x+e))^(2/3)*(a+b*sec(f*x+e))^(2/3)*Unintegrable((b+a*cos(f*x+e))
^(2/3)/(d+c*cos(f*x+e))^(2/3),x)/(b+a*cos(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2
/3)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(2/3)*(a + b*Sec[e + f*x])^(2/3)*Defer[Int][(b + a*Co
s[e + f*x])^(2/3)/(d + c*Cos[e + f*x])^(2/3), x])/((b + a*Cos[e + f*x])^(2/
3)*(c + d*Sec[e + f*x])^(2/3))
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \frac{\left((d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\right)\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}\,dx}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

Mathematica [A]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(fx + e))^{\frac{2}{3}}}{(c + d\sec(fx + e))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(e + fx))^{\frac{2}{3}}}{(c + d\sec(e + fx))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(2/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="giac"
)
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3),x)
[Out] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3), x)
```

3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Mathematica [A]
time = 49.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(e + fx))^{\frac{2}{3}}}{(c + d\sec(e + fx))^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(5/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(5/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3),x)
[Out] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3), x)
```

3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Mathematica [A]
time = 84.59, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(8/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(8/3),x)
[Out] \text{Hanged}
```

3.227 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=89

$$\frac{(d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\mathrm{Int}\left(\frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}},x\right)}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

```
[Out] (d+c*cos(f*x+e))^(4/3)*(a+b*sec(f*x+e))^(4/3)*Unintegrable((b+a*cos(f*x+e))
^(4/3)/(d+c*cos(f*x+e))^(4/3),x)/(b+a*cos(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4
/3)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] ((d + c*Cos[e + f*x])^(4/3)*(a + b*Sec[e + f*x])^(4/3)*Defer[Int][(b + a*Co
s[e + f*x])^(4/3)/(d + c*Cos[e + f*x])^(4/3), x])/((b + a*Cos[e + f*x])^(4/
3)*(c + d*Sec[e + f*x])^(4/3))
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \frac{\left((d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\right)\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}\,dx}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

Mathematica [A]
time = 56.51, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{\frac{4}{3}}}{(c+d\sec(e+fx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(4/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(4/3)/(c + d*sec(e + f*x))**(4/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{4/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3),x)
```

```
[Out] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3), x)
```

3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal. Leaf size=32

$$\operatorname{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Mathematica [A]
time = 91.26, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Mupad [F(-1)]

time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(7/3),x)
[Out] \text{Hanged}
```

3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}},x\right)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Mathematica [A]
time = 130.04, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(10/3),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="giac
")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Mupad [F(-1)]

time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(10/3),x)
```

```
[Out] \text{Hanged}
```

3.230 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m dx$

Optimal. Leaf size=106

$$-\frac{F_1\left(np;\frac{1}{2},\frac{1}{2}-m;1+np;\sec(e+fx),-\sec(e+fx)\right)(c(d\sec(e+fx))^p)^n(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(\ldots}{fnp\sqrt{1-\sec(e+fx)}}$$

```
[Out] -AppellF1(n*p,1/2-m,1/2,n*p+1,-sec(f*x+e),sec(f*x+e))*(c*(d*sec(f*x+e))^p)^
n*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/p/(1-sec(f*x+e)
)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4033, 3913, 3912, 138}

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(np;\frac{1}{2},\frac{1}{2}-m;np+1;\sec(e+fx),-\sec(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d*
Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan
[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx$$

$$= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n(1+\sec(e+fx)\right.$$

$$= -\frac{\left(d(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n(1+\sec(e+\right.}{}$$

$$= -\frac{F_1\left(np;\frac{1}{2},\frac{1}{2}-m;1+np;\sec(e+fx),-\sec(e+fx)\right)(c}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2425 vs. $2(106) = 212$.
time = 14.54, size = 2425, normalized size = 22.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(m + n*p)*(c*(d*Sec[e + f*x])^p)^n*(a*(1 + Sec[e + f*x]))^m*Tan[(e +
 f*x)/2])/(f*(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p
, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2)*((3*2^m*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(m + n*p))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan
```

```
[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p,
1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
 + (3*2^(1 + m)*(-1 + n*p)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m + n*p
, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*Ap
pellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2
)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]*(
-1/3*((1 - n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n*p)*Appell
F1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n*p, 1 - n*p
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/
2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n
*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1
+ n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]*(2*((-1
 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/
3*((1 - n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n*p)*AppellF1[
3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n*p)*((
-3*(2 - n*p)*AppellF1[5/2, m + n*p, 3 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n*p)*Appel
lF1[5/2, 1 + m + n*p, 2 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n*p)*((-3*(1 - n*p)*Appell
F1[5/2, 1 + m + n*p, 2 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m + n*p)*AppellF1[5/2, 2
+ m + n*p, 1 - n*p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n
*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*Appe
llF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n*p)*AppellF1[1/2, m + n*p, 1
 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^
(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n*p)*Tan[(e + f*x)/2
]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*S
ec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 -
```

```
n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/
2, 1 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (c(d\sec (fx+e))^{p})^{n} (a+a\sec (fx+e))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (e+fx)+1))^{m} (c(d\sec (e+fx))^{p})^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(c*(d*sec(e + f*x))**p)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^p\right)^n\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m, x)
```

3.231 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=275

$$\frac{a^3(7+4np)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}}-\frac{a^3(1+4np)\cos(e+fx}{}$$

```
[Out] a^3*(4*n*p+7)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*se
c(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2)-a^3*(4*n*p+1)*
cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d
*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(2*n*p
+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)/(n*p+2)+(c*(d*sec(f*x+e))^p
)^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)
```

Rubi [A]
time = 0.30, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4033, 3899, 4082, 3872, 3857, 2722}

$$-\frac{a^3(4np+1)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4np+7)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp(np+2)\sqrt{\sin^2(e+fx)}}+\frac{a^3(2np+5)\tan(e+fx)(c(d\sec(e+fx))^p)^n}{f(np+1)(np+2)}+\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)(c(d\sec(e+fx))^p)^n}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(7 + 4*n*p)*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*
x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 + n*p)*Sqrt[Sin[e +
f*x]^2]) - (a^3*(1 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2
, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1
 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a^3*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)
^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)) + ((c*(d*Sec[e + f*x])^p)^n*(a^3 +
 a^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int (d\sec(e+fx)$$

$$= \frac{(c(d\sec(e+fx))^p)^n\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{f(2+np)} + $$

$$= \frac{a^3(5+2np)\,(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{(c(d\sec(e}{}$$

$$= \frac{a^3(5+2np)\,(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{(c(d\sec(e}{}$$

$$= \frac{a^3(5+2np)\,(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{(c(d\sec(e}{}$$

$$= \frac{a^3(7+4np)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.08, size = 343, normalized size = 1.25

$$-i2^{-3+np}a^3\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{np}\left(\frac{12e^{2i(e+fx)}\,{}_2F_1\left(1,-\frac{np}{2};2+\frac{np}{2};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)f(2+np)}+\frac{8e^{3i(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-1-np);\frac{1}{2}(5+np);-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2 f(3+np)}+\frac{6e^{i(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(1-np);\frac{1}{2}(3+np);-e^{2i(e+fx)}\right)}{f+fnp}+\frac{\left(1+e^{2i(e+fx)}\right){}_2F_1\left(1,1-\frac{np}{2};1+\frac{np}{2};-e^{2i(e+fx)}\right)}{fnp}\right)\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^{-3-np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n(1+\sec(e+fx))^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-I)*2^(-3 + n*p)*a^3*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(n*p)*((1
2*E^((2*I)*(e + f*x))*Hypergeometric2F1[1, -1/2*(n*p), 2 + (n*p)/2, -E^((2*
I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))*f*(2 + n*p)) + (8*E^((3*I)*(e +
f*x))*Hypergeometric2F1[1, (-1 - n*p)/2, (5 + n*p)/2, -E^((2*I)*(e + f*x))]
)/((1 + E^((2*I)*(e + f*x)))^2*f*(3 + n*p)) + (6*E^(I*(e + f*x))*Hypergeome
tric2F1[1, (1 - n*p)/2, (3 + n*p)/2, -E^((2*I)*(e + f*x))])/(f + f*n*p) + (
(1 + E^((2*I)*(e + f*x)))*Hypergeometric2F1[1, 1 - (n*p)/2, 1 + (n*p)/2, -E
^((2*I)*(e + f*x))])/(f*n*p))*Sec[(e + f*x)/2]^6*Sec[e + f*x]^(-3 - n*p)*(c
*(d*Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^3
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (c(d\sec(fx+e))^p)^n\,(a+a\sec(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \left(c\left(d\sec \left(e+fx\right)\right)^p\right)^n\, dx+\int 3\left(c\left(d\sec \left(e+fx\right)\right)^p\right)^n\sec \left(e+fx\right) dx+\int 3\left(c\left(d\sec \left(e+fx\right)\right)^p\right)^n\sec^2\left(e+fx\right) dx+\int \left(c\left(d\sec \left(e+fx\right)\right)^p\right)^n\sec^3\left(e+fx\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**3,x)
[Out] a**3*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(3*(c*(d*sec(e + f*
x))**p)**n*sec(e + f*x), x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e +
 f*x)**2, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^p\right)^n\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3, x)
```

3.232 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx$

Optimal. Leaf size=205

$$\frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a^2(1+2np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\right.}{}$$

```
[Out] 2*a^2*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e)
)^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a^2*(2*n*p+1)*cos(f*x+e)*hyper
geom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^
n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^2*(c*(d*sec(f*x+e))^p)^n
*tan(f*x+e)/f/(n*p+1)
```

Rubi [A]

time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4033, 3873, 3857, 2722, 4131}

$$-\frac{a^2(2np+1)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}}+\frac{a^2\tan(e+fx)(c(d\sec(e+fx))^p)^n}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^2*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 +
 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e
 + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin
[e + f*x]^2]) + (a^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))^2\,dx = \left((d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx)$$

$$= \left((d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx)$$

$$= \frac{a^2(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)} + \frac{\left(2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}(\cdots}{}$$

$$= \frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$

$$= \frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.82, size = 299, normalized size = 1.46

$$-\frac{i2^{-2+np}a^2e^{-i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{1+np}\left(4e^{2i(e+fx)}np(1+np)\,{}_2F_1\left(1,-\frac{np}{2};2+\frac{np}{2};-e^{2i(e+fx)}\right)+\left(1+e^{2i(e+fx)}\right)(2+np)\left(4e^{i(e+fx)}np\,{}_2F_1\left(1,\frac{1}{2}(1-np);\frac{1}{2}(3+np);-e^{2i(e+fx)}\right)+\left(1+e^{2i(e+fx)}\right)(1+np)\,{}_2F_1\left(1,1-\frac{np}{2};1+\frac{np}{2};-e^{2i(e+fx)}\right)\right)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^{-2-np}(e+fx)\,(c(d\sec(e+fx))^p)^n\,(1+\sec(e+fx))^2}{fnp(1+np)(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((-I)*2^(-2 + n*p)*a^2*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1 + n*p
)*(4*E^((2*I)*(e + f*x))*n*p*(1 + n*p)*Hypergeometric2F1[1, -1/2*(n*p), 2 +
 (n*p)/2, -E^((2*I)*(e + f*x))] + (1 + E^((2*I)*(e + f*x)))*(2 + n*p)*(4*E^
(I*(e + f*x))*n*p*Hypergeometric2F1[1, (1 - n*p)/2, (3 + n*p)/2, -E^((2*I)*
(e + f*x))] + (1 + E^((2*I)*(e + f*x)))*(1 + n*p)*Hypergeometric2F1[1, 1 -
(n*p)/2, 1 + (n*p)/2, -E^((2*I)*(e + f*x))]))*Sec[(e + f*x)/2]^4*Sec[e + f*
x]^(-2 - n*p)*(c*(d*Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^2)/(E^(I*(e + f*x
))*f*n*p*(1 + n*p)*(2 + n*p))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec \left(f x +e \right)\right)^{p}\right)^{n} \left(a +a \sec \left(f x +e \right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*((d*sec(f*x + e))^
p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \left(c \left(d \sec{\left(e + f x \right)}\right)^{p}\right)^{n}\, dx + \int 2 \left(c \left(d \sec{\left(e + f x \right)}\right)^{p}\right)^{n} \sec{\left(e + f x \right)}\, dx + \int \left(c \left(d \sec{\left(e + f x \right)}\right)^{p}\right)^{n} \sec^{2}{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(2*(c*(d*sec(e + f*
x))**p)**n*sec(e + f*x), x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f
*x)**2, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^p\right)^n\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2, x)
```

3.233 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx$

Optimal. Leaf size=156

$$\frac{a\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3\right.}{f(1}$$

```
[Out] a*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)
^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/2*
n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(
-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4033, 3872, 3857, 2722}

$$\frac{a\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Se
c[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*
x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*S
ec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))\,dx = \left((d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx))$$

$$= \left(a(d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx$$

$$= \left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(c(d\sec(e+fx))^p)^n\right)\int\left(\frac{\cos(e+fx}{d}\right.$$

$$= \frac{a\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.22, size = 124, normalized size = 0.79

$$\frac{a\csc(e+fx)\left((1+np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};1+\frac{np}{2};\sec^2(e+fx)\right)+np\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sec^2(e+fx)\right)\right)(c(d\sec(e+fx))^p)^n\sqrt{-\tan^2(e+fx)}}{fnp(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Csc[e + f*x]*((1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 +
 (n*p)/2, Sec[e + f*x]^2] + n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*
p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n
*p*(1 + n*p))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (c(d\sec(fx+e))^p)^n\,(a+a\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(c(d\sec\left(e+fx\right))^{p}\right)^{n}\,dx+\int\left(c(d\sec\left(e+fx\right))^{p}\right)^{n}\sec\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e)),x)
[Out] a*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral((c*(d*sec(e + f*x))**
p)**n*sec(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(c\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{p}\right)^{n}\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)), x)
```

3.234 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=208

$$\frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))}-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)}{af\sqrt{\sin^2(e+fx)}}$$

```
[Out] (c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*hypergeom([
1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(
f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)+(-n*p+1)*cos(f*x+e)^2*hypergeom([1/2, -1/2*
n*p+1],[-1/2*n*p+2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/a/f/(-n
*p+2)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4033, 3905, 3872, 3857, 2722}

$$-\frac{\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{af\sqrt{\sin^2(e+fx)}}+\frac{(1-np)\sin(e+fx)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2-np);\frac{1}{2}(4-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{af(2-np)\sqrt{\sin^2(e+fx)}}+\frac{\sin(e+fx)(c(d\sec(e+fx))^p)^n}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e +
 f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 - n*p)*
Cos[e + f*x]^2*Hypergeometric2F1[1/2, (2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x
]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*(2 - n*p)*Sqrt[Sin[e + f*x
]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{a+a\sec(e+fx)}\,dx$$

$$= \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{(d(1-np)(d\sec(e+fx))^{-np}\,(c(d\sec(e+$$

$$= \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{((1-np)(d\sec(e+fx))^{-np}\,(c(d\sec(e+f}{a}$$

$$= \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(c(d\sec(e+fx))^p)\right.}{a}$$

$$= \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\mathrm{c}\right.}{af\sqrt{\sin}}$$

Mathematica [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]), x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(d \sec \left(f x +e \right)\right)^{p}\right)^{n}}{a +a \sec \left(f x +e \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(c \left(d \sec{\left(e + f x \right)}\right)^{p}\right)^{n}}{\sec{\left(e + f x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e)),x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)), x)
```

3.235 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=248

$$\frac{2(2-np)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(3-2np)\cos(e+fx)\,{}_2F_1\left(\right.}{}$$

```
[Out] 2/3*(-n*p+2)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec
(f*x+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(-2*n*p+3)*cos(f*x+
e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x
+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-2/3*(-n*p+2)*(c*(d*sec(f*x+
e))^p)^n*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*(c*(d*sec(f*x+e))^p)^n*tan(f*x
+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.31, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4033, 3902, 4105, 3872, 3857, 2722}

$$\frac{2(2-np)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(3-2np)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{2(2-np)\tan(e+fx)(c(d\sec(e+fx))^p)^n}{3a^2f(\sec(e+fx)+1)}-\frac{\tan(e+fx)(c(d\sec(e+fx))^p)^n}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(2 - n*p)*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2
]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (
(3 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, C
os[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e +
 f*x]^2]) - (2*(2 - n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*a^2*f*(1
 + Sec[e + f*x])) - ((c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(3*f*(a + a*Sec
[e + f*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{(a+a\sec(e+fx))^2}\,dx$$

$$= -\frac{(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)}{3}$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(c(d\sec(e+fx))^p)^n\tan(e+fx}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(c(d\sec(e+fx))^p)^n\tan(e+fx}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{2(2-np)\left(c(d\sec(e+fx))^p\right)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(c(d\sec(e+fx))^p)^n\tan(e+fx}{3f(a+a\sec(e+fx))^2}$$

$$= \frac{2(2-np)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [F]
time = 9.83, size = 0, normalized size = 0.00

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(c(d\sec(fx+e))^p)^n}{(a+a\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e)
+ a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c(d\sec{(e+fx)})^p)^n}{\sec^2{(e+fx)}+2\sec{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
 x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,{\left(\frac{d}{\cos(e+f\,x)}\right)}^p\right)^n}{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2, x)
```

3.236 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$

Optimal. Leaf size=56

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \operatorname{Int}((d\sec(e+fx))^{np}(a+b\sec(e+fx))^m, x)$$

```
[Out] (c*(d*sec(f*x+e))^p)^n*Unintegrable((d*sec(f*x+e))^(n*p)*(a+b*sec(f*x+e))^m
,x)/((d*sec(f*x+e))^(n*p))
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$$

Verification is not applicable to the result.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] ((c*(d*Sec[e + f*x])^p)^n*Defer[Int][(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e +
f*x])^m, x])/(d*Sec[e + f*x])^(n*p)
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \left((d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n\right) \int (d\sec(e+fx))$$

Mathematica [A]
time = 2.46, size = 0, normalized size = 0.00

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\sec (e+fx))^p)^n\,(a+b\sec (e+fx))^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c \left(\frac{d}{\cos \left(e + f\,x\right)}\right)^p\right)^n \left(a + \frac{b}{\cos \left(e + f\,x\right)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m, x)
```

3.237 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx$

Optimal. Leaf size=296

$$\frac{b(b^2(1+np)+3a^2(2+np))\ {}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}}-\frac{a(3b^2n$$

```
[Out] b*(b^2*(n*p+1)+3*a^2*(n*p+2))*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*
x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2
)-a*(3*b^2*n*p+a^2*(n*p+1))*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*
n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(si
n(f*x+e)^2)^(1/2)+a*b^2*(2*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+
1)/(n*p+2)+b^2*(c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)
```

Rubi [A]

time = 0.35, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4033, 3927, 4132, 3857, 2722, 4131}

$$-\frac{a(a^2(np+1)+3b^2np)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}+\frac{b(3a^2(np+2)+b^2(np+1))\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp(np+2)\sqrt{\sin^2(e+fx)}}+\frac{ab^2(2np+5)\tan(e+fx)(c(d\sec(e+fx))^p)^n}{f(np+1)(np+2)}+\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(c(d\sec(e+fx))^p)^n}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] (b*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hypergeometric2F1[1/2, -1/2*(n*p), (2
- n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2
+ n*p)*Sqrt[Sin[e + f*x]^2]) - (a*(3*b^2*n*p + a^2*(1 + n*p))*Cos[e + f*x]*
Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[
e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a*b^
2*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)
) + (b^2*(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2
+ n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))^3\,dx = \left((d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx)$$

$$= \frac{b^2(c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{(}{}$$

$$= \frac{b^2(c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)} + \frac{(}{}$$

$$= \frac{ab^2(5+2np)\,(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)} + \frac{b^2(c(d\,\mathrm{se}}{}$$

$$= \frac{b(b^2(1+np)+3a^2(2+np))\ {}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e}{fnp(2+np)\sqrt{\sin^2(e+}}$$

$$= \frac{b(b^2(1+np)+3a^2(2+np))\ {}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e}{fnp(2+np)\sqrt{\sin^2(e+}}$$

Mathematica [A]
time = 1.09, size = 278, normalized size = 0.94

$$-\frac{\csc^3(e+fx)\left(a^3(6+11np+6n^2p^2+n^3p^3)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};1+\frac{np}{2};\sec^2(e+fx)\right)+bnp\left(3ab(3+4np+n^2p^2)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sec^2(e+fx)\right)+(2+np)\left(3a^2(3+np)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sec^2(e+fx)\right)+b^2(1+np)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3+np);\frac{1}{2}(5+np);\sec^2(e+fx)\right)\right)\right)\right)(c(d\sec(e+fx))^p)^n\left(-\tan^2(e+fx)\right)^{3/2}}{fnp(1+np)(2+np)(3+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n*p + 6*n^2*p^2 + n^3*p^3)*Cos[e + f*x]^3*Hy
pergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(3*a*b*(
3 + 4*n*p + n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (
n*p)/2, Sec[e + f*x]^2] + (2 + n*p)*(3*a^2*(3 + n*p)*Cos[e + f*x]^2*Hyperge
ometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2] + b^2*(1 + n*p)*H
ypergeometric2F1[1/2, (3 + n*p)/2, (5 + n*p)/2, Sec[e + f*x]^2])))*(c*(d*Se
c[e + f*x])^p)^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*p*(1 + n*p)*(2 + n*p)*(3 + n
*p)))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sec\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**3,x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3, x)
```

3.238 $\int \left(c(d\sec(e+fx))^p\right)^n (a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=211

$$\frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}-\frac{\left(b^2np+a^2(1+np)\right)\cos(e+fx)\,{}_2}{}$$

```
[Out] 2*a*b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e)
)^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-(b^2*n*p+a^2*(n*p+1))*cos(f*x+
e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x
+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(c*(d*sec(f*x+
e))^p)^n*tan(f*x+e)/f/(n*p+1)
```

Rubi [A]
time = 0.17, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4033, 3873, 3857, 2722, 4131}

$$-\frac{\left(a^2(np+1)+b^2np\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{fnp\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)\left(c(d\sec(e+fx))^p\right)^n}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(
d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - ((b^2*n*p
 + a^2*(1 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p
)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2
)*Sqrt[Sin[e + f*x]^2]) + (b^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1
 + n*p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \left((d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n\right) \int (d\sec(e+fx)$$

$$= \left((d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n\right) \int (d\sec(e+fx)$$

$$= \frac{b^2(c(d\sec(e+fx))^p)^n \tan(e+fx)}{f(1+np)} + \frac{\left(2ab\left(\frac{\cos(e+fx)}{d}\right)^{np} (c$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right) (c(d\sec(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right) (c(d\sec(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.51, size = 200, normalized size = 0.95

$$\frac{\csc(e+fx)\left(a^2(2+3np+n^2p^2)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};1+\frac{np}{2};\sec^2(e+fx)\right)+bnp\left(b(1+np)\,{}_2F_1\left(\frac{1}{2},1+\frac{np}{2};2+\frac{np}{2};\sec^2(e+fx)\right)+2a(2+np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sec^2(e+fx)\right)\right)\right)\sec(e+fx)\left(c(d\sec(e+fx))^p\right)^n\sqrt{-\tan^2(e+fx)}}{fnp(1+np)(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(a^2*(2 + 3*n*p + n^2*p^2)*Cos[e + f*x]^2*Hypergeometric2F1[1
/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(b*(1 + n*p)*Hypergeometr
ic2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2] + 2*a*(2 + n*p)*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]))*S
ec[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n*p*(1 + n*p
)*(2 + n*p))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(c(d\sec\left(fx+e\right))^{p}\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*((d*sec(f*x + e))^
p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c(d\sec\left(e+fx\right))^{p}\right)^{n}\left(a+b\sec\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**2,x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^p\right)^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2,x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2, x)
```

3.239 $\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=156

$$\frac{b\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3\right.}{f(1}$$

```
[Out] b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)
^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/2*
n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(
-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4033, 3872, 3857, 2722}

$$\frac{b\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{fnp\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(e+fx)\right)\left(c(d\sec(e+fx))^p\right)^n}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Se
c[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*
x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*S
ec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (c(d\sec(e+fx))^p)^n\,(a+b\sec(e+fx))\,dx &= \left((d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx))\\ &= \left(a(d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\right)\int (d\sec(e+fx)\\ &= \left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(c(d\sec(e+fx))^p)^n\right)\int\left(\frac{\cos(e+fx}{d}\right.\\ &= \frac{b\,{}_2F_1\left(\frac{1}{2},-\frac{np}{2};\frac{1}{2}(2-np);\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 125, normalized size = 0.80

$$\frac{\csc(e+fx)\left(a(1+np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};1+\frac{np}{2};\sec^2(e+fx)\right)+bnp\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sec^2(e+fx)\right)\right)(c(d\sec(e+fx))^p)^n\sqrt{-\tan^2(e+fx)}}{fnp(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (Csc[e + f*x]*(a*(1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 +
 (n*p)/2, Sec[e + f*x]^2] + b*n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 +
n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f
*n*p*(1 + n*p))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (c(d\sec(fx+e))^p)^n\,(a+b\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c(d\sec{(e+fx)})^{p}\right)^{n}\left(a+b\sec{(e+fx)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e)),x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^p\right)}^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)), x)
```

3.240 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=206

$$-\frac{bF_1\left(\frac{1}{2};\frac{np}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{(a^2-b^2)\,f}+\frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1\right.}{}$$

```
[Out] -b*AppellF1(1/2,1/2*n*p,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(cos
(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/f+a*Appell
F1(1/2,1/2*n*p-1/2,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*cos(f*x+e
)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/
f
```

Rubi [A]
time = 0.28, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4033, 3954, 2902, 3268, 440}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{np}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] -((b*AppellF1[1/2, (n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]
)/((a^2 - b^2)*f)) + (a*AppellF1[1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2,
 (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p
)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx &= \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{a+b\sec(e+fx)}\,dx \\
&= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{1-np}(e+fx)}{b+a\cos(e+fx)}\,dx \\
&= -\left(\left(a\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right)+\left(b\cos^{n}\right. \\
&= \frac{\left(b\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{(1-x^2)^{-\frac{np}{2}}}{-a^2+b^2+a^2x^2}\,dx,x,\sin(e+fx)\right)}{f} \\
&= -\frac{bF_1\left(\frac{1}{2};\frac{np}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n}{(a^2-b^2)f}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5411 vs. $2(206) = 412$.
time = 25.23, size = 5411, normalized size = 26.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sec\left(fx+e\right)\right)^{p}\right)^{n}}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sec\left(e+fx\right)\right)^{p}\right)^{n}}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e)),x)
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)),x)
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)), x)
```

3.241 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=322

$$-\frac{2abF_1\left(\frac{1}{2};\frac{1}{2}(-2+np),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{(a^2-b^2)^2 f}+\frac{a^2}{}$$

```
[Out] -2*a*b*AppellF1(1/2,1/2*n*p-1,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2)
)*(cos(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)^2/f+
a^2*AppellF1(1/2,1/2*n*p-3/2,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/
(a^2-b^2)^2/f+b^2*AppellF1(1/2,1/2*n*p-1/2,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e
)^2/(a^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)
^n*sin(f*x+e)/(a^2-b^2)^2/f
```

Rubi [A]

time = 0.38, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4033, 3954, 2903, 3268, 440}

$$-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-2),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)^2}+\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-3),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)^2}+\frac{b^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\left(c(d\sec(e+fx))^p\right)^n F_1\left(\frac{1}{2};\frac{1}{2}(np-1),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*b*AppellF1[1/2, (-2 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*
x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[
e + f*x])/((a^2 - b^2)^2*f) + (a^2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, Sin[
e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)
^((-1 + n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f) +
(b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^
2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p)/2)*(c*(d*Sec[e +
f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)^2*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
```

```
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4033

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n])), Int[(a + b*Sec[e + f*x
])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \left((d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{(d\sec(e+fx))^{np}}{(a+b\sec(e+fx))^2}\,dx$$

$$= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{2-np}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \left(\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \left(\frac{b^2\cos^{2-np}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} - \frac{2ab\cos^{3-n}}{(b^2-a^2\cos}\right.$$

$$= \left(a^2\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\right)\int \frac{\cos^{4-np}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx - (2ab\,c$$

$$= -\frac{\left(2ab\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1}{2}(2-np)}}{\left(-a^2+b^2+a^2x^2\right)^2}\,dx, x, \sin(e\right.}{f}$$

$$= -\frac{2abF_1\left(\frac{1}{2}; \frac{1}{2}(-2+np), 2; \frac{3}{2}; \sin^2(e+fx), \frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(\right.}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14108 vs. $2(322) = 644$.
time = 44.06, size = 14108, normalized size = 43.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d \sec \left(f x+e\right)\right)^{p}\right)^{n}}{\left(a+b \sec \left(f x+e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
+ a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d \sec \left(e+f x\right)\right)^{p}\right)^{n}}{\left(a+b \sec \left(e+f x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,{\left(\frac{d}{\cos(e+f\,x)}\right)}^p\right)^n}{{\left(a+\frac{b}{\cos(e+f\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2,x)
```

```
[Out] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 1290
4.2 Listing of Grading functions 1290

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
       if expr.args[1] == Rational(1,2):
          return True
       else:
          return False
    else:
       return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
       if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
       else:
         return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```